Course Name: Design of Electric Motors

Professor Name: Dr. Prathap Reddy B

Department Name: Electronic Systems Engineering

Institute Name: Indian Institute of Science Bengaluru

Week: 10

Lecture: 53

Title: Switched Reluctance Machine Sizing Equations-Output Power and Volume (D^2L) Product Equation

Greetings to all of you. In this lecture, we will discuss the design of switched reluctance machine with respect to given specifications like power, torque, speed and efficiency and electric loading or magnetic loading, etcetera. The first step to design the any type of electrical machine is to analyze the sizing equations and find the volume product for a given power rating, like the volume is nothing but in terms of bore diameter and length of the core. First, we will discuss the principle of switched reluctance machine. The working principle of switched reluctance machine is nothing but electromagnetic attractions and repulsions of magnetic fields. Let us consider a simple electromagnet.

Here we have a coil and we are placing a simple iron piece. This is the iron piece. So, depends upon the magnetic flux lines with respect to the electromagnet, the iron piece will attract towards the electromagnet, right. It is same as a permanent magnet and an iron piece.

This is the iron piece. This iron piece will attract towards the magnet, right, whether it is permanent magnet or electromagnet. The magnetic domains with respect to the iron piece will align in one particular direction and the induced magnetic poles with respect to the iron piece are exactly opposite to the source magnetic poles. So, here induced poles will be, you can see, this will be north and this will be south, same way here also, if this is south and north. For symmetry, we can consider here also, south and north.

Then this side induced poles in the iron piece will be like this manner. If we will see the principles of electromagnets, what we have discussed in the earlier lectures, the study one represents the switched reluctance machine. From this electromagnetic circuit principles only, the switched reluctance machine is realized. Here, let us consider the C type core and on top of C type core, a coil is placed and it is excited with a current I and it is fixed core. This one is fixed core and the bottom one is the moving I core.

With respect to the current in the coil, the magnetic field will induce in the core and it is in the direction of in this manner and the magnetic poles will be this side north and this side south. North to south, the magnetic flux lines are flowing. The induced magnetic poles in the moving I bar are simple iron pieces exactly opposite with respect to the source. We can see here, now there is an attraction of magnetic poles because these two are in opposite poles right north south and south north. So, there will be attraction and based upon the attraction, this will moving I bar will attract towards the C type core.

The same principle is applicable in switched reluctance machine and the torque depends upon the reluctance and torque equation and forth equation we have derived already. There is no second coil. The current in the second coil is equals to 0 and mutual inductance term also will not be there. This term and this term also will not be there. Only the reluctance term torque is equals to half I square d L by d theta.

We can see and if you are exciting multiple windings like let us say the circuit consist of multiple windings and if you are exciting all windings, then this mutual term and the reluctance term with respect to the other windings also we have to consider. If we will see the structural view of switched reluctance machine, it is shown here. The stator consist of 8 poles. 8 poles are at the stator side and windings are placed on 8 poles and a phase winding we can see the red color one and it is having a current I. We are exciting the a phase with current I and it consist of number of turns n.

Then the magnetic flux lines will flow in this fashion. If you will excite a automatically based upon the reluctance, the rotor will try to align with respect to the a phase winding that we can see in this figure or in the other words similar to the electromagnet. This iron pole that is rotor side, we have iron poles. There is no winding that is the advantage with respect to the switched reluctance machine. This nearest pole near to the excited stator winding will attract.

Based upon the reluctance also, it will move towards the excited winding. Next if you will excite the B phase winding, then nearest rotor pole will align towards the B phase. Then, the same way C phase like that based upon the reluctance principle as well as the magnetic attractions and repulsions, this mission will work. If you will see the flux linkage versus current wave forms for electromagnet, this is flux linkages psi versus current I. If the air gap is large, then we will see the flux linkage versus current wave form near to the x axis.

Here flux linkage psi is equals to l into I that is equals to n into phi as per the faraday's law and N into flux is nothing but flux density into area. So, l into I that is flux linkages, we can represent with psi or lambda both are fine. So, the flux linkage versus current wave form for larger air gap, we can see like this fashion and for smaller air gap, it will be near to the y axis and this is in the linear zone. If you will see the flux linkage versus current wave forms in the non-linear region, that will be in this fashion. Same way, if you will vary the air gap, the flux linkage versus current wave forms will come in this manner.

So, here the air gap is increased in this manner, top one having the less air gap and bottom one consist of larger air gap 1, 2, 3, 4, 5 and so on. lg with respect to the case 1 consist of small value and lg 2 and so on, lg 5. So, these curves are applicable for switched reluctance machine also, where the inductance or flux linkages are varying with respect to the rotor position and with respect to the current also. If you will see the flux linkage versus current wave forms for switched reluctance machine, it will be in this manner. So, we can see here, the flux linkages versus current wave forms for switched reluctance machine.

This is with respect to the higher air gap, that is nothing, but unaligned position, where the rotor pole is unaligned with the stator pole. This is the rotor pole. Here one more stator pole is there. The rotor pole is unaligned with respect to the stator pole, that is case 1. If rotor pole is exactly aligned with respect to the stator pole, this is the rotor pole, then this is case 2, where air gap value is small.

So, the flux linkage versus current wave forms with respect to the electromagnets are applicable and it is same as the same for switched reluctance machine also. The inductance with respect to the case 1 that is unaligned condition, is represented with L u that is inductance with respect to the unaligned position. The inductance with respect to the aligned position and with no saturation in this region. The inductance with respect to the aligned position and there is no saturation that is represented with L a u. And in this non-linear region or saturating region, the inductance will be maximum that is represented with l a s.

So, here flux linkage is always same l into i is equals to flux linkages and L a s is nothing but the maximum flux linkage to the maximum current we can define. The aligned and saturated inductance we can define as the ratio of flux linkages and current at maximum points. With respect to the inductance values like L a u sorry L a s and L a u and L a L u. So, this is unaligned induction inductance and this is aligned and unsaturated inductance and this is aligned and saturated inductance. So, from these three terms we can define the inductance ratios that is sigma s is equals to L a aligned inductance with respect to the saturation divided by aligned inductance with respect to the unsaturated condition.

And sigma u with respect to the unsaturated inductance is nothing but L a that is inductance with respect to the unsaturated condition to inductance at the unaligned condition.

So, at the unaligned condition the inductance value is minimum at the aligned and saturated condition the inductance value is maximum. The area under this curve represent the developed magnetic or developed mechanical energy with respect to one particular phase excitation or with respect to one particular stroke. In switched reluctance machine we are exciting only one particular phase in each and every instant. The inductance versus theta waveforms or angular position of rotor if you will draw, then the inductance at the unaligned position will be minimum where the rotor pole is exactly unaligned with respect to the stator pole.

And after this point with respect to the excitation the rotor pole will try to align with respect to the rotor pole. The moment the alignment or overlapping starts then the inductance will rise linearly and once it is completely aligned then the inductance will be constant. After that inductance will come down with respect to the excitation de-energization of the winding like this fashion here l maximum that is l's aligned condition. But saturated condition the inductance with respect to the aligned position will be higher as compared to this l max. And same way this inductance will waveform will vary symmetrically.

And torque waveform if you will observe for switched reluctance machine each and every condition we are exciting only one particular phase. So, the torque developed by one particular phase is this one like one phase to other phase the torque shifting is happening because of that reason huge ripple we will see during the transition of from one phase to other phase this is the transition. This is the one drawback with respect to the switched reluctance machine we are exciting only one particular phase at a time that results in discontinuity in torque. We can see here a phase is excited for an angle of theta and b phase also excited in the next instant and c phase is excited next instant and D phase is excited simultaneously at one after another. So, at one particular instant only one phase is conducting that means, torque developed by the machine is with respect to one phase only.

Next the torque will shift from this a phase to b phase like torque shifting is happening from one phase to other phase because of that reason we can see the discontinuity in torque. So, this is the basic principle with respect to the switched reluctance machine. Now, we will discuss the sizing equations of the switched reluctance machine. The sizing equations for any type of machine will give the volume product with respect to the bore diameter. D I s is the bore diameter with respect to the air gap or inner diameter of the stator we can consider and l e is the length of the stator core.

So, this product will give the volume for a given power rating and the torque developed by the machine with respect to the SRM is directly related to the or it is equals to I square by 2 into d L by d theta. This is with respect to the electrical quantities like change in inductance with respect to the rotor position and current and this torque is also dependent on the iron component that is with respect to magnetic loading similar to the induction machines. So, the magnetic loading is nothing but b g flux density at the air gap and it depends upon the saturation and type of material and losses and also type of coolant and inductance variation. Although, these parameters will limit the flux density at the air gap and the flux density at the air gap value is in terms of Weber per meter square or Tesla. Here, B g is equals to flux per unit area and the torque developed by the machine is also proportional to the amount of copper we are utilizing.

So, the amount of iron and amount of copper we are utilizing to generate the required torque and power will directly related. The amount of copper represents the electric loading. Electric loading is nothing but surface current density or ampere conductors per unit length. Ampere conductors is nothing but z into I divided by length. In some cases, we can represent mmf by length also.

mmf is nothing but n phase into I. So, here instead of N phase, we can select 2 into N phase. So, with respect to the mmf, we can represent the electric loading 2 into mmf divided by length also. Again, this electric loading depends upon the I square or losses and type of winding and coolant or thermal system, what we are utilizing for the machine and skin effects and proximity effects and other limitations, which are limiting the electric loading value, we have to consider appropriately to achieve the required torque. Now, let us consider a switched reluctance machine. This one, which is having a length core length of l e and inner diameter of the stator is D I s.

This value is D I s inner diameter of the stator and inner diameter of the rotor. This is inner diameter of the rotor and outer diameter of the stator will be this thing. So, the volume product is nothing but D I s square into l e. This volume product, we have to find with respect to the required power rating or given power rating. For this, let us consider the switched reluctance machine.

This is the machine input power. We are giving p in and output power mechanical power that is p mechanical is equals to 2 pi speed of the rotor into developed torque divided by 60. This is the standard equation with respect to the mechanical power and the input power of the switched reluctance machine. We have to define with respect to the duty cycle. It is not a AC machine like in terms of power factor and phase RMS voltages and other things. It is a type of machine, where the excitation current pulse is defined with respect to the duty cycle similar to the DC-DC converters or DC machines control, but it is not a type of DC machine.

In between, losses will be there in the system like iron losses, copper losses and mechanical losses. The losses are iron losses in both stator and rotor and copper losses with respect to the stator only. That is the advantage with respect to the switched reluctance machine and it will eventually, increase the efficiency and mechanical losses. Mechanical losses will be slightly higher because of the higher torque ripples and the input power P in is equals to the number of phases conducting simultaneously. Here, only one phase is conducting at a time, then m is equals to 1 and if two phases are conducting at a time, then m is equals to 2.

Here, the number of phases is nothing but number of poles by 2. We can see here, 8 poles are there at the stator side. If you will place the a phase winding and 2 poles, then total number of phases will be 4 only. m into v phase peak value, then I phase peak value into duty cycle. This duty cycle will represent the duration of current pulse or excitation pulse with respect to the windings.

Let us say, a phase winding were exciting for a duration of theta 1. Next, b phase winding were exciting after theta 1 from theta 1 to theta 2 duration. Next, c phase were exciting in this fashion. So, the duty cycle represents with respect to the excitation pulse. This is the input power equation and efficiency of any system, we can represent in terms of output power and input power or output power divided by output power plus losses.

So, with respect to this equation, second equation, output power is equals to efficiency into input power. Here, efficiency into m into v phase peak, this phase voltage, peak value, phase current peak value into duty cycle. Here, the v phase peak, we have to represent in terms of magnetic loading and I phase peak, we have to represent in terms of electric loading. Then, we can get the final power equation in terms of magnetic loading and electric loading. In that process, we will get the other term, D square le term also, that is, volume product.

First, we will find the voltage per phase equation as per the Faraday's law. For a simple RL circuit, apply a KVL and the voltage equation is equals to I into r plus d lambda by dt, that is, flux linkages either with respect to lambda or psi. Any parameter, we can utilize it here for representing the flux linkages. If we neglect the I into r drop, it is a small one and for ideal system, I am neglecting this drop and v phase peak is equals to d lambda by dt, the change in flux linkages with respect to the change in time. So, the change in flux linkages is equals to delta lambda.

The flux linkages with respect to the inductance terms like lambda is equals to L into I. So, if I is equals to constant, then inductance is varying from the aligned position to unaligned position, that is, nothing but L into I. So, with respect to the aligned position, maximum inductance with respect to the saturation minus unaligned position, that is, minimum inductance into current. This is the change in flux linkages equation.

This is equation number 3 and equation number 4. Similarly, the change in time delta t is nothing but time is equals to distance by speed. Here, how much distance rotor is moved? Let us consider the image of stator. You can see here, this is the rotor, a stator pole arc, the blue color one. So, wherever it may be the rotor, if I will excite the a phase winding, rotor will move this much length from aligned position, unaligned position to aligned position. So, rotor is moving from this position to this position.

This is unaligned position and it is aligned position. So, the distance traveled or distance the rotor is rotated in the radiance is nothing but stator angle with respect to the pole arc, that is, beta s divided by the beta s is in radians. You can see here, this is the beta s value, the arc length with respect to the angle b s. I am not representing the arc length directly. The distance with respect to the radiance, I am representing as beta s, beta s divided by speed in terms of radiance per second, that is, speed of the rotor omega m. So, the change in time also we have calculated and change in flux linkages also we have calculated.

From equation 4 and 5, we can get the voltage equation b is equals to L a inductance with respect to the aligned position and saturated value, saturated condition and inductance with respect to the unaligned condition into current divided by beta s by omega m. This is time and numerator is change in flux linkages. Here we have already discussed the flux like inductances with respect to the saturated and unsaturated terms, that ratios are we can see here. I am just copying this inductance ratios with respect to the saturated and unsaturated conditions. We will utilize these two equations to represent the voltage equation in the form of inductance ratios.

Use this equation. This is equation number 7, 6 and equation number 7. If we will substitute equation number 7 in equation number 6, then voltage is equals to omega m divided by speed of the rotor divided by angle with respect to the stator pole arc into 1 minus L u by L a s with respect to the unaligned position to the aligned position inductance into current into L a s. I have taken inductance with respect to the aligned position and saturated value taken out. Now, if we will multiply these two terms, then we will get the sigma s into sigma u is equals to L a s, the inductance with respect to the aligned position and saturated value and unaligned position. So, if we will substitute that term here, sigma s into sigma u into L a s inductance with respect to the saturated and aligned condition into current and remaining terms will be as it is, omega m by beta s.

We have to represent the V phase peak value in terms of magnetic loading. So, L into I as per the Faraday's law, L into I is equals to n into phi. We have to represent V phase peak in terms of flux density at the air gap. For that, we will utilize L into I is equals to N into phi relation and phi is nothing but flux at the air gap that is B g into A g with respect to one pole. The area cross section of area with respect to one particular pole, this is the rotor pole and here flux lines are flowing in this direction and the perpendicular area with respect to these flux lines.

So, we can see here, this yellow color area with respect to the perpendicular to the flux lines, we have to calculate that is nothing but A g. So, in order to calculate that thing from the center point, this angle is nothing but beta s and radius is nothing but D by 2. So, arc length into y axis length is nothing but core length that is l e, x axis length is nothing but arc length. So, this length we are considering and y axis length is nothing but l e, n into flux density at the air gap, N is nothing but number of turns per phase, A g is nothing but arc length that is r into theta.

Here, r is nothing but d by 2 and theta is nothing but beta s. So, beta s into D I s by 2 into length of the core. So, this is nothing but L a inductance with respect to the aligned condition and saturated position or saturated region into I is equals to this one. Start this equation in the voltage equation, then final voltage V phase peak value is equals to omega m that is rotor speed divided by angular speed divided by beta s that is angle with respect to the stator pole arc 1 minus 1 divided by inductance ratio product into n phase into B g flux density at the air gap into beta s that is angle with respect to the stator pole arc into D I s by 2 into l e. So, this angle will cancel each other and the final voltage equation we can see here. V phase peak is equals to omega m into 1 minus the constant terms will come as it is N phase number of turns per phase flux density at the air gap bore diameter or stator inner diameter into le.

This is final equation of voltage equation number 9. Now, we will find the I phase peak with respect to the electric loading. Electric loading is equals to ampere conductors per unit length z into I divided by length or surface current density. The length with respect to the stator inner diameter is nothing but pi into d I s and total number of conductors are nothing but n phase is the total number of turns per phase into 2 will give the total conductors into how many number of phases are conducting simultaneously that is m and I phase peak value. This term represents the total number of conductors and this term represents the current and the denominator term represents the length.

So, this is equation number 10. If you will substitute the equation number 9 and 10 in the power equation that is this one that is equation number 2 a. If you will substitute here, this is 2 a in equation number 2 a, then power is equals to efficiency into duty cycle into omega m b phase peak. I am substituting here into 1 minus 1 by inductance ratio product sigma s into sigma u into n phase into B g D I s by 2 into l e and I phase peak is equals to what electric loading into pi D I s by 2 into n phase into m and in the power equation m also will be there in the equation 2 a m into efficiency into v phase peak I phase peak into duty cycle. So, this number of phases conducting simultaneously will cancel each other and number of turns also will cancel.

Now, in this equation, we have to represent the angular speed in terms of rpm. This is revolution radians per second and we have to represent the rotor speed in rpm. If we will do the conversion, then we can get the omega r is equals to radians per second to rpm. If we will convert omega m is equals to 2 pi N r by 60, substitute this omega m value in the above power equation that is equation number 11, equation number 12. So, equation number 12 in equation number 11, then final power equation p naught is equals to efficiency into duty cycle into omega m. We are replacing with speed of rotor speed is 2 pi Nr by 60 into 1 by 1 minus the inductance ratio product into v g into D I s by 2 into l e electric loading by 2 into pi D I s.

These are the terms with respect to the equation number 11. Here, these two terms will cancel 2 2 and final equation is efficiency into duty cycle into pi square by 120 into 1 minus 1 by inductance ratio product inductance ratio terms product into v g into v g is nothing but magnetic loading into electric loading into D I s square that is bore diameter or straighter inner diameter square into le. This term represents the volume into speed of the rotor. So, this is the final equation of the power. Here, the first term represents the efficiency, then duty cycle. These are the constant terms and this term represents the magnetic loading as well as electric loading and this is the volume product.

So, for a given power rating in order to generate this much of power, how much volume of the machine is required with respect to the bore diameter? This is inner diameter of the straighter. If total volume requires outer diameter of the straighter, we have to consider that is D I s plus the slot and back iron thickness. We have to consider, let us say d s s is the slot depth or pole height and d c s is the back iron depth. Then, this thing will give the outer diameter of the straighter. So, D naught s square into le will give the volume with respect to the outer diameter.

Generally, for any type of machine, we will consider the volume product with respect to the bore diameter and length of the core. This is the final power equation, equation number 13 and p naught is equals to output coefficient into magnetic loading into electric loading into volume product into speed. This is the final power equation with respect to the output coefficient. Here, output coefficient c naught is equals to efficiency into duty cycle and other constant pi square by 120 into 1 minus the inductance ratio coefficients.

This is sigma s sigma u into sigma s everywhere. So, this is the output coefficient. So, from the equation 13 and 14, we can see that output power is directly proportional to the electric loading, but electric loading depends upon the iron losses and winding type and type of coolant and saturation effects and change in variation and etcetera. The electric loading, we have to select appropriately in the range of 25000 to 90000 ampere per ampere tons per meter because each phase is delivering or generating a torque with respect to the individual phases like at a time, only one phase is excited. Because of that reason, electric loading with respect to the machine or current carrying capability of the winding, we have to select more in order to generate the more torque.

Then, power is directly related to the magnetic loading also that is B g value. It depends upon the saturation and inductance variation and type of coolants and losses and type of materials and etcetera. So, the flux density at the air gap, we have to select in the range 0.7 tesla to 1.2 tesla. Similarly, the output power is directly related to the speed of the rotor and volume product.

So, if you will increase the volume product, automatically power also will increase. So, then it is directly related to the duty cycle like excitation pulse or excitation depends upon the excitation current also. The output power will vary and it is directly related to the efficiency also. In order to decouple the D square l products, volume product, how to select the aspect ratio that is l e by D I s and how to select the outer diameter and length of the core, we will discuss in the next class.

With this, I am concluding this lecture. In this lecture, we have discussed the principle of switched reluctance machine and sizing equations in terms of volume product with respect to a given power rating. Thank you.