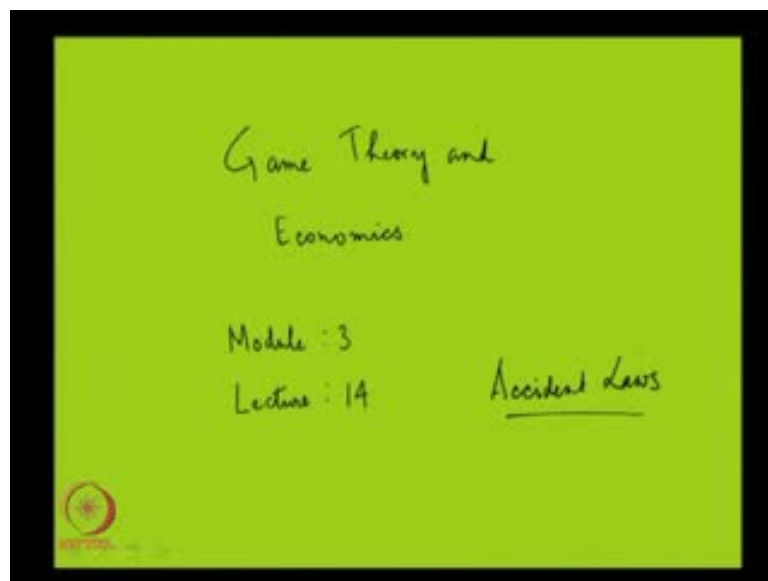


Game Theory and Economics
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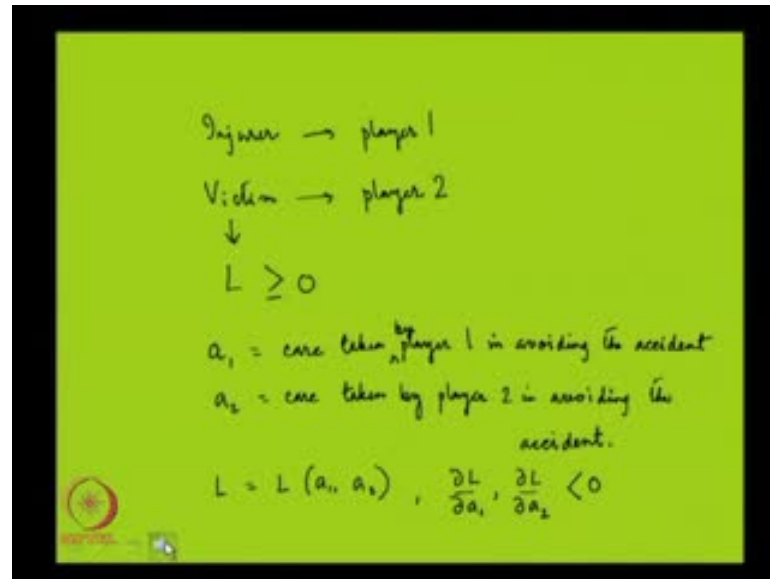
Module No. # 03
Illustrations of Nash Equilibrium
Lecture No. # 14
Accident Laws

Welcome to lecture 14 of module 3 of this course, called game theory and economics. Before we start this lecture, let me just take you through what we have been discussing so far. We have been discussing the topic of what is known as accident laws.

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Accident laws are something we have been talking about. Typically, we are going to look at a situation where there are two parties involved, one party will be called an injurer, this is our player 1, and the other is called a victim, this is player 2.

Now, typically, in a case of accident, what we shall assume is that when the accident is happening, both these parties are involved. The loss that is occurring due to the accident is falling on this person - the victim - and this law we shall call it as L capital; L capital, L is either 0 or positive.

Now, this is the loss, but behind the loss, it is possible that both these parties have been responsible. It may happen that the injurer has been careless in avoiding the accident that is why the accident happened or it might be the case that the victim has been careless in avoiding the accident and that is why the accident has happened.

This L can be thought of as a function of two variables, this will be called a 1 and a 2. Now, what kind of function is L of these two variables a 1 and a 2. It is reasonable to assume that if a 1 or a 2 go up that is the players are taking more care in avoiding the accident, then the loss that is suffered by the victim should reasonably go down. It may also be interpreted - this L might also be interpreted, in the sense that it is the expected value of the loss.

It can be imagined in the sense that if more care is taken, then there is less probability for the accident to happen. If there is less possibility that the accident happens, in that case, the expected loss due to extend that is L also goes down.

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L, a_1, a_2 are measured in the same unit, money (say).
 p = fraction of the loss to be compensated by the injurer.
 $p = p(a_1, a_2)$
 If a_1 rises $\rightarrow p$ declines.
 If a_2 rises $\rightarrow p$ rises.
 Payoff to pl. 1 : $-a_1 - p(a_1, a_2) \cdot L(a_1, a_2)$
 Payoff to player 2 : $-a_2 - (1-p(a_1, a_2)) \cdot L(a_1, a_2)$

So, L can be thought of as a function of a_1 and a_2 , it is a declining function in both the variables. I can write it like this, it is a continuous function I am going to assume, which means that these two things exist and they are negative. If more care is taken, there is less loss incurred by the victim, so this is one part and this is the loss incurred by the victim. We are going to assume that L, a_1, a_2 are measured in the same unit, let us call this may be money.

So, which means I can add and subtract L with a_1 and a_2 . Now, if there is a loss, then what is the rule of law in this case? The rule of the law is the following that law can say that if there is a loss, then the entire loss should not be borne by the victim. The law might say that a part of the loss, fraction of the loss, has to be borne by the injurer who has caused the accident. That means there is a fraction of the loss which has to be compensated by the injurer.

Let us call that fraction as ρ . Now, ρ can also be thought of as a function of a_1 and a_2 , so I can say the ρ is a function of a_1 and a_2 . Now, what kind of function this will be? If a_1 rises, this means that the injurer is taking more care, in that case, it is reasonable to assume that ρ should go down, because he is taking more care, so he is

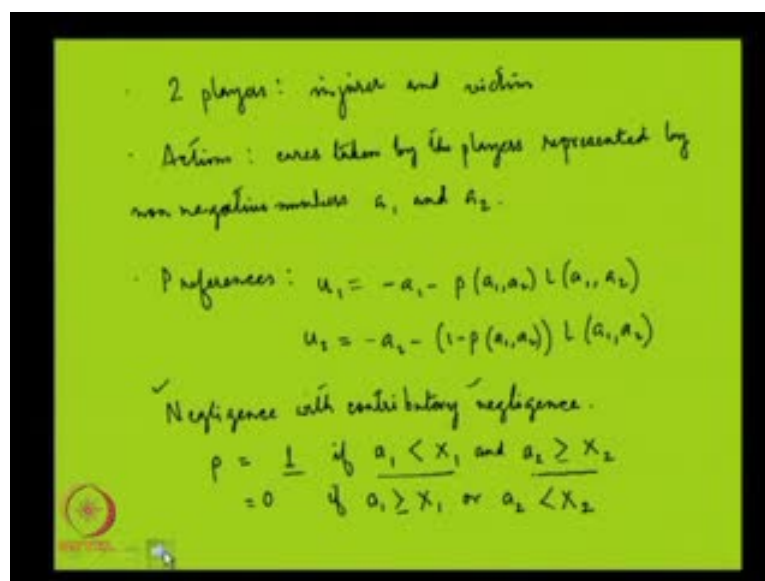
less responsible for the accident. Therefore, he should bear less fraction of the loss that has to be compensated, so it may be reasonable assume that if a_1 rises; a_2 rises what happens? (Refer Slide Time: 07:32) If a_2 rises that means victim is taking more care, if a victim is taking more care, it can be safely assume that he is less responsible for the accident. So, in that case, ρ should go up, because ρ is the fraction that is compensated by the injurer.

So, I can say that it appeals to logic that has a a_2 rise; that is, if the victim is taking more care, he should be compensated more, because he is not responsible that is why ρ should rise. So, this is more or less the story, now in this story, what is the payoff of the injurer when the accident has happened? When it has been decided, what is the value of ρ ?

Payoff to player 1 that is injurer is how much? Remember, this he has to take care of a 1; in fact he is taking the care a_1 . So, minus a_1 is the initial payoff, because we are going to assume that when one takes care, it is expensive, it is costly for him to take that care. So, minus of a_1 is the payoff, because the person is taking care, minus he is going to pay the compensation, so this is the loss to him due to the fact that he has to pay ρ fraction of the loss to the victim.

What is the payoff to player 2? He takes a care a_2 , so minus a_2 , minus a part of the compensation, a part of the loss will not be covered up by the compensation that part of the loss he has to bare. This part of the loss, which is not covered by the injurer, is $1 - \rho$ multiplied by L . So, this is the loss due to accident and this is the care that he is taking, so both of them are being added up.

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This is the setting in terms of language of game theory, it has become very clear; what are the parameters of the setting. First, the players, so a 1, a 2 are the cares, these are the actions that are taken by player 1, player 2. Remember, these actions can be a 0 or positive, which means that they cannot be negative. So, if someone is taking care close to 0, very small amount of care, which basically means he has been negligent.

So, negative does not arise. If I have to show that person has been negligent, just tell that the a that his taking, is very low close to 0. So, the preferences are represented by payoff functions. This is the payoff function of player 1 that is the injurer, now there can be different kinds of laws that are enshrined in the legal system of the country. What we are going to look at is the particular kind of law, which was in practice in United States for a long time and this law is called negligence with contributed negligence.

So, what is the idea? The idea is that this law fixes certain critical level of care, a threshold level of care that has to be undertaken by each of these two players. If it is found that the injurer has not been careful enough that is his level of care is below this critical level of care specified by the law. Simultaneously, it is the fact that the victim has been careful that the victim has taken sufficient care, which has been specified by the law; only in that case, the injurer will pay compensation to the victim.

So, there are two conditions that have to be satisfied; first, injurer has been negligent and the victim has not been negligent. If these two conditions are met, only in that case, the

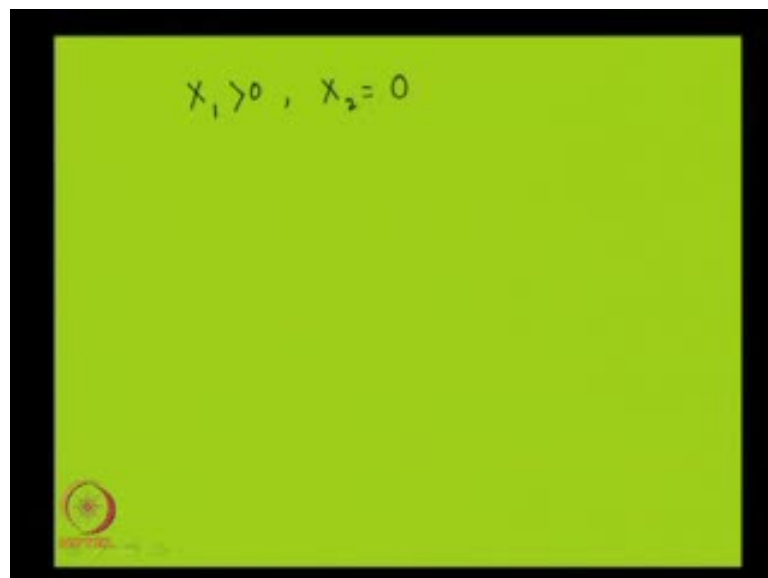
injurer will pay compensation to the victim. The compensation is such that it is the full compensation, which means that the entire loss is then borne by the injurer.

So, in that case, ρ is equal to 1. If any of these two conditions is not satisfied, which means that the injurer has been careful or the victim has been careless. If any of them is satisfied, in that case, the injurer does not pay anything, the entire loss is borne by the victim. This is what this meant by negligence with contributed negligence. The first negligence is the negligence that is cost by the injurer and the second negligence is contributed negligence, here we are looking at the negligence by the victim.

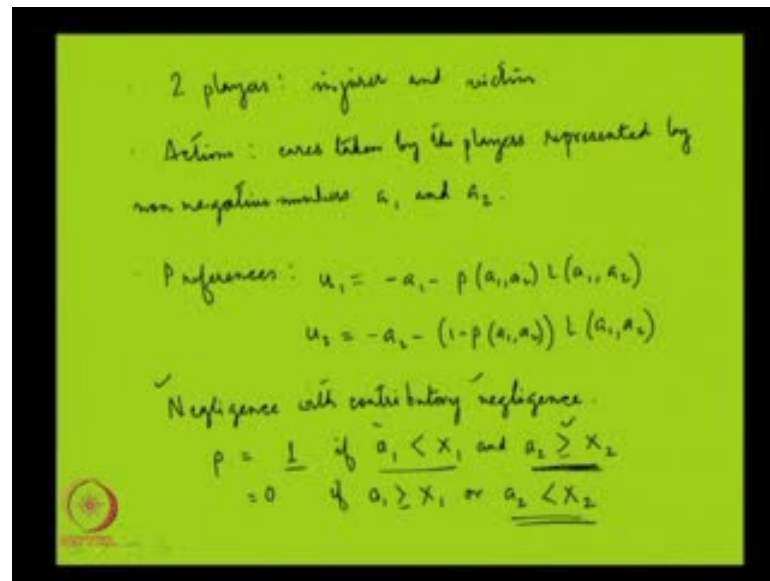
So, in terms of mathematics symbols, what it means is that ρ is equal to 1. Here, X_1 and X_2 are the threshold level of cares that is specified by the law, they are given from outside, these are the parameters. If this is not satisfied, not this condition, which means this.

Again, just to repeat what you have been saying is that if the injurer has been careless that is a 1 is less than some value and the victim has been careful that is a 2 is equal or more than a particular value, in that case, ρ is equal to 1, the injurer bares the entire a loss. If any of them is not satisfied, if a 1 is greater than equal to X_1 that is injurer is careful or the victim has been careless, then ρ is equal to 0.

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$$X_1 > 0, X_2 = 0$$

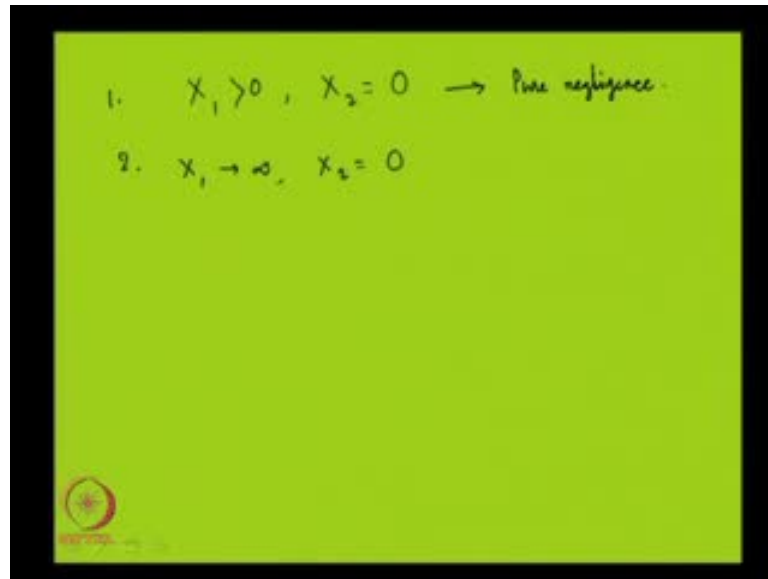
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Now, this may seem little harsh on the victim, because he is getting compensated only if both these conditions are satisfied, but it is not so, I can play around with X_1 and X_2 and get different kinds of laws. For example, suppose X_1 is greater than 0 and X_2 is equal to 0, in this case, if X_2 is equal to 0, then what is happening is that this is never satisfied; this condition is never satisfied, this is always satisfied. Because, if you remember a_1 and a_2 are positive or 0, they can be either positive or 0.

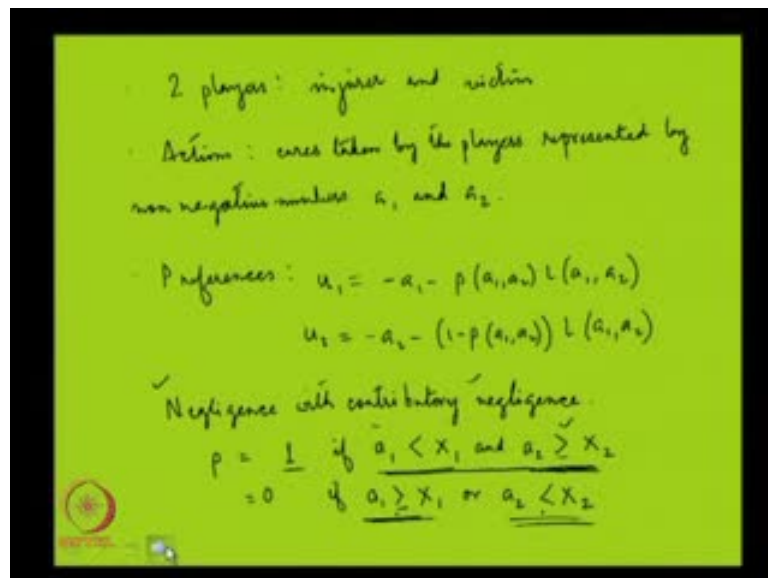
So, this is always satisfied if X_2 is equal to 0, which means that whether the injurer will pay something or not, it depends purely on his action. If he is less than X_1 , if a_1 is less than X_1 , then he pays; if X_1 is less than a_1 , then he does not pay.

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So, this is the case where the entire owner is right now on the injurer - the amount of care that he takes. So, this is called pure negligence, so we do not have negligence with contributed negligence, but pure negligence. One can do another trick, one can say that X_1 is infinity - close to infinity and X_2 is equal to 0.

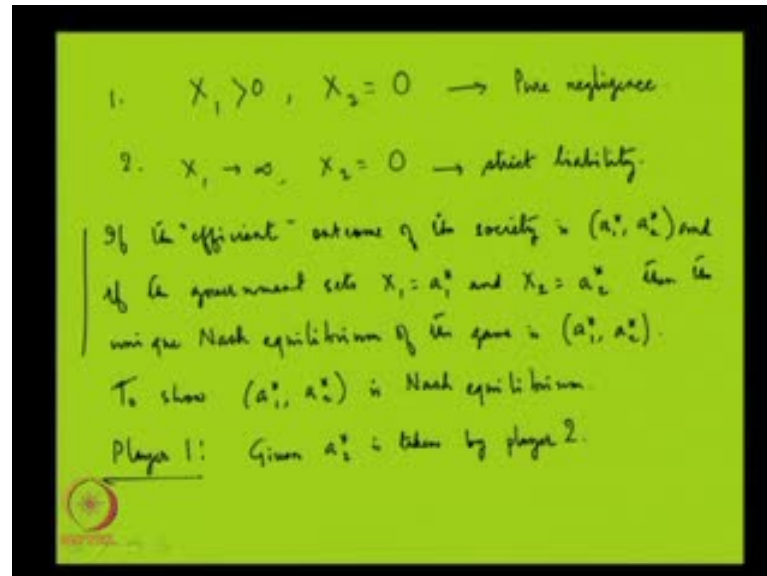
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Here, what is happening is that this is never going to be satisfied, because X_1 is infinity, so you cannot have any care which is equal to infinity; this is not going to be satisfied. Obviously, this is not going to be satisfied, none of them is going to be satisfied, so this

condition is automatically satisfied, which means that here whenever the accident occurs, the injurer pays the entire compensation to the victim.

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So, this is the case of strict liability. So, one can play around with these threshold levels of X_1 and X_2 , and get different kinds of laws, which can be thought to be more just to the injurer or to the victim. Now, this is the general setting, now the question is what kind of laws that are most efficient for the society? If certain law is imposed by the government, is that law efficient? Means, if some law is imposed by the government, then will the people generate an outcome? Which outcome is efficient? Because, it may happen that the government wants the people to take some actions, but when it sets out the guidelines, people play some game with in themselves and the outcome is such that is not according to the wishes of the government.

So, one is, what is the specification by the government, which is X_1 and X_2 ? And what is the outcome of the game, which is being played by the players? These two can be different. In particular, we are going to look at the conditions, under which the outcome of the game that is Nash equilibrium is an efficient outcome for the entire society.

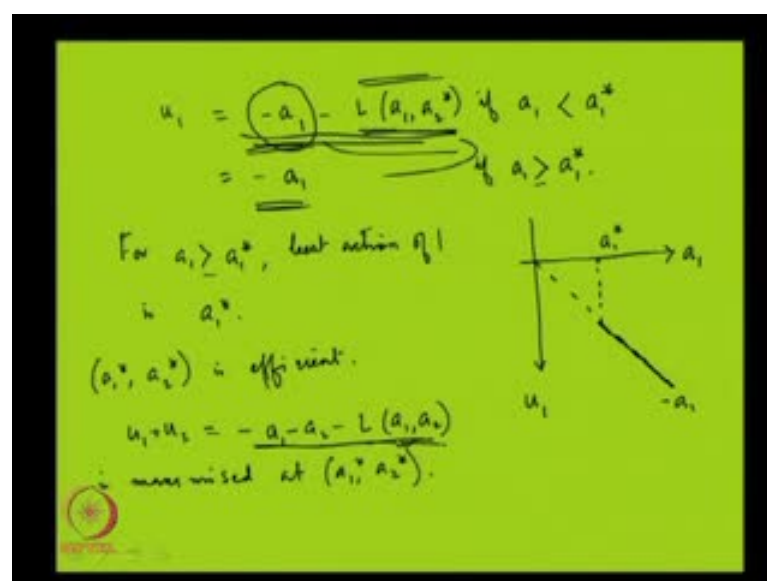
Our proposition is the following that if the efficient outcome for the entire society – now, efficient will be defined in a particular sense that we are going to look at. So, what one is saying is that if player 1 takes the action a_1^* and player 2 takes the action a_2^* that is best for the society. If the government - then - so this is the result or the proposition

one is putting forward (Refer Slide Time: 23:39). So, what is being said is the following. Suppose for the society a 1 star and a 2 star is the best, the best possible outcome. The government knowing that a 1 star and a 2 star - the best out, if a 1 star and a 2 star are played by player 1 and player 2 that is best for the society, sets X 1 that threshold level of care by player 1 to be equal to a 1 star and threshold level of player by player 2 to be a 2 star. If the government sets these values, then when the players play their games without the direct intervention of the government, the Nash equilibrium that they generate is in fact a 1 star and a 2 star, which means that the society is reaching an efficient outcome; so this is what is being proposed.

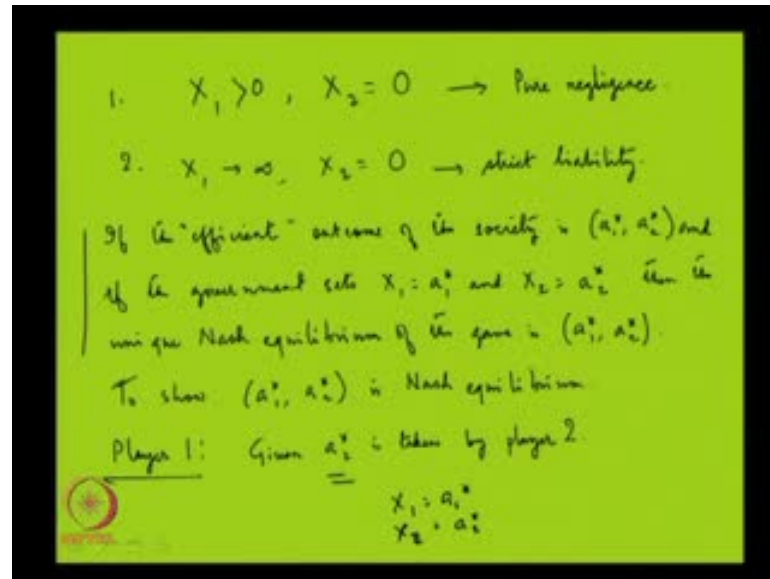
What we are going to do now is that we are going to first show that a 1 star and a 2 star is Nash equilibrium. The second step will be to show that it is unique; it is the only Nash equilibrium. Now, how to show that this is Nash equilibrium? The method is, as we have been doing before, with given a 2 star we are going to prove that a 1 star is the best action for player 1. With given a 1 star, a 2 star is the best action for player 2, if that is shown then you know that a 1 star and a 2 star is Nash equilibrium.

Now, to say that a 1 star is the best action for player 1, I have to know what the payoff function of player 1 is, given that player 2 is taking a 2 star action. So player 1, let us look at the game from his point of view. Given, player 2 is playing the action a 2 star, what is the action? What is the payoff function of player 1?

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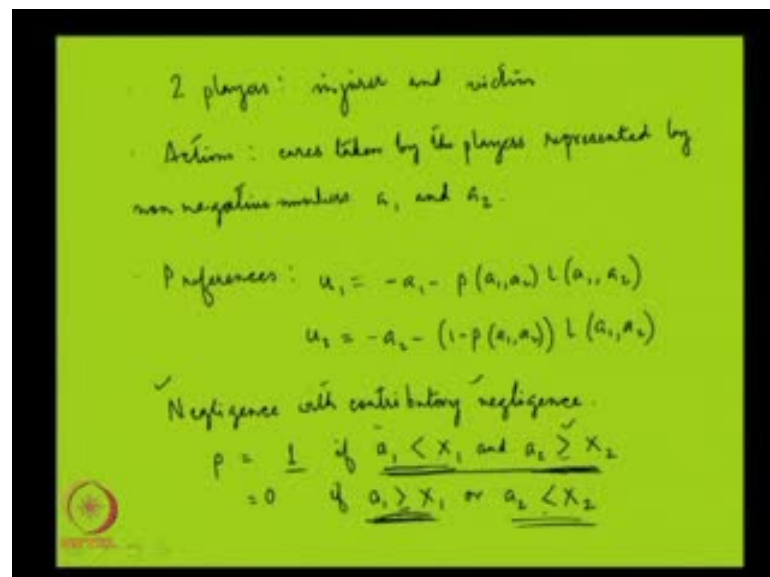


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Now, remember, payoff function of player 1 depends upon two things. If player 1 is taking sufficient care, then he does not pick to pay anything. If he does not take the sufficient care, then he is in trouble, because player 1 has been taking the sufficient action that is mention by the government, because I know that X_1 is equal to a 1 star and X_2 is equal to a 2 star. Here, player 2 is taking this action, a 2 star which is equal to X_2 .

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So, it is now entirely upon player 1, what he will do? If we take any action which is greater than or equal to a 1 star, then he does not need to pay anything, because this

condition will then be satisfied. If he takes an action less than a 1 star, then this becomes operative and therefore, he will have to pay the entire compensation.

So, this is equal to minus a 1 - minus - is equal to minus a 1 - if it takes sufficient precaution, sufficient care (Refer Slide Time: 28:16). So there are two ranges of value that has to be considered as far as a 1 is concerned. Suppose this is a 1, this is u_1 ; this is the 45 degree line, so this is minus a 1. Suppose, here I have a 1 star, now if a 1 is greater than a 1 star, then we are talking about this portion, payoff function is given by minus a 1. In that case, what is the best action for player 1? The best action obviously will be to maximize minus a 1, which means that he will choose just a 1 star. This is also true for a 1 is equal to a 1 star, because in that case also this is operative.

Now, the problem is if a 1 is less than a 1 star, then I do not have this function any more, I have this function. In this function, I know that as a 1 falls, this L thing rises; so, if this L rises, the entire thing becomes low, it falls, the u_1 falls. In that case, I will like to keep a 1 as high as possible, because if a 1 is high, then L is low, so u_1 is high. So, the player 1 going by this L part will like to keep a 1 as high as possible, but if a 1 is kept as high as possible, this part becomes very high that means the negative becomes very low. That means there has to be a balance, which has to be looked at. One has to see, if I keep a 1 very high, how much is this benefiting me? And how much is this causing me a loss that has to be looked into?

But, one has to remember that a 2 is equal to a 2 star in this case. So, for this range, how do I find out at what level of a 1 this is going to be maximized? Now, to understand that let us start from one basic principle, which is how a 1 star and a 2 star are defined? We have said that this is efficient, now by that what one meant is that the combined payoff functions of these two players is being maximized at a 1 star and a 2 star. That is u_1 plus u_2 , what is the value of u_1 plus u_2 ? This is the value of u_1 plus u_2 - the total loss; part of it will be borne by the injurer, part of it will be borne by the victim.

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$$\text{At } (a_1^*, a_2^*), \quad -a_1 - a_2^* - L(a_1, a_2^*) \text{ is minimised.}$$

$$\text{Or at } a_1 = a_1^*, \quad \underline{-a_1 - L(a_1, a_2^*)} \text{ is minimised.}$$

The cares that have been taken by these two players, total payoff to both this players is maximized at a 1 star and a 2 star. So, this is the meaning of efficiency one is invoking in this case. Now, if at a 1 star and a 2 star this is being maximized, what is the inference from that? Which means that I am just putting a 2 star is equal to a 2, or this is maximized.

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$$u_1 = \begin{cases} -a_1 - L(a_1, a_2^*) & \text{if } a_1 < a_1^* \\ -a_1 & \text{if } a_1 \geq a_1^* \end{cases}$$

For $a_1 \geq a_1^*$, best action of 1 is a_1^* .

(a_1^*, a_2^*) is efficient.

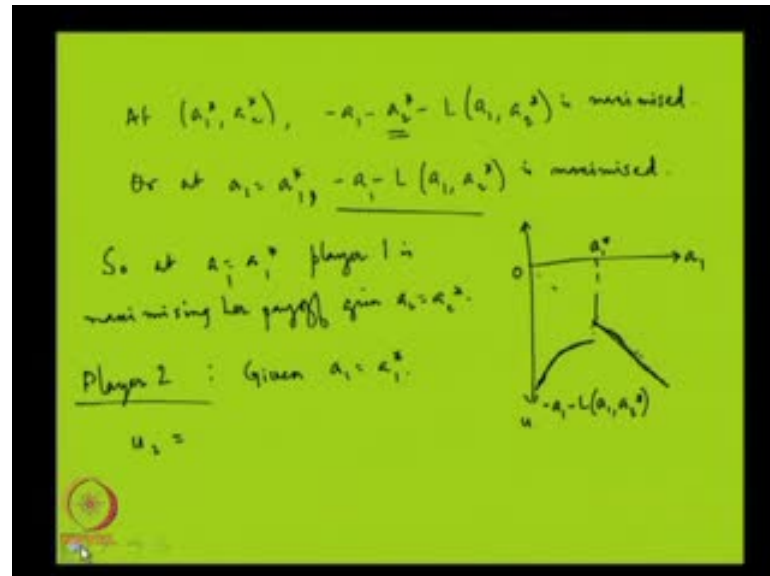
$$u_1 + u_2 = -a_1 - a_2 - L(a_1, a_2)$$

minimised at (a_1^*, a_2^*) .

So, better to write it this way, at a 1 is equal to a 1 star, this is being maximized, I am just removing this a 2 star point, because that is a constant. So, at a 1 is equal to a 1 star, this

function is being maximized and this function is something which we have seen before, this is the function.

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I have got the result that if I want to maximize this function, what I should said is that I should said a 1's is equal to a 1 star, which means that the shape of the curves that we have here will be something like this. This is a 45 degree line, we are talking about this parts and this is the function, which is minus a 1 minus L a 1 a 2 star. Therefore, the maximum value that can happen is only at this point, at a 1 is equal to a 1 star.

Now, we have to get the complete picture that only at a 1 is equal to a 1 star the payoff function of player 1 is maximized. So, given a 2 is equal to a 2 star, what about player 2 from player 2's point of view? Is a 2 star the best action for him, given player 1 is taking a 1 star?

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- 2 players: injurer and victim
- Action: care taken by the players represented by non-negative numbers a_1 and a_2 .
- Preferences: $u_1 = -a_1 - p(a_1, a_2) L(a_1, a_2)$
 $u_2 = -a_2 - (1-p(a_1, a_2)) L(a_1, a_2)$
- Negligence with contributory negligence.

$$p = \begin{cases} 1 & \text{if } \underline{a_1} < x_1 \text{ and } a_2 \geq x_2 \\ 0 & \text{if } \underline{a_1} > x_1 \text{ or } \underline{a_2} < x_2 \end{cases}$$

Now, the proof here is all like before that given a 1 is equal to a 1 star, we are going to show that at a 2 is equal to a 2 star the payoff function of player 2 is being maximized. So, what is the payoff function of player 2? Since player 1 is taking a 1 star action, which means that this is true, so player 1 is sufficiently careful, in that case, the compensation the player 2 gets is 0.

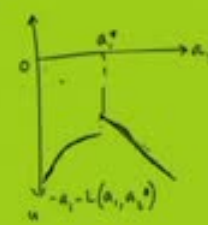
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At (a_1^*, a_2^*) , $-a_1 - \underline{a_2} - L(a_1, a_2^*)$ is minimised.
 Or at $a_1 = a_1^*$, $-a_1 - L(a_1, a_2^*)$ is minimised.

So at $a_1 = a_1^*$ player 1 is minimising his payoff given $a_2 = a_2^*$.

Player 2: Given $a_1 = a_1^*$
 $u_2 = -a_2 - L(a_1^*, a_2)$

We know at (a_1^*, a_2^*) , $-a_1 - a_2 - L(a_1, a_2)$ is minimised.



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$$\therefore \text{At } a_2 = a_2^*, -a_1 - a_2 - L(a_1, a_2) \text{ is minimised.}$$

$$\text{So at } a_1 = a_1^*, -a_1 - L(a_1, a_2) \text{ is minimised.}$$

So, I do not have two ranges, like I had for player 1. It will be just minus a 2 minus L a 1 star and a 2, because a 1 is equal to a 1 star. Now, the demonstration is just as before, so we know, since at a 1 star and a 2 star this minus a 1, minus a 2, minus L of a 1 a 2 is maximized, it means that this is maximized. That is, I am just putting a 1 is equal to a 2 star, because I know that is the action taken by player 1, which means that at a 2 is equal to a 2 star, this is also being maximized.

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$$\text{At } (a_1^*, a_2^*), -a_1 - a_2 - L(a_1, a_2) \text{ is minimised.}$$

$$\text{Or at } a_1 = a_1^*, -a_1 - L(a_1, a_2) \text{ is minimised.}$$

So at $a_1 = a_1^*$ player 1 is minimising his payoff given $a_2 = a_2^*$.

Player 2 : Given $a_1 = a_1^*$

$$u_2 = -a_2 - L(a_1^*, a_2)$$

We know at $(a_1^*, a_2^*), -a_1 - a_2 - L(a_1, a_2) \text{ is minimised.}$

So, I am going to remove this constant part and that is the proof. What we have found is that this is the payoff function of player 2, this is maximized that a 2 is equal to a 2 star.

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\therefore At $a_2 = a_2^*$, $-a_1 - a_2 - L(a_1, a_2)$ is maximised.
 So at $a_2 = a_2^*$, $-a_1 - L(a_1, a_2)$ is maximised.
 \therefore If $a_1 = a_1^*$, $a_2 = a_2^*$ is the best action for pl. 2.
 $\therefore (a_1^*, a_2^*)$ is a Nash equilibrium.
Uniqueness: Player 1:
 $u_1 = -a_1 - L(a_1, a_2)$ if $a_1 < a_1^*$ and $a_2 \geq a_2^*$
 $= -a_1$ if $a_1 \geq a_1^*$ or $a_2 < a_2^*$

If a 1 is equal to a 1 star, a 2 is equal to a 2 star is the best action - optimal action for player 2. Therefore, a 1 star, a 2 star is Nash equilibrium, so we have proved that this is Nash equilibrium. What is the proof that this equilibrium is unique? Now, to do that what one needs to do is just the usual way, we are going to find out the best response functions. If we have two best response functions, which intersect each other at a unique point, at a single point, then we have got the proof that there is a unique Nash equilibrium, so uniqueness.

Now, first again try to find out the best response function of player 1. What is the payoff function of player 1; let us try to remember that. Player 1 pays full composition, if he is careless and player 2 is careful, and he does not pay if he is careful or player 2 is careless.

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$\text{If } a_2 > a_2^*, a_1 \text{ cannot be more than } a_1^*$
 $\text{optimal } a_1 \leq a_1^*$
 $\text{If } a_2 = a_2^*, \text{ the optimal } a_1 = a_1^*$
 $\text{If } a_2 < a_2^*, \text{ optimal } a_1 = 0$
Player 2: For $a_1 = a_1^*$, best response of 2 is a_2^* .
 $\text{If } a_1 < a_1^*$
 $u_2 = -a_2, \text{ if } a_2 \geq a_2^*$
 $= -a_2 - L(a_1, a_2) \text{ if } a_2 < a_2^*$

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$\therefore \text{At } a_2 = a_2^*, -a_1 - a_2 - L(a_1, a_2) \text{ is maximised.}$
 $\text{So at } a_1 = a_1^*, -a_1 - L(a_1^*, a_2) \text{ is maximised}$
 $\therefore \text{If } a_1 = a_1^*, a_2 = a_2^* \text{ is the best action for p 2.}$
 $\therefore (a_1^*, a_2^*) \text{ is a Nash equilibrium.}$
Uniqueness: Player 1:
 $u_1 = -a_1 - L(a_1, a_2) \text{ if } a_1 < a_1^* \text{ and } a_2 \geq a_2^*$
 $= -a_1 \text{ if } a_1 \geq a_1^* \text{ or } a_2 < a_2^*$

One can now think of different ranges of values for a_2 , correspondingly one can talk of different best response functions. Suppose a_2 is greater than a_2^* , then what happens? If a_2 is greater than a_2^* , then I have this case. In that case, if player 1 is taking an action which is less than a_1^* , then his payoff function is this.

Now, in that case, is it possible that he takes an action greater than a_1^* ? The answer is no. If he takes an action greater than a_1^* , because if he takes action more than a_1^* , in which case, the optimal think for him to do will be to set a_1 is equal to minus a_1

star. So, he can never set an action which is greater than a 1 star, at most it can be a 1 star. It is possible that he takes an action which is less than a 1 star, it may happen that if a 2 is greater than a 2 star, at some value a 1 less than a 1 star, this function is being maximized that is possible. But, what is being ruled out is that if a 2 is greater than a 2 star, a 1 can be more than a 1 star that is being ruled out.

So, if a 2 is greater than a 2 star, a 1 cannot be more than a 1 star, because if it is more than a 1 star, this becomes operative, in which case, player 1 can take the optimal action for player 1, is minus and is a 1 star. But, what we are saying is that it is possible that a 1 is less than a 1 star? That is entirely possible.

So, a 1 is less than equal to a 1 star, if a 2 is equal to a 2 star. This is what we have just seen that if a 2 is equal to a 2 star this is optimal - the optimal a 1 is equal to a 1 star, because this is the Nash equilibrium. If a 2 is less than a 2 star, then what happens? If a 2 is less than a 2 star, so I have this case. In that case, obviously player 1 will take just action, which is equal to 0, because at 0 this is being maximized, so this is very simple.

We have got our best response function of player 1 for a 2 less than a 2 star it is 0, for a 2 is equal to a 2 star it is equal to a 1 star. For a 2 greater than a 2 star we can say that player 1's optimal action should be either equal to a 1 star or less than a 1 star, it cannot be more than a 1 star.

Now, let us look at player 2. Now, to analyze the best response of player 2, first let us gather from the analysis (()) that a 1 never exceeds a 1 star. The ranges of value that I have to look into for a 1 is that it is equal to a 1 star or less than a 1 star. Now, for a 1 star, for a 1 is equal to a 1 star, the best response I know is a 2 star that has been established.

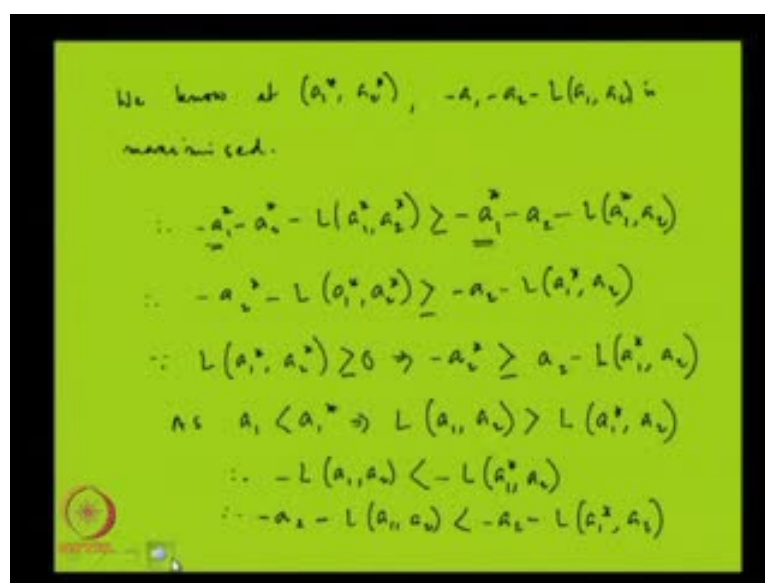
The only thing I have to look at is that if a 1 is less than a 1 star, then what happens? What should be the best action of player 2 that one has to look at? Now, for player 2, what is the payoff function of player 2? Payoff function of player 2 is the following that given a 1 is less than a 1 star, he will get the compensation. If he gets the compensation his payoff is minus a 2.

This happens if he is careful that is a 2 is greater than equal to a 2 star and if he is not careful, he is not going to get any compensation, in that case his payoff is (Refer Slide

Time: 48:04). This is because I know that player 1 has been negligent, so it entirely depends on player 2 now. If he is careful, then he gets compensation, if he is not careful, then the second thing becomes operating that he does not get compensation.

Now, what we are going to show now is the following that a 2 star is the best response for this, sorry this is u 2, what we are going to show is that a 2 star is the best response for player 2, if a 1 is less than a 1 star that is what we are going to show.

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We know at (a_1^*, a_2^*) , $-a_1 - a_2 - L(a_1, a_2)$ is maximised.
 $\therefore -a_1^* - a_2^* - L(a_1^*, a_2^*) \geq -a_1^* - a_2 - L(a_1^*, a_2)$
 $\therefore -a_2^* - L(a_1^*, a_2^*) \geq -a_2 - L(a_1^*, a_2)$
 $\therefore L(a_1^*, a_2^*) \geq 0 \Rightarrow -a_2^* \geq a_2 - L(a_1^*, a_2)$
 As $a_1 < a_1^* \Rightarrow L(a_1, a_2) > L(a_1^*, a_2)$
 $\therefore -L(a_1, a_2) < -L(a_1^*, a_2)$
 $\therefore -a_2 - L(a_1, a_2) < -a_2 - L(a_1^*, a_2)$

Now, to show that first, let us again remember at a 1 star and a 2 star this function is going to be - is being maximized. Now, if this function is being maximized at this values that means it is being maximized at the values (Refer Slide Time: 50:01). This also given a 1 is equal to a 1 star, I am just taking different values of a 2. I know that at a 2 is equal to a 2 star this being maximized, so this being maximized that this value also.

Now, I can say from this that I can just forget about this minus a 1 star part on both sides. Since, this is greater than 0, which means that minus a 2 star will be greater than equal to minus a 2 minus a 1 star a 2. Now, remember, the a 1's that we are considering are less than a 1 star, as a 1 is less than a 1 star, which means that L a 1 a 2 is more than L a 1 star a 2.

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$$\therefore -a_1^* >$$

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We know at (a_1^*, a_2^*) , $-a_1 - a_2 - L(a_1, a_2)$ is maximised.

$$\therefore -a_1^* - a_2^* - L(a_1^*, a_2^*) \geq -a_1^* - a_2 - L(a_1^*, a_2)$$

$$\therefore -a_2^* - L(a_1^*, a_2^*) \geq -a_2 - L(a_1^*, a_2)$$

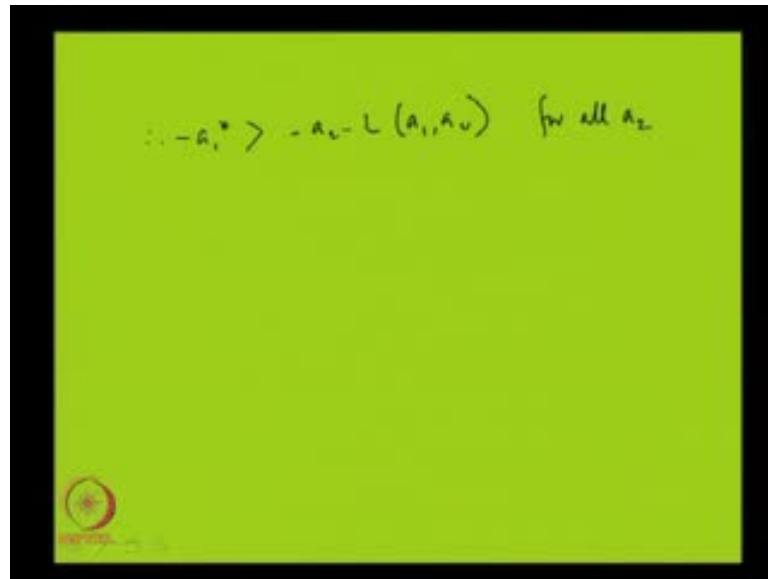
$$\therefore L(a_1^*, a_2^*) \geq 0 \Rightarrow -a_2^* \geq a_2 - L(a_1^*, a_2)$$

As $a_1 < a_1^* \Rightarrow L(a_1, a_2) > L(a_1^*, a_2)$

$$\therefore -L(a_1, a_2) < -L(a_1^*, a_2)$$

$$\therefore \underline{-a_2 - L(a_1, a_2)} < -a_2 - L(a_1^*, a_2)$$

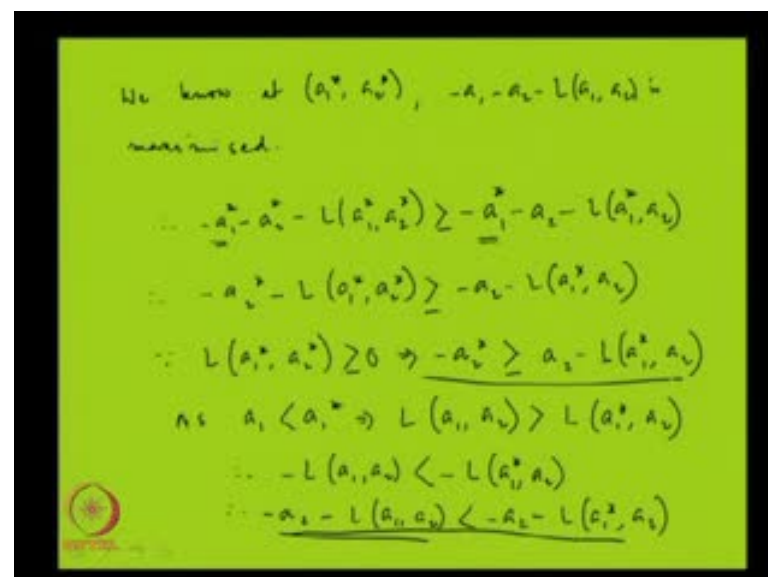
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$$\therefore -a_1^* > -a_2 - L(a_1, a_2) \text{ for all } a_2$$

This means that this, therefore it means that a 1 star is greater than this value; this value and this is for all a 2.

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We know at (a_1^*, a_2^*) , $-a_1 - a_2 - L(a_1, a_2)$ is maximised.

$$\therefore -a_1^* - a_2^* - L(a_1^*, a_2^*) \geq -a_1^* - a_2 - L(a_1^*, a_2)$$

$$\therefore -a_2^* - L(a_1^*, a_2^*) \geq -a_2 - L(a_1^*, a_2)$$

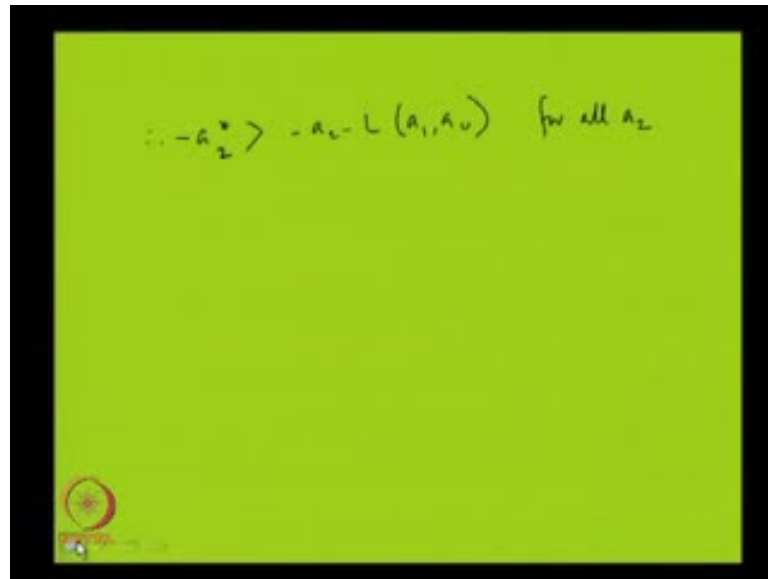
$$\therefore L(a_1^*, a_2^*) \geq 0 \Rightarrow -a_2^* \geq a_2 - L(a_1^*, a_2)$$

As $a_1 < a_1^* \Rightarrow L(a_1, a_2) > L(a_1^*, a_2)$

$$\therefore -L(a_1, a_2) < -L(a_1^*, a_2)$$

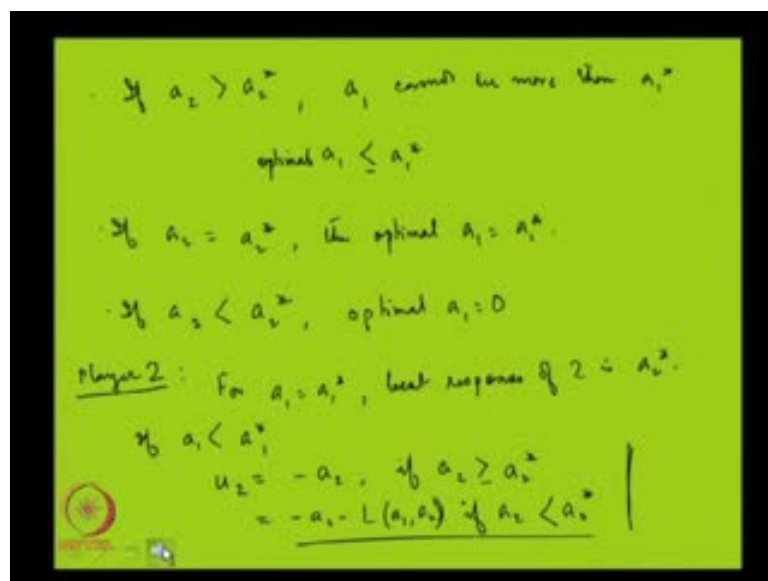
$$\therefore -a_2 - L(a_1, a_2) < -a_2 - L(a_1^*, a_2)$$

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$$\therefore -a_2^* \geq -a_2 - L(a_1, a_2) \text{ for all } a_2$$

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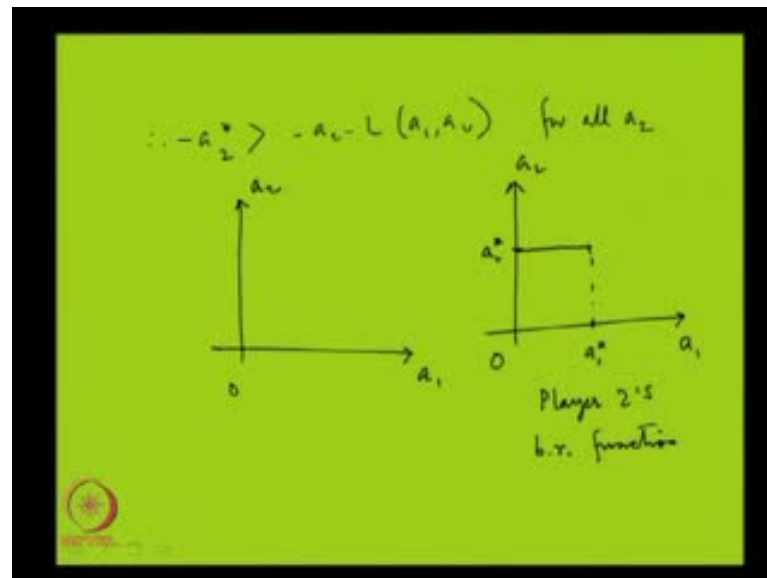


If $a_2 > a_2^*$, a_1 cannot be more than a_1^*
 optimal $a_1 \leq a_1^*$
 If $a_2 = a_2^*$, the optimal $a_1 = a_1^*$
 If $a_2 < a_2^*$, optimal $a_1 = 0$
Player 2: For $a_1 = a_1^*$, best response of 2 is a_2^* .
 If $a_1 < a_1^*$

$$u_2 = \begin{cases} -a_2, & \text{if } a_2 \geq a_2^* \\ -a_1 - L(a_1, a_2) & \text{if } a_2 < a_2^* \end{cases}$$

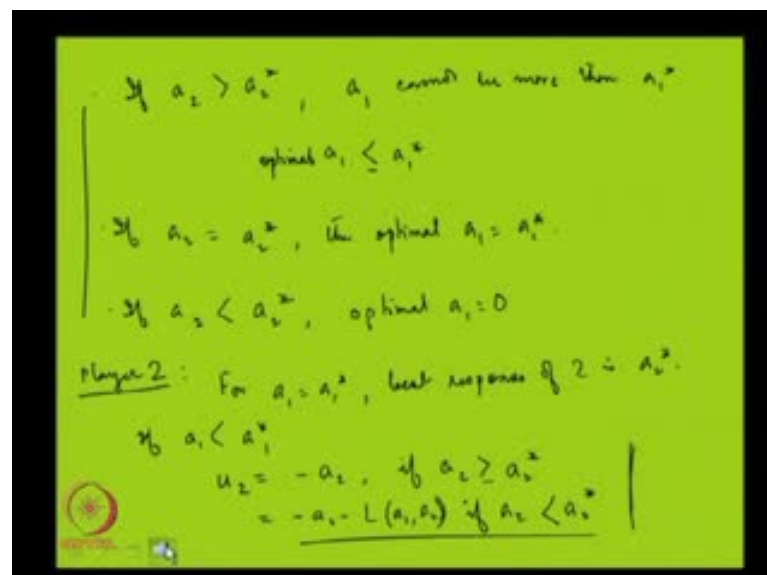
So, I have just combined this fact with this fact to get - so this will be minus a 2 star minus a 2 star, will be greater than equal to this, which means that at a 2 star this payoff function is being maximized. This is the payoff function that we are considering at a 2 star, this is being maximized and that is what we wanted to prove. If player 2 takes the action a 2 star that is the payoff that he gets is higher than any other payoff that he can get, with given different values of a 2.

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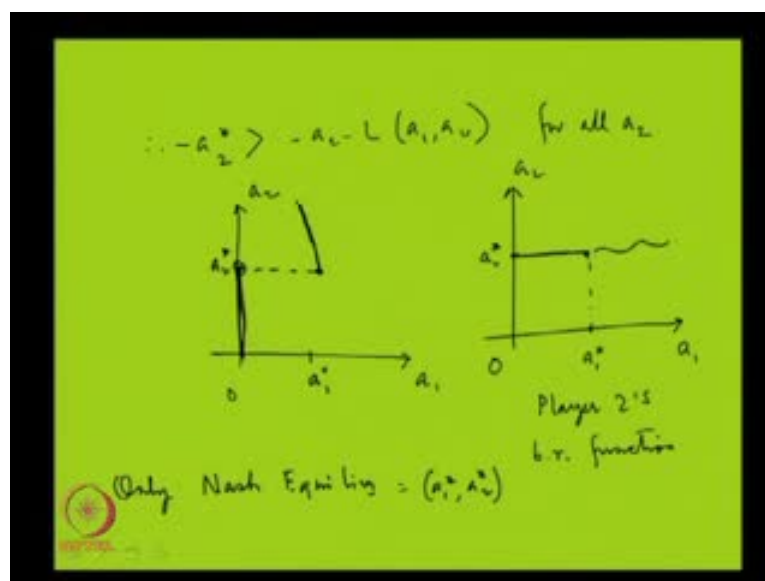


Now, this means that how will the best response function of a 1 and a 2 will look like? We have seen already that this is player 2's payoff function; player 2's payoff function I have seen is a 2 star, which is constant. This is a 1 star suppose, till this we consider and after that we are not concerned. So, this is player 2's best response function.

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What about player 1's best response function? This is what we have seen as the player 1's best response function, it is given by this line 0. This is a 2 star, at a 2 it is equal to a 1 star and if a 2 is greater than a 2 star, it can be less than a 1 star.


Now, if I combined this with this, remember this point is not included, this point is included. If I combined this with this, the only Nash equilibrium - the unique Nash equilibrium is at a 1 star and a 2 star, because this is the point where the functions are meeting. Otherwise, there is no meeting point, because this point is not included here or where this point is included here. After this, it can take any shape I am not concerned.

So, we have proven the uniqueness and the fact that a 1 star and a 2 star is Nash equilibrium. This is the end of the lecture; we have finished this section of applications of Nash equilibrium. In this particular lecture, we have covered accident laws, in the next lecture we shall talk about what is known as make strategy Nash equilibrium; thank you.

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Lecture 26

1. What is the general framework of Accident Law games?
2. Explain the legal rules known as *negligence with contributory negligence*. What is the unique equilibrium if social welfare is assumed to be summation of individual payoff functions and welfare maximising care levels are set as norms?



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1. 2 players: injurer and victim

Loss to the victim = $L(a_1, a_2)$

a_i = care taken by player i in avoiding the accident


$i=1, 2$, 1 = injurer, 2 = victim.

$\frac{\partial L}{\partial a_i} < 0$

So payoff to 1, $u_1(a_1, a_2) = -a_1 - p(a_1, a_2)L(a_1, a_2)$

p = fraction of loss that 1 bears by law.

$u_2(a_1, a_2) = -a_2 - (1-p)L(a_1, a_2)$



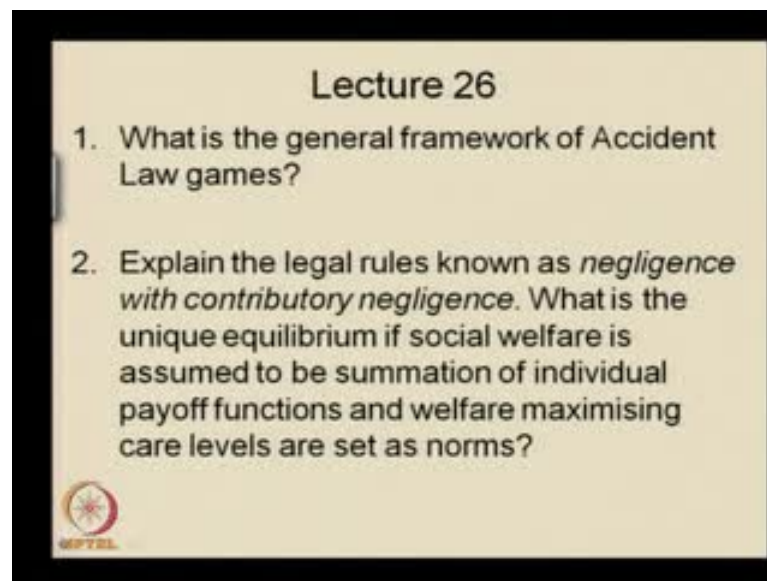
What is the general framework of accident law games? In accident law games what we have is 2 players, the injurer and the victim. So, the injurer has caused some injury to the victim and the loss to the victim due to this accident, is given by minus or if we forget about the minus sign, the loss is given by L of a_1 and a_2 . Where a_1 and a_2 , a_i is the care taken by player i in avoiding the accident, i could be 1, 2, where 1 is injurer, 2 is the victim. We can realistically assume that if a_i rises L , the loss will go on declining, because when more care you take to avoid the loss, to avoid the accident, the amount of loss is assumed to fall.

We can interpret in another way also that more care you take to avoid the accident, the probability of accident occurring that declines, so the expected loss from the accident that declines. So, in terms of probability also this can be in terms to expected value, also this can be interpreted.

So, what is the payoff to player 1, which we write by u_1 ? First, minus a_1 , remember, a_1 is the care that she takes in avoiding the accident and taking care is costly. So, minus a_1 minus ρ which is a function of a_1 and a_2 multiplied by a_1 multiplied by L , it is a function of a_1 and a_2 , where ρ is the fraction of loss, which I bare by law.

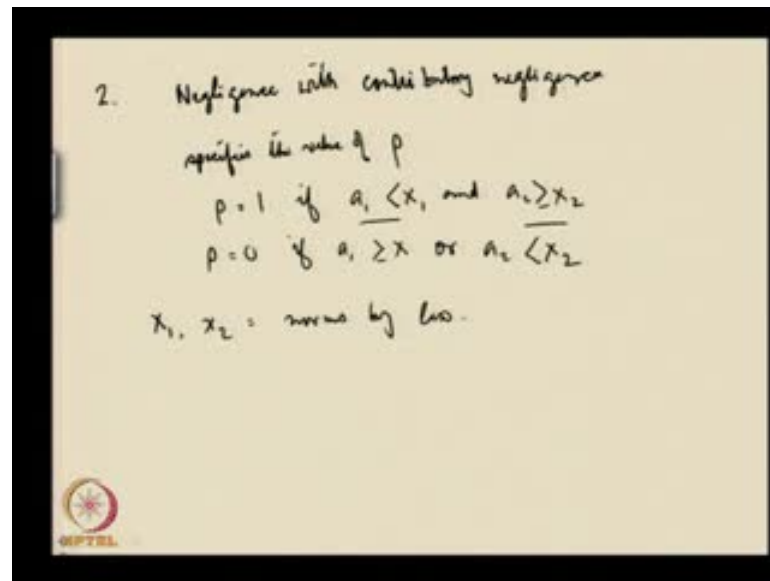
Similarly, u_2 will be this part, which is not covered by the injurer that has to be borne by the victim, so we have 1 minus ρ multiplied by L .

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This is the general framework of accident law games, the law specifies ρ . ρ is specified by the law, given the different ways in which ρ is specified, the players decide how much of care that they will undertake. So, this is essentially the framework of the accident law games.

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Explain the legal rules known as negligence with contributory negligence. What is the unique equilibrium if social welfare is assumed to be summation of individual payoff functions and welfare maximizing care levels are set as norms?

So, negligence with contributory negligence; it basically specifies the value of ρ and it is given as the following; ρ is equal to 1; where x_1 and x_2 are the norms by law (Refer Slide Time: 61:34). So, if the injurer has been negligent, the victim has not been negligent and then the injurer pays the full compensation. Otherwise, in the negative of these cases, the victim bares the complete loss and the injurer does not pay any compensation.

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Lecture 26

1. What is the general framework of Accident Law games?
2. Explain the legal rules known as *negligence with contributory negligence*. What is the unique equilibrium if social welfare is assumed to be summation of individual payoff functions and welfare maximising care levels are set as norms?



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2. Negligence with contributory negligence

specifies the value of p


$$p = 1 \text{ if } a_1 < x_1 \text{ and } a_2 \geq x_2$$

$$p = 0 \text{ if } a_1 \geq x_1 \text{ or } a_2 < x_2$$

x_1, x_2 = norms by law.

$$S = u_1 + u_2 = -a_1 - a_2 - L(a_1, a_2)$$

Suppose \bar{a}_1, \bar{a}_2 maximise S , then if $x_1 = \bar{a}_1$,
 $x_2 = \bar{a}_2$ an equilibria $a_1 = \bar{a}_1, a_2 = \bar{a}_2$



The last part of the question is that what is the unique equilibrium if social welfare is assumed to be summation of individual payoff functions? If social welfare, let us call this S , social welfare is u_1 plus u_2 , then we can see that this is nothing but this. Suppose \bar{a}_1, \bar{a}_2 maximize S , then if x_1 is equal to \bar{a}_1 and x_2 is equal to \bar{a}_2 , then equilibrium a_1 will be equal to \bar{a}_1 and equilibrium a_2 will be \bar{a}_2 , so that is the result; thank you.