Course Name: Design of Electric Motors

Professor Name: Dr. Prathap Reddy B

Department Name: Electronic Systems Engineering

Institute Name: Indian Institute of Science Bengaluru

Week: 02

Lecture: 07

Title: Analysis of Magnetic Circuits With and Without Air Gaps

Greetings to all, in the last lecture we have discussed the magnetic materials and BH curves right. In this lecture, we will discuss the magnetic circuits analysis. The conclusion from the last class, ferrimagnetic materials and ferromagnetic materials we will utilize to design the magnetic circuits right, because these materials will offer the higher permeability values and higher flux densities or we can say higher magnetic capacitance to store the energy. The magnetic circuits we can classify as with single excited systems and multiple excited systems, single winding and multiple winding. Again this single winding based magnetic circuits we can classify as with air gap and without air gaps. Same thing in multiple excited systems, there also with and without air gap based circuits will be there.

In most of the electromagnetic systems, we will see the air gaps right. For example, in inductor for storing the energy, we will utilize the core along with the air gaps and in rotating machines, air gap will play a crucial role with respect to the rotating object and flux linking. To analyze this type of magnetic circuits with single excited, multiple excited or with air gap or without air gap, we will follow the same approach to analyze any type of magnetic circuit like the approach is as follows. Step one, for a given magnetic circuit, we will apply the Orsted magnetic principles to find the B fields.

In the B fields directions, we will find with respect to the right hand thumb rule. This is step one. Step two, identify the flow variable and effort variable, draw the equivalent circuit based upon the gyrator capacitor models like MMF capacitance model d phi by dt and it is Cm. We should not consider the flow variable phi. In step three, we have to apply the Ampere's law and find the relation with respect to the H fields, B fields and MMF n into i.

Step four, analyze the equivalent circuit realized in step two with respect to the network theory. Based upon the network theory principles, we have to analyze the equivalent circuit and find the effective capacitance value or effective reluctance values, different reluctance or different capacitances we have to calculate. Step five, we have to apply the Faraday's law to find the relation with respect to the inductance and MMF and flux with respect to the Faraday's law. First thing at the end, after analyzing all these steps, performance curves like B versus H, how the curve looks like with respect to the magnetic circuit, then inductance versus current. Flux linkage is nothing but l into i flux linkage versus current, how the waveforms looks like.

Different curves we have to analyze as per the magnetic circuit at the end of the circuit analysis. So, first I will start with magnetic circuit without air gap. Let us consider a magnetic circuit with no air gap. This core is designed with ferromagnetic materials or ferrimagnetic materials. Let us consider simple iron and permeability of mu and the cross sectional area of the core is Ac and length of the core is lc, mean length of the core and the excited winding with respect to the electrical side.

This is the coil with current i and number of turns n and effort variable voltage at the electric domain. Now, we will analyze this circuit based upon the approach what we have discussed. Same approach is valid for all type of circuits. First step is what we have to apply the Arstert magnetic principles and we have to find the B fields. Let us consider the thumb rule.

With respect to the coil, thumb represents the magnetic fields and fingers represent the current in either way. Thumb representing the B fields means fingers should represent the current or B is representing with fingers means current should represent with the thumb either way. So, apply thumb rule here and find the B fields. So, fingers here I am considering as a current and B fields will be in this fashion. B fields are in this direction.

This B fields we can represent either with flux or d phi by dt, but flow variable should be always d phi by dt and effort variable is n into i. The representation purpose, the flux lines we can represent with phi or d phi by dt. There is no issue in it. Now, we will realize that in second step, we have to realize the equivalent circuit with respect to the gyrator capacitor model. The source here is mmf or effort variable is mmf and flow variable is d phi by dt that is im and the magnetic core I am considering the same cross sectional area and uniform core and flux is flowing uniformly across the core.

In complete core, I am representing with one capacitance that is c m is equals to mu naught mu r into cross sectional area of the core divided by length of the core. Mean length of the core or mean magnetic length of the path is same. Here, l length should be with respect to the length of the magnetic path. So, here both are same. We can consider either lc or lm.

Second step is done. Now, in third step, we have to apply the Ampere's law. Integral h dot dl is equals to n into i. Apply a closed loop integral along with this flux path. This flux path length is lm.

Apply a closed loop integral hc into lm is equals to n into i. So, hc is equals to ni by lm. The relation with respect to the magnetic fields b and h fields is b is equals to mu naught mu r into hc and there will be one more flux component in the circuit that is phi l. Here, I considered leakage flux is negligible. Leakage flux, we are neglecting it for the analysis.

So, three steps are done. First step, we have to analyze the equivalent circuit and we have to relate the voltage equations with respect to the capacitor. Apply a KVL in this loop. The MMF should match with respect to the voltage of the capacitor, right, Vcm. That equation already we have derived.

If not, we can apply the same voltage equation 1 by Cm integral Im dt. Im is nothing but d phi by dt flow variable. So, we will get flux divided by magnetic capacitance. This is the equation which relates the MMF and magnetic capacitance in terms of reluctance flux into reluctance, we can call it and permeance also same as the magnetic capacitance that is equals to 1 by reluctance. Any variable we can utilize here to analyze the magnetic circuit.

Next step, we have to apply the Faraday's law in step 5 and find the inductance relation in terms of n and B or flux in terms of magnetic fields, we have to find the relation. Induced EMF as per the Faraday's law, E is equals to n d phi by dt. The voltage equation with respect to the inductor V is equals to L di by dt, right. If we will equate these two equations, n d phi by dt is equals to L di by dt. So, we will see the final equation n into phi, the number of turns into flux is equals to inductance into current.

Inductance into current, we can represent it as a flux linkages psi. So, L is equals to n phi by I. Flux if we will replace with MMF, then n into MMF by reluctance, right, from this equation. Just substitute that equation here, then we will see inductance relation n square by reluctance or n square into permeance. Any one equation we can utilize it to find the inductance value.

Now, we will analyze the performance curves B versus H. B is related to the n item, sorry, H is related to the n item and B is related to the flux. The relation for the B-H curve will be B equals to mu naught, mu r, H. In a linear magnetic system, if there is no saturation, then we will see the linear characteristics between the B and H. If the mu value is very high, we will see the B curve near to, sorry, B-H curve near to the y axis.

If mu r value is small, that B-H curve will be near to H axis or horizontal axis. Here the mu is slope, it is related to the reluctance, 1 by reluctance. In all practical applications, we will see the non-linear magnetic characteristics and there will be a saturation because of that reason, we will see the non-linear characteristics here where B equals to constant, H is varying and mu value is almost equals to 0. Reluctance will be infinite in the saturation region from this point to this point is the saturation region and this side is the non-saturation region or linear zone.

This is non-linear. Next inductance versus current. The relation for inductance versus current is L is equals to n phi divided by I or n square by reluctance, we have derived. So, 1 by x curve, how it looks like? It will be exponential kind of decay, right? It will be in this manner in a linear zone. In a non-linear zone, it will be almost approaching to 0 value.

We can see like this manner. So, up to this point, linear variation of inductance in the starting case where I value is small, but N square term is dominant in this region. N square term is dominant because of that reason, we will see some constant value of inductance. After that, it will vary with respect to the 1 by I, 1 by x manner in a non-linear region where mu value is equals to 0 and current is very high. We will see the almost inductance is equals to 0 or very small value. Next in flux linkage versus current waveform, the flux linkage psi is nothing but L into I.

This is I. So, the flux linkage versus current waveform also will vary in a linear manner in the linear system or linear region because flux is equals to the relation L into I. L is nothing but n phi by I into I. N phi, we can write it as N B A. It will follow with respect to the B.

So, here the slope will be mu. In terms of L, if you will represent L also, we can represent that is proportional to 1 by reluctance in a non-linear zone. We will see in this fashion where mu equals to 0, the lambda versus or psi versus current. With this, the analysis with respect to the magnetic circuit without air gap is done. Now, we will consider the magnetic circuit with air gap as a second type of circuit.

Let us consider a magnetic circuit. One side we have a C core, other side we have I core. The core permeability is mu and the effective length of complete core is lc. It is a combination of lc type core plus LI type core combination of these two things. Both lengths will be lc, I am considering and then length of air gap is lg by 2 this side, lg by 2 this side and core cross sectional area is Ac with respect to the both sides. Both C type core and A type core, I type core consist of area Ac.

Now, we will see the electrical side, the winding carrying a current I and having a number of turns N. We will follow the same approach to analyze this type of circuit also, where magnetic circuit consists of air gaps also. First step, we have to apply the Arsturd magnetic principles to find the B fields. So, apply the thumb rule. We can see here with respect to the winding represents the fingers as a current and thumb is B fields.

Here current is coming out from this side and going in this side, then magnetic fields are in this direction. This is the flux loop d phi by dt. We have to find the magnetic fields and then direction in step 1. Step 2, we have to draw the equivalent circuit. Before discussing the equivalent circuit, here how the flux lines in air gap we will discuss.

Let us consider this is the air gap length lg by 2. Magnetic fields are coming in like this and going in a I core like this fashion. So, the flux lines if you will see at the air gap like this fashion. Here the entire flux will represent it as a phi g. Phi g is equals to b into a phi equals to bg is nothing but magnetic flux density at the air gap and ag is the cross sectional area with respect to the air gap flux.

So, this is the complete cross sectional area perpendicular cross sectional area ag because of this fringing flux, the flux lines which are going out are other than the core area. This area we can represent it as a ac with respect to the core. This area is ac. Greater than this ac, there is a flux lines are coming out from the teeth or core and going in back to the core. This portion we will call it as a fringing flux because of that fringing flux ag is greater than the ac.

If area of the air gap is more, then flux value will be more or with compared to the core flux, it will be same phi g equals to phi c. Whatever the flux inside the core is there, it is same as the flux with respect to the air gap. If the ag is increasing, bg value will come down flux density at the air gap. So, in most of the electromagnetic systems or in electrical machines, in order to establish the required flux density at the air gap, we have to pump the higher magnitudes of magnetizing current to establish the same magnitude because of this fringing flux and higher area of the air gap with respect to the flux.

Now, we will discuss the equivalent circuit. In this circuit analysis, we have neglected the fringing flux. We are considering ag is equals to ac. That means, bg value is equals to bc. The equivalent circuit will be mmf with respect to the source effort variable and then, the core capacitance will be cmc, I am representing by including both c type core plus i type core. The equivalent capacitance is cmc and capacitance with respect to the air gap, cmg.

Here the flow variable is d phi by dt. We can represent the flux lines with either d phi by dt or flux, but in the equivalent circuit analysis, we have to represent with d phi by dt that is the flow variable. Next step, we have to apply the Ampere's law integral h dot dl is equals to n into i. If we will apply a closed loop integral along with this magnetic flux path, we have two mmf drops. That is one is with respect to the core that is hc into lc plus hg into lg. Here length of the core, I am considering the effective length, the combination of c type core and i type core that is equals to n into i.

From here, we can calculate either hc or hg, if we know the number of turns and currents and other things. Now, we have to analyze the equivalent circuit with respect to the network theory concepts. Let us consider the same equivalent circuit. Apply the KVL. The input voltage should be equivalent to the vc with respect to the core, vc with respect to the air gap, cm with respect to the core and cm with respect to the air gap.

Equivalent capacitance is what? cm equivalent is equals to 1 by cm capacitance with respect to the core capacitance with respect to the air gap. Equivalent capacitance by substituting the cmc is equals to 1 by reluctance plus cmg is equals to 1 by reluctance of the air gap. Then we will see rc plus rg reluctance with respect to the core and reluctance with respect to the air gap. So, cm that is equivalent capacitance is equals to 1 by rc plus rg reluctance terms. So, from here, we can get mmf is equals to flux into reluctance term, flux by cm term, flux into reluctance is fine or flux by magnetic capacitance.

If we will substitute this cm equivalent thing effective voltage, we can get it. mmf is equals to flux divided by 1 by rc plus rg, then flux into reluctance with respect to the core and reluctance with respect to the air gap, we can get it. This equation we can realize it. We can derive from the voltage equations also 1 by cmc integral im dt plus 1 by cmg integral idt from here also we can derive it. Next we will analyze the flux at the different parts of iron and air gap.

phi g is equals to bg into ag and phi c is equals to bc into ac and effective reluctance of the magnetic path is nothing but 1 by cm equivalent or we can calculate it r equivalent is equals to rc that is reluctance of the core plus reluctance of the air gap that is l by mu naught mu r into ac. This is lc plus lg divided by length of air gap divided by mu naught into ag. This is the effective reluctance equation in terms of mu naught and mu r terms. Now, in step 5, we will calculate the relation with respect to the inductance versus reluctance and flux or B fields etcetera.

As per the Faraday's law, l into i is equals to N into phi. So, l is equals to N into phi by i. Here flux is equals to mmf by reluctance. If we will substitute that thing N square divided by reluctance, we will get it. Here reluctance will be effective reluctance that is N square by Rg reluctance with respect to the air gap and reluctance with respect to the core. We will see some example depends on this air gap based magnetic circuits, how the reluctance with respect to the air gap and reluctance with respect to the core variation with respect to the air gap.

In step 6, we have to analyze the performance curves. The performance curves will not change with respect to the without air gap. So, I will copy the same things. So, B versus H curves will not change. It will be same as the single excited without air gap magnetic circuit, but here mu we have to consider B with respect to the core will be mu naught mu r into H, but the BH curves for the air gap magnetic fields, if we will draw Hg and Bg, B equals to mu naught into Hg.

Mu naught is a constant value. The b h curves for the air gap magnetic fields will be in this fashion, but for the complete magnetic circuit will not change. In a linear region, it will be proportional to the mu permeability value, where it is a combination of both reluctance's Rg and Rc core and air gap reluctance's and in the non-linear region, it will saturate. Similarly inductance versus current waveform also same and the flux linkage versus inductance waveform also same. It will be in this fashion.

Only the magnitudes will vary, but wave shape will not change. With this, I am concluding this lecture. In this lecture, we have discussed the magnetic circuits with and without air gaps and we have discussed the magnetic equivalent circuits with respect to the gyrator capacitor models also. Thank you.