Course Name: Design of Electric Motors

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Title: Thermal Equivalent Circuit- 2

Greetings to all, in the last lecture we have discussed the thermal equivalent circuit only for stator circuit by considering the 5 thermal nodes. In this lecture, we will discuss the thermal equivalent circuit by considering multiple thermal nodes and all type of losses existed in the machine. We can see here, the thermal equivalent circuit realization for induction machine, the 14 thermal nodes are considered to realize the thermal model of a induction machine. These thermal nodes are ambient and frame axial duct and stator winding, stator core and stator teeth and air gap and rotor winding and end windings and same way, rotor teeth and rotor core and shaft and bearings at the bearings and mechanical losses and representation of the nodes. So, if we will see the complete thermal nodes for the induction machine, here stator and rotor windings 1 and 5 and stator teeth and rotor teeth 3 and 7 and air gap is 10 and stator core is 5 and rotor core is 8 like this. So, these are the 14 nodes we have considered.

These are the 14 nodes with respect to these 14 thermal nodes, we will realize the thermal equivalent circuit. The assumptions, but we have considered to realize the thermal equivalent circuit for induction machine, mechanical losses are modeled at the bearing point. Let us say here, bearings are presented at this point, at the end of the shaft. So, at this node, bearing losses are incorporated and mechanical losses like which friction and windage losses are represented at the end ring point and air at this particular point, where the end ring portion of the rotor and air gap region overlap that at that particular point the remaining friction and windage losses are considered, but the bearing losses are incorporated at the bearings node.

The two dimensional heat flow is considered both radial as well as axial direction of heat flow and heat flow between the stator copper portion to the stator iron is considered convection plus conduction through the insulating paper and heat flow with respect to the windings along the direction of copper winding in axial direction is significant as compared to the radial direction through the insulating paper and heat flow through the various parts of the machine is analyzed either by visualizing as a cylinder or rectangular cuboid. The heat flow is assumed in the radial direction with respect to the frame and air gap. In order to achieve the higher accuracy with respect to the thermal equivalent circuit, we have to consider the heat flow in other direction also, axial direction also and radial ducts are not considered even though the thermal node here represented radial ducts are not considered and overhangs air space also not considered, this node also node number 11 and node number 18 also not considered to make the equivalent circuit for with respect to 14 thermal nodes. Let us consider the first thermal nodes that is rotor core node where the power losses are rotor core losses and these losses are incorporated as a heat generated source at this particular node that is T temperature at the rotor core and this heat will flow towards the shaft or towards the rotor teeth. Let us consider heat is flowing from rotor core to shaft side where the temperature at the rotor core is greater than temperature at the shaft.

Then what is the thermal resistance offered by the shaft from the starting point of the rotor core to midpoint of the shaft? What is the thermal resistance? That is nothing but R theta SH to RC, the thermal resistance with respect to the conductive heat transfer in radial direction between shaft to the rotor core or rotor core to the shaft. From the midpoint of the shaft to starting point of the rotor core that is nothing but l by lambda A. Here l represents the thickness of the material with respect to the heat flow and the thickness with respect to the shaft is nothing but Dir by 2 inner diameter of the rotor divided by 2. The total diameter of the shaft is nothing but Dir. So, half of the thickness is nothing but Dir by 2 divided by the thermal conductivity of the shaft material into area with respect to the shaft material.

So, the area with respect to the shaft material we can calculate x axis direction x axis length into y axis length. x axis length by utilizing the perimeter that is pi into Dir y axis length is nothing but l e we can see in this image. So, the total area is nothing but pi Dir into l e and the thermal conductivity for shaft material that is steel 35 to 50 watt per meter kelvin. Same way the thermal resistance between the rotor core to rotor teeth with respect to the conductive heat transfer in radial direction. So, from this point to this point what is the thermal resistance that is R theta RC to RT rotor core to rotor teeth.

So, first we have to see the thickness of the stator core material. This is this area, this region, this is the back iron portion from this point to this point. This is the back iron portion that green color area. So, the thickness with respect to the back iron is nothing but d c r that is l equivalent to l divided by thermal conductivity of the core material into this area we have to calculate with respect to the heat flow. So, x axis length will be pi into mean diameter of this portion.

If I will calculate the mean diameter here, so with respect to this inner circle, it is diameter will be Dir with respect to the outer circle, it is Dir plus 2 into dcr So, mean diameter we can calculate by adding these two terms divided by 2 into length in y direction. So, then we can find the thermal resistance between rotor core to rotor teeth.

This is nothing but the thermal resistance offered by rotor core and here with respect to the rotor core, the power losses PRC are incorporated here, rotor core losses at the rotor core thermal node. Then how to realize the thermal equivalent circuit, we can see now. So, this heat generating point or we can consider the losses with respect to the shaft side also the core losses.

So, if we will consider the heat generation at the midpoint of the circuit or hollow cylinder, solid cylinder not hollow cylinder. So, for solid cylinder heat generating source is there at the center point, then this T type of building block we can utilize it. So, if we will incorporate that t type of thermal equivalent circuit building block with respect to the rotor shaft circuit that will be in this form. So, from the midpoint of the shaft to axial direction, how the heat is transferring? So, this is the shaft resistance and these also shaft resistance terms with some magnitudes. So, heat transfer with respect to the radial direction is representing r theta shaft to core and with heat transfer with respect to the axial direction that thermal resistance is nothing but R theta SH.

This resistance is in radial direction, this resistance is an axial direction. So, the axial direction thermal resistance is nothing but in this direction heat is flowing. So, the thickness will be length of the core in this direction heat is flowing. So, from this point to this point, so, the length of the core is nothing but le. We have to consider as a thickness and shaft thermal conductivity that is lambda shaft into area with respect to this solid cylinder kind of structure.

Shaft is nothing but solid cylindrical structure. So, that area in the direction of heat flow is nothing but pi r square. So, from this pi r square here r is nothing but Dir by 2, then we can calculate the area. So, based on that thing the thermal resistance with respect to the shaft in axial direction is this one. So, we have to substitute R theta SH in these three places.

Here I am assuming heat generating point at the middle point of the shaft also. If we will not consider heat generating point at the midpoint of the shaft, then this t type of network no need to represent directly. We can represent in this fashion the building block 1 where this is R theta SH by 2 and this is R theta SH by 2. I am assuming at the center point there is a heat generating source that is why T type of network I am representing here in red color. If heat generating source is not there at the midpoint of the shaft directly, we can represent the thermal network in this fashion and this node is rotor core and this node is midpoint of the shaft.

Next the thermal resistance with respect to the end point of the rotor to center point of the bearing. Let us consider the length from the midpoint of the end ring to midpoint of the bearing is nothing but lx. Based upon this length and through the shaft heat is flowing in an axial direction. What is this thermal resistance? That thermal resistance is nothing but R theta x in this portion. The distance between the center point of the bearing to center point of the end ring in this portion also heat is flowing in an axial direction and how to calculate this thermal resistance we will see now.

The mechanical losses are incorporated at the bearing as well as end point of the rotor at the air gap. Let us say this is the air gap region. At this point I am representing the windage losses and at the bearing node we are representing the bearing losses. With respect to this loss representation, heat will be there and heat generation we have to consider. Here directly I am neglecting the T type network for simplicity with respect to the mechanical losses.

Only with respect to the main nodes, stator core and stator winding and rotor core and rotor winding I am representing the T type network to minimize the complexity. With respect to the node 1, node 5, stator and rotor winding, stator and rotor teeth and stator core and rotor core I am representing the thermal network as a T type where the heat generating source is there at the center point. Even though for windage losses and bearing losses with respect to the mechanical losses, heat generating source will be there, but to minimize the complexity I am not considering the T type network directly. There is two thermal resistances in series fashion I am representing. So, R theta x that is thermal resistance between the center point of the bearing to center point of the end ring we can represent with R theta x that is nothing but l divided by lambda A.

Here thickness of the shaft between these two points that is midpoint of the bearing to midpoint of the end ring is nothing but lx divided by thermal conductivity of the shaft into area with respect to the heat flow direction. Heat flow is happening in this direction. The area of the shaft in axial direction is nothing but pi r square, r is nothing but Dir by 2. Just from the bearing point, heat is flowing towards the ambient in this portion. Heat is flowing towards the ambient by radiation as well as convection manner.

So, that resistance with respect to the convection is represented with R theta shaft to ambient in a convection manner 1 by hc into a. Here hc represents the convective heat coefficient and based upon the type of cooling arrangement whether it is air or liquid, we have to select this convective heat coefficient and area with respect to the heat flow is nothing but heat is flowing in this direction. Area will be perimeter with respect to the y axis and length with respect to the x axis that is lshk shaft length from the center point of the bearing to end of the shaft point that will give the length of this portion. So, if we will multiply the x axis length into y axis length is nothing but pi into Dir, then we will get the surface area which is related to the convective as well as radiative heat transfer. So, these two resistances we can calculate by utilizing the convective heat transfer equation as well as radiative heat transfer equation.

So, the radiative heat transfer coefficient we have to select from this table. So, with respect to the temperature change, let us say temperature change is 100 degrees and ambient temperature is 30 degree Celsius, then the hc, hr value is nothing but 18, 18 we have to select. Same way for different temperature rise, what is the heat transfer or heat temperature radiative coefficient we can select from this table. So, after knowing r theta, the thermal resistance between the bearing point to the ambient point with respect to this point to this point, how the heat is transferred, then we have to analyze this resistance that is the resistance between the bearing node to end cap. End cap is nothing but the end support which is supporting the rotor structure coupled to the stator.

So, from bearing to end cap through the conduction manner, heat is flowing in a radial direction. So, this is the bearing, here rotor shaft will be there and on top of this thing, this end cap will be there. So, from bearing center point of the bearing to end cap, how the heat is flowing. So, we can see the thermal resistance here that is l by lambda A. Again, the thickness of the bearing is nothing but from this point to this point.

Center point in radial direction that is this line to this line represents the thickness of the bearing that is d and in order to calculate the area for heat transfer in radial direction, length with respect to the x axis, length with respect to the y axis is this one. This is nothing but pi into D that is mean diameter of the bearing and length of the bearing will be lb, l bearing and D bearing. So, by considering the thickness of the bearing and mean diameter of the bearing into length of the bearing, we can calculate the thermal resistance offered by the bearing towards the end cap. This side we have the end cap, here end cap will be there and here shaft is there at the center point center of the bearing. So, from shaft to end cap, what is the thermal resistance that we can calculate by utilizing this equation.

First from the end cap to this is the end cap of the machine. So, from end cap to ambient, how the heat is flowing? Heat is transferred that is with respect to the radiation and convection. So, for convection, basic thermal resistance in axial direction is nothing but 1 by h c into A, same equation. Here D naught s is nothing but outer diameter of the stator into length of the end cap in mm. So, this we have to consider length of the end cap and the other direction length will be pi into D naught s.

So, based upon the velocity of coolant, either it is air or liquid, we can consider the velocity and then we can find the convection heat transfer coefficient. Then radiative based heat transfer from end cap to ambient, we can calculate by utilizing this equation. Area will be same and heat transfer coefficient with respect to the radiation, we have to consider from this table. Depends upon the ambient temperature and depends upon the temperature rise, we can calculate the heat temperature, heat coefficient for radiation. So, this resistance also we have calculated.

Now, with respect to the rotor core node, this is the rotor core node in axial direction, we are considering the T type network here, heat generating source is there. So, because of that reason, we have to consider here with respect to the heat generating source. In axial direction, T type network is considered. So, radial direction, there is no T type network, but in axial direction, I am considering the T type network that is this one, rotor core t type building block. R theta, R c represent the rotor core thermal resistance in axial direction with respect to the conductive heat transfer.

So, the rotor core will be in this fashion, this is inner diameter Dir and depth of the rotor core will be dcr. Then how to calculate the area with respect to the axial heat transfer, axial direction of heat transfer is nothing, but l by lambda A. Here l is the thickness, that thickness will be in this direction, heat is transferring through the rotor core. So, here thickness will be le divided by hollow cylinder area. So, this is outer diameter, this is inner diameter with respect to the rotor core.

For this thing, Dir plus 2 into dcr, for inner circle it will be Dir. So, hollow cylinder area we can calculate pi d 1 square minus d 2 square divided by 4, d 1 is nothing, but this thing and d 2 is nothing, but Dir. So, based upon hollow cylinder area, we can calculate the thermal resistance offered by the rotor core in axial direction. So, that axial direction heat flow, we have to represent the T type manner. Same way for radial direction also, we have to represent in a T type network, but radial direction I am neglecting to minimize the complexity.

Only in axial direction of heat flow, I am considering and representing the T type network. Then from rotor core to end cap, in between air will be there, between the end cap to this is end cap and this is the rotor core, in between this is the air gap, air region. So, air region resistance also I am not considering. If you will consider here, one more resistance we have to add, then it will reach to the temperature at the end cap. So, directly I am neglecting this resistance representation and represented with end cap temperature.

The thermal resistance between the rotor core to end cap are how the heat is transferred through the radiation as well as convection. Through the air gap only, heat is transferred that means, radiation as well as convection. So, if you will see the convection based heat transfer that is 1 by hc into A, here also area with respect to the convective heat transfer method. So, the hallow cylinder area we can consider because only that portion is responsible with respect to the heat flow in this direction axial direction. In axial direction of heat flow from rotor core point to the end cap, if you will consider that is in terms of radiation and convection is nothing but this equation.

The thermal resistance between the rotor core to end cap through the air presented in between the end cap to the rotor core, we can represent as a radiation based heat transfer and thermal resistance r theta rotor core to end cap. Radiative transfer is nothing but 1 by hr into A, here area with respect to the radiative heat transfer also same hallow cylinder area and hr is the heat transfer coefficient for radiation.

Next at the rotor teeth, I am considering as one more thermal node and axial direction of heat flow. Again, we have to represent with one t type network and this network representing the rotor teeth to end cap thermal resistance with respect to convection as well as radiation. In the rotor teeth axial direction, conduction based heat transfer is happening.

Here conduction Qc, we can say here heat flow is in convection as well as radiation that is from here. Let us say this is the rotor teeth. Here end cap will be there on both sides. From here, heat is transferred through the air towards the end cap.

This is the end cap. This heat transfer is happening with respect to the radiation as well as convection from rotor teeth end point to the end cap of the machine. So, first we will analyze the thermal resistance with respect to the rotor teeth in axial direction. So, the rotor teeth area is nothing but the area with respect to the single teeth and into number of rotor slots. Qr represents the number of rotor slots.

tr represents the width of the each rotor teeth. This is the width of rotor teeth into height of the rotor teeth is nothing but dsr. So, we are calculating the area with respect to the single teeth into total number of slots with respect to the rotor. We will give the total teeth area and width of the rotor teeth in the direction of axial heat flow is nothing but le rotor core length divided by thermal conductivity of the rotor core. For steel core, it is 35 to 50 watt per meter Kelvin. So, with respect to that thermal resistance, we can calculate these three thermal resistances and thermal resistance between rotor teeth end portion to the end cap by convection and radiation.

Area will be same. Only the heat transfer coefficient or thermal coefficients will change. Next, the thermal resistance offered by the rotor winding from rotor teeth to the rotor winding is nothing but the rotor teeth thermal resistance in radial direction. That is thermal resistance between rotor core to rotor teeth is nothing but R RT to RW. This one here again in the radial direction, the thickness of the teeth will be these are the two slots and thickness of the teeth in the radial direction of heat flow is in this direction. Thickness will be here this thing that is dsr and area will be Q into tr into length of the core into stacking factor.

We can neglect also this stacking factor directly. We can consider the le value if requires exert iron length by excluding the ducts radial ducts. Then, we have to consider the ks value also. So, the total area with respect to the radial direction of heat flow is nothing but tr into le with respect to one teeth portion into total number of slots. This will give the total area offered in the direction of radial heat flow. Then, thermal resistance is with respect to the rotor winding.

So, here we can see the thermal resistance and heat flow in axial direction. Also, I am representing with simple two resistance model, not the T type model. Here, the heat generation point PRW and this is R theta R winding by 2 rotor winding R rotor winding by 2. We can see here instead of representing with T type network to make the simplicity. I have neglected this point and directly I am considering the temperature at the midpoint of the circuit is T12 and the power generation heat generation at the node midpoint of the circuit is nothing but Pi.

This is T 1 and this is T 2. You can see here this is T1 and T2 if I will consider directly I am representing in this fashion. The other resistance in the T type network I have neglected for simplicity and the thermal resistance of the rotor winding in axial direction. We can calculate by utilizing this equation and rotor end winding resistance is this one. This resistance that also we can calculate with the same formula the rotor end winding thickness divided by winding conductivity into A r is nothing but area of the each bar into total number of slots. Here also same thing thermal conductivity of the aluminum bars we have to consider here.

Here, I presented for steel for aluminum. It will be different for steel. I presented here aluminum. It will be different. So, the winding thermal conductivity we have to select appropriately and same manner from rotor winding to this resistance we have calculated.

This resistance also we have calculated. So, from rotor winding end point to the end cap, this is the rotor winding end point and end cap through the air heat is transferred. So, by conduction, convection and radiation, there is no conduction thing in the air. We have assumed that through radiation as well as convection, it is happening in the axial direction. It is in axial direction.

In the radial direction, we have considered as a conduction manner. In axial direction, we are considering through convection as well as radiation. So, here area with respect to the rotor winding, we have to calculate in this fashion. Area is equal to rotor end winding number of end coils into perimeter of the coil into length of the coil. So, the perimeter of the end winding, we can calculate with respect to the thickness.

Let us consider this is the end winding. So, the thickness will be te and depth will be de. Then the perimeter will be 2 into te plus de. That is what we have calculated during the induction motor rotor core design and number of end rings with respect will be 2 in a squirrel cage induction motor rotor. If we will split into two parts, one side will be one end ring and end ring length will be mean diameter of the end ring is nothing but pi into d 1 plus d 2 by 2. So, with that thing, we can calculate the resistance with respect to the convection as well as resistance in this network.

Same way, the rotor winding thermal resistance is nothing but R theta RW g air gap. It is this one. So, the thickness of the rotor winding and number of bars or number of slots and mean width of the rotor winding into length will give the thermal resistance with respect to the rotor winding to gap and the thermal resistance offered by the air gap in radial direction in conduction manner. I am considering here that is l by lambda A thermal conductivity with respect to the air and area with respect to the air gap is nothing but pi Ds into le and thickness of the air gap is nothing but lg.

Same manner, we can calculate the thermal resistance for the stator winding in terms of stator copper area for total number of slots will be Qs into Ac, Ac copper area.

Then from stator winding to end cap in radiation as well as convection manner, we can see here through insulation like conductor has insulation, right. That insulation thermal conductive thermal resistance and radiative thermal resistance. Similarly, for convection manner convection thermal resistance plus insulating thermal resistance, the through insulation with respect to the conduction heat is transferred. These two terms are with respect to the conduction and these two terms are with respect to the insulation and convection and length of the end ring. We can calculate for stator winding arc length is nothing but this thing, right.

Let us say this is the total coil. So, this coil arc length is nothing but pi Dis by P plus 10 to 30 percent. We are considering the length which is coming outside the stator core. The actual core length will be this thing and the length which is extra portion, the straight portion of the conductors is nothing but 10 to 30 percent of the length and perimeter of the coil. We can calculate by considering the mean width of the winding as well as depth of the winding with respect to the stator slot.

So, the stator slot top side width is w 1 and bottom side width is w 2. Depth of the slot is dss. Then the perimeter is nothing but 2 into mean width of the stator slot plus height of the stator slot. Same way, we can calculate the stator winding to the stator teeth thermal resistance by considering the thermal resistance with respect to the copper, thermal resistance with respect to the insulating sheet that is mica sheet.

So, for mica sheet, it will be 0.6 watt per meter Kelvin thermal conductivity. For copper, it is 360 watt per meter Kelvin because of this thermal conductivity numbers. The heat flow in the stator winding in axial direction is significant as compared to the radial direction. Radial is in this direction compared to this thing. The axial direction of heat flow is significant. Then the resistance between the stator teeth to stator core and stator core to axial duct and stator core axial direction of thermal resistance and that is this one and stator core to axial duct in radial direction these two.

So, the first term stator teeth to stator core is this term in radial direction. Second term or second equation stator core to axial duct in radial direction, what is the thermal resistance and the thermal resistance with respect to the stator core in axial directions and thermal resistance from stator core point to the end cap. This is the end cap of the motor or end surface of the motor and this is the end point of the stator core.

So, what is the thermal resistance through the air in convection manner and radiation manner, but air gap resistance I have neglected here and these two are the resistances and from the axial duct to ambient axial duct to frame the thermal resistance. We can consider in radiation manner as well as convection manner and then frame to ambient also in a convection as well as radiation manner.

So, those equations we can see here in terms of convection and radiation. So, with respect to these equations, we can find the thermal resistances between the all 14 nodes. We can see here 14 nodes of the machine and other nodes with respect to the bearings and windage losses and at the end cap. So, these are the 14 nodes and thermal resistance and thermal network, we can see complete thermal network. So, from shaft to ambient, bottom side also same circuit will be there for with respect to the symmetry. I am representing the half portion of the machine from shaft to ambient in radial direction means in this manner and axial direction from shaft to end cap and rotor core to end cap, rotor teeth to end side support and rotor winding to end supports in both directions in all cases.

How the equivalent thermal circuit and by incorporating the thermal capacitances in the circuit, we can see the thermal equivalent circuit of the induction machine at various heat generating points with respect to the losses, stator losses, rotor losses and etcetera and these are the mechanical loss components. This is the complete thermal equivalent circuit realization of an induction machine. So, how to calculate the thermal capacitance? Again it will be the same equation. Capacitance is equals to density into specific heat capacity into volume of the core and for stator winding, it will be copper density into specific heat capacity of the copper into volume of the copper. This A into Qs represents the area and le represents the length of the stator winding and same way thermal capacitance with respect to the stator core node also we can calculate here and thermal capacitance with respect to the stator teeth node also we can see here.

Here ks represents the stacking factor. Generally, if you will incorporate the radial ducts, then this stacking factor will come down. Otherwise, we can consider always it is 1. The transient equation for thermal network for 14 number of thermal nodes will be this thing. The heat generation with respect to the losses at node 1 is nothing but Pi and Cth into capacitance into change in temperature plus the difference in temperature divided by thermal resistance will give the heat generation at various nodes. From this equation, we can calculate the temperatures at various nodes of the equivalent circuit of a machine.

Once we have the 14 equations and 14 unknowns, then we can identify the temperature. For attaining higher accuracy, higher number of thermal nodes has to be considered. This is another type of thermal equivalent circuit which is presented in this literature where the axial as well as radial direction of heat flow is considered and 16 thermal nodes are considered. Same way, in order to see the different thermal networks in various literatures, this is another literature where the thermal nodes are 10 nodes and 10 equations will be there. This is high accurate, accurate thermal equivalent circuit of induction machine where 107 thermal nodes are considered.

We can see here the stator frame itself consists of n number of nodes. Same way, stator yoke, stator winding also consists of different nodes. Like that 107 nodes are realized in an electrical machine and 107 nodes will come. So, 107 unknowns will be there and 107 equations we have to solve to get the different temperatures at these nodes. This is the respective literature for this network. Another type of thermal network, instead of representing the thermal network with respect to the individual resistances, we can represent the complete network as a lumped parameter thermal model that circuit with respect to the 10 nodes and 10 equations lumped model of electrical machines for totally enclosed fan cooling based design.

This is the circuit and the respective literature we can see here. The example related to heat distribution of induction machine, anyone is interested, they can go through this literature and they can see how to calculate the temperature at the various parts of the machine. With this, I am concluding this lecture. In this lecture, we have discussed the thermal equivalent circuit of an induction machine by considering the multiple nodes and also various thermal networks which are existed in the literature also we have seen. Thank you.