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Title: Stator MMF and Magnetizing Current Equations of Induction Machine

Greetings to all, in the last lecture we have discussed the Carter coefficients and how to find the effective length of air gap as well as some empirical formulas to find the effective length of a core, we have discussed. In this lecture, we will discuss the magnetizing current equations with respect to the MMF drops at the various parts of the machine. If we will see here, for a three phase machine with the balanced three phase winding and the winding is symmetrical and we have derived the MMF equation, MMF is equals to N into I. The fundamental stator MMF Fs1 is equals to N into I, here N is the number of turns per phase and the constants Kp1 is nothing but pitch factor, Kd1 is nothing but distribution factor, Ksq1 is nothing but skewing factor, like rotor skewing thing we have discussed already and K psi 1 is nothing but slot opening factor. Here, depends upon the number of turns per phase and the current we are calculating the peak fundamental stator MMF.

Now, we will calculate the MMF based upon the different drops at the various parts of the machine, like a core at this particular point, what is the MMF drop, at the teeth what is the MMF drop, at the air gap and then in the rotor teeth and then in the rotor core.

If we will see the flux lines in a magnetic circuits are passing through a two stator teeth and two rotor teeth and two air gaps, a portion of stator core like this one, this is nothing but a portion of stator core and this is a portion of rotor core, but here in between two teeth are covered at the stator side and two teeth are covered at the rotor side and two air gaps also. Based on this flux path, if we will apply a ampere's law, the peak fundamental MMF is nothing but drop with respect to the teeth, drop with respect to the stator teeth and drop with respect to the rotor teeth and MMF drop with respect to the air gap and MMF drop with respect to the core and MMF drop with respect to the rotor core, here Fcr. By calculating all these five types of MMF drops, we can find the peak fundamental MMF. So, by equating the equation 1 and equation 2, we can find the magnetizing current required to establish this much MMF at the air gap.

Now, we will discuss what is the reluctance equation and MMF drop with respect to the air gap. In general, if we will see the flux density waveform will be in a sinusoidal nature with respect to the pole axis, it will be cosine side and with respect to the inter pole axis, it may be sinusoidal. Let us consider the cosine side waveform. In general, for a ideal machines where there is no saturation like in iron parts, there is no saturation means we can see the exact sinusoidal waveform of the flux density that is BG 1 waveform, we can see sinusoidal, but most of the machines will exhibit the non-linear or saturation at the iron parts. Because of that reason, the tapped red color waveform we can observe here that is the actual flux density waveform at the air gap.

This flux density waveform consists of all type of harmonics like fundamental third, fifth, seventh respectively. The dominant harmonics will be third, fifth, seventh. If we will see for a p number of poles with respect to the spatial distribution, the flux density equation we can write it like B equals to Bg1 into cos theta. Here, Bg1 is the peak fundamental flux density at the air gap and P theta by 2 is nothing but an electrical angle. Generally, theta is equals to theta electrical is equals to theta into P by 2.

That is what is written here. The mechanical angle is nothing but theta and P by 2, if we will multiply, then we will get the electrical angle. Suppose, if I take the crossover points, this is point 1 and this is point 2, 30 degrees leading with respect to the reference axis, 30 degrees lagging with respect to the reference axis, we can see the maximum points of the flux densities. Let us consider the flux density value at 30 degrees with respect to the BG 1 value. The flux density at 30 degrees is equals to Bg1 is the peak flux density at the air gap into cos 30 degrees.

If we will incorporate the third harmonic also, there is no effect or there is no change. The Bg value at 30 degrees is equals to Bg 1 into cos 30 only, because cos 90 value is equals to 0. This term will be cancelled. Only Bg is equals to Bg1 into cos theta electrical. This is the equation to find the magnetic flux density at the air gap with respect to the non-linear flux waveform like core is saturated slightly.

So, the MMF is necessary to overcome the air gap reluctance is nothing but f is equals to H into l. So, from here, magnetic field intensity H is nothing but B by mu. We have calculated the maximum flux density at the 30 degrees already, that is the value of magnetic flux density and mu naught is the permeability of the free space into l. It will give the MMF drop at the air gap. In order to find the air gap effective length, in the last lectures, we have discussed the Carter coefficient based effective length of air gap equation.

Here, Lge is nothing but effective length of air gap, Kcs is the Carter coefficient with respect to the stator and Kcr is nothing but Carter coefficient with respect to the rotor side and Lge is the actual length of air gap. So, we can use either this equation, equation 1 or equation 2 to find the Carter coefficient. The air gap reluctance refers to one stator tooth. Generally, reluctance is equals to l by mu a. Based on that equation, effective length is now Lge and reluctance is equals to l by mu naught into a.

Here, area of the 1 stator tooth is equals to tau s into l e. Tau s is nothing but slot pitch in mm and l e is nothing but length of the core. Effective length of a core, we have to calculate based upon the empirical formulas, what we have discussed earlier. Similarly, with the air gap reluctance referred to one rotor tooth is nothing but simply replace the stator slot pitch with rotor slot pitch, then we will get the air gap reluctance referred to one rotor tooth. So, with this, we can conclude that how to calculate the MMF drop at the air gap and reluctance with respect to the air gap.

Next, we will calculate the MMF drops as well as reluctance term with respect to the tooth or multiple teeth. If you will see here, the blue color dotted points, the first one is nothing but Bg average, the second one at the top of the tooth is nothing but B top and at the middle, it is B mid and at the bottom side of the stator tooth is nothing but B root. Similar same kind of analysis, we can do it for rotor tooth also. If we will see the flux with respect to the one slot pitch at the air gap, flux is equals to B into a and Bg average, we know already, we can take the maximum flux density value at the air gap will be Bg1 at 30 degrees, we have seen that thing and area is nothing but slot pitch into length. If there is no saturation, the flux over one slot pitch will completely enter to the teeth.

The pink color lines, we can see here, if there is no saturation, all flux lines will flow through the stator tooth and the tooth flux, if we will calculate it again, flux is equals to B into A. The tooth single tooth area, how we can calculate is t into t is the width of the one stator tooth at the top side into Ks is the stacking factor into le. This will give the area with respect to one stator tooth at the top side position. Next, the flux with respect to the air gap and with respect to the top side of the tooth, if we will consider same under the condition, where there is no saturation, then this equation, we can rewrite it based upon the equation 9 and 10.

Then, the flux density at the top side of the tooth is nothing but this value B top in terms of Bg average will give the equation number 11.

Bt top is equals to Bg average into slot pitch into core length divided by ks is the stacking factor into tt is nothing but width of the stator tooth and le. So, finally, we can get the equation with respect to the B top and Bg average flux density at the tooth top and flux density with respect to the air gap relation. We can get it based upon the equation number 11. Now, we will calculate the flux at the middle point of the tooth. Flux at the middle point of the tooth also we can calculate it based upon the equation flux is equals to B into A.

The area of the tooth at the middle is equals to t m into ks into le, t m is the width of the stator tooth at the middle portion of the tooth, t m is equals to t t plus t r by 2, t t is nothing but the width of the stator tooth at the top side of the stator tooth and t r is nothing but at the bottom side of the stator tooth, this one and this one. If you know the width at the top side of the tooth and bottom side of the tooth, in average we can take it to find the tooth width at the middle portion of the tooth. Next the flux at the bottom side of the tooth also based upon the same equation we can calculate it, but the area of the tooth at the bottom side is nothing but A root. A root is equals to t r into ks into le, this one. If we consider there is no saturation thing, the flux entering from the air gap and flowing through the B t top like at the top portion and then middle portion and then bottom or root portion, then linking to the rotor side.

This flux will be same in the entire path from the air gap, then at the top side of the tooth and the middle side of the tooth and then bottom side of the tooth that is same. Flux will be same in the entire path. The flux density is varying with respect to the areas of the different portions of the tooth. If you want to relate or to calculate the flux densities at the different parts of the tooth like B root and B mid with respect to the flux density at the air gap or flux density at the top value, we can get it based upon the equation 14 and equation 15.

Here, by equating the equation number 11 and 12, equation number 10, sorry, by equating equation number 10 equals to 12, then we will get the B root that is flux density at the bottom side of the tooth is equals to flux density at the top side of the tooth into the width factor.

Similarly, by equating equation number 10 is equals to equation number 13, then we will get the relation with respect to the flux density at the middle side of the tooth to flux density at the bottom side of the tooth. So, we can represent the flux density at the top of the tooth and the bottom of the tooth and middle of the tooth by utilizing these equations.

All these flux densities, we can calculate based upon the flux density at the air gap. This is the maximum value of the flux density at the air gap, where the flatted flux density waveform will get the maximum value and there is no third harder harmonic with respect to the 30 degree position. Now, in order to calculate the MMF drop, we require the flux intensity.

So, to find the flux intensity thing, we require the permeability. If the magnetic circuit is linear and there is no saturation, then mu r value we can consider depends upon the permeability of the material like for iron, different type of value and for steel or some other material, we will get the different value of BH curves. If it is linear, then we will see the BH curve in this manner, where mu is constant, but practical machine exhibits the saturation. So, the permeability values will be estimated based upon the BH curves of that particular material. If we will see the BH curves for different materials, in the next slides, we will see.

We can see here the BH curve with respect to the different materials. This is with respect to the 29 gauge M27 fully processed steel material BH curve and here with respect to the cast iron, cold rolled steel and cast steel and sheet steel also, we can see different BH curves. We can observe from this slide based upon the value of B. Let us say B top, we are getting 60 kilo lines per square inch. Then if we will consider, it is cast steel.

Cast steel will be this one. So, according to that, we have to consider the H value for top portion of the stator tooth and bottom portion of the stator tooth and middle portion of the stator tooth. That means, H top, H middle and H root. These three values we have to calculate based upon the BH curves of the material. Once we know the different values of the H magnetic flux intensity at the various parts of the iron or tooth like at the bottom middle top, we have to find the average value of the flux intensity over a period. So, in order to find the flux intensity average value, the Simpson rule, we can utilize it.

The Simpson rule will give the approximation of the integral of a function f of x over the measure of x. The rule is nothing but this one Simpson rule, where the integral a to b f of x dx is equals to delta x by 3 into f of x naught plus 4 into f of x 1 2 into f of x 2 plus and so on. Here, delta x is nothing but the limits b minus a by n and n is the number of points or number of instants to find the average of the different values of the function and x i is nothing but lower limit plus i into delta x. Why we are utilizing this rule means to find the average flux density with respect to the complete tooth.

So, this is the Simpson rule. Here, the function is nothing but now flux intensity with respect to the length and dx is equals to dl and lower limits a equals to 0 and ds is the depth of the slot and n equals to 2. We are considering two instants like three instants at the bottom, at the H mid and then H root. This is H top we can consider and then x naught is equals to 0 based upon this equation i equals to 0 and substitute all these values and then we can get the flux intensity average value is equals to 1 by ds period and 0 to ds is nothing but the limits and ds by 2 is nothing but delta x and 1 by 3 as it is will be there and f of x naught is nothing but h t of 0 and f of x 1 is nothing but H t of ds by 2 and f of x 2 is nothing but H t of ds with respect to the slot depth or tooth depth. So, if you will replace this H t of 0 and H t of ds by 2 and H t of ds, these three functions with h top that is h field intensity at the top side of the stator tooth and the flux intensity at the middle portion of the stator tooth and flux intensity at the bottom side of the stator tooth, then flux intensity average value we can get it by utilizing the equation number 19. Once we know the magnetic field intensity, we can find the mmf drop.

mmf is nothing but H into l. Here, H t average we have calculated based upon the Simpson rule and ds is nothing but length that is slot depth or tooth depth is equals to one sixth of the same thing H t top plus 4 into flux intensity at the middle portion of the tooth 2 into flux intensity at the bottom portion of the tooth into l. So, based on this equation, we can calculate the mmf drop at the stator tooth for the rotor also. Here, mistake is there. It is rotor.

So, ds r is presented here. This is with the equation number 20 is with respect to the one stator tooth and equation 21 is with respect to one rotor tooth. So, here flux intensity at the bottom side of the rotor tooth and middle portion of the rotor tooth and top portion of the rotor tooth we have to consider by utilizing the BH curves and substitute in the Simpson rule and here ds r is the depth of the one rotor tooth. Based on these two equations, we can calculate the mmf drop at the stator tooth and mmf drop at the rotor tooth. Now, we will calculate the reluctance terms with respect to the stator and rotor. So, reluctance is nothing but mmf per flux.

So, we know the flux value at the tooth and we know the mmf drop at the tooth, f t also we know and flux also we know. Based on that, we can calculate the reluctance. So, the flux and reluctance term with respect to the stator tooth are this one with respect to the rotor tooth is this one. 24 and 25 will give the stator tooth flux and reluctance values and 20s equation number 26 and 27 will give the tooth flux and reluctance with respect to the rotor core. Here, flux at the stator tooth is equals to B into A and that is equals to Bg at 30 degrees into air gap area with respect to the one slot pitch.

From this equation, we can get the flux value and from this earlier equation phi t s average, we can find it from the earlier equation. Then, we will get the reluctance terms. After knowing the reluctance and mmf drop at the tooth, now we will calculate the reluctance and mmf drops at the core side because as of now we have discussed the mmf drop at the air gap and mmf drop at the tooth portions of the stator as well as tooth portions of the rotor. Now, we will calculate the mmf drop at the core portion. So, the flux lines are flowing in the core here as well as here rotor core.

So, by applying it, let us consider the three phase distributed winding and it is forming a four pole based upon the thumb rule for each and every conductor we can see here. If we will see here, the flux density at the air gap and flux density at the core, this is the flux density at the core and this is the value flux density at the air gap. Both are 90 degree phase shifted and if we will consider the Bg value is cosine sidal and the flux density at the core will be sinusoidal. So, the flux density equation with respect to the fundamental Bg of theta is equals to peak flux density value that is Bg 1 into cos theta electrical that is p into theta by 2 and core flux density is nothing but Bc of t theta. Bc 1 is the peak core flux density into sin theta electrical.

From this figure, if we can observe that at an angle theta is E equals to 0, the flux density at the air gap will be maximum, this portion and after 90 degrees, the flux density at the air gap is coming down to 0 and flux density at the core will be maximum. So, flux density at the core, we can calculate with respect to the integral of B into r l theta d theta, where r represents the radius from the rotor center line to the cylindrical surface defined by the midpoint of air gap that is Dis by 2, r is equals to Dis by 2 and L e is the length of the core. B into A, this equation that t is nothing but phi is equals to integral of B A d theta. Based on that equation, we can rewrite this thing flux value with respect to the core. So, phi c is equals to Bg into sin theta integration of cos will be sin into r l and 2 by p also will come by solving the equation number 30.

Now, we will rearrange the terms theta E equals to electrical P into theta by 2 at 90 degrees, it will be like this Bg1 into sin 90 r into le into 2 by P and flux in the core with respect to the 90 degree point, we are calculating that is this one. If we will rearrange the terms and if we will write in terms of pole pitch that is tau p pi into Dis by 2 and r is equals to Dis by 2. If we will see based upon this equations, if we will rewrite the flux in the core at 90 degrees, then we will get this equation. Here, we have to find the flux at the different parts of the core and then we will calculate the flux densities at the different parts of the core and followed by flux intensity at the different parts of the core. Now, the flux value at the 90 degree angle, we have calculated that is equals to this one Bc is equals to phi c is equals to Bc at 90 degree into dc is nothing, but back iron depth.

This length is nothing, but dc into k s into le. The flux density at 90 degrees is equals to we have calculated already that is nothing, but 2 by pi Bg1 into tau p by 2 into 1 by dc into k s. Then, flux density at an angle 60 degrees is equals to this one flux density at an angle equals to Bc at 90 degrees into cos 30. We know the flux density at angle 90 degrees, flux density at an angle 60 degrees, flux density at an angle 30 degrees, and then 90 degrees. This is Bc of flux density at 30 degrees, flux density at 60 degrees, this value is flux density at 90 degrees.

All these three instants we have calculated. For this instance, what will be the average flux intensity? We will calculate based upon the B-H curves. So, B equals to mu into H. If it is linear, then we can consider directly some relative permeability value and then, we can find the flux density. So, for practical machines, we have to see the B-H curves. Based upon the B-H curves, we have to select the mu value and we have to find the H value, flux intensity at 90 degrees, flux intensity at 60 degrees, flux intensity value at 30 degrees.

After knowing the flux intensities at the various points, H c at 90 degrees, 60 degrees and 30 degrees and so on, we have to calculate the average flux intensity. Again, we will find the average flux intensity by utilizing the Simpson rule, that is equation number 36. That Simpson rule will give the average flux intensity of a core with respect to the different points of the flux densities. So, here function will be H c of l, f of x is equals to flux intensity, where the variable will be length of a core and then, d x equals to d l and we are varying the limits from 30 degrees to 150 degrees. The number of instants or points have been considered 4.

It depends upon that thing. We can see H c at 0, H c at 30 degrees, H c at 60 degrees, 90 degrees, 120 degrees and so on. Substitute all these values in the equation number 36. Then, we can get the H c average. 1 by l c is the period integral 0 to l c H c of flux intensity function into d l. Now, the average flux intensity with respect to the core by utilizing the Simpson rule will be this one, that is H c is equals to 1 by 12 H c of 30 degrees, H c of 60 degrees, H c of 90 degrees and then, H c of 120 and then, flux intensity at 150 degrees.

For a sinusoidal function, if we will consider at 150 degrees and 30 degrees, both are same and 120 degrees and 60 degrees, both are same. So, it depends upon that thing, if we will rewrite the equation number 38. So, H c 30 degrees and flux intensity at 150 degrees will be same. So, 3 into H c 30 degrees will come. Based on that thing, we can rewrite the final equation, one-sixth of flux intensity at 30 degrees, two-third of flux intensity at 60 degrees, one-sixth of flux intensity at 90 degrees.

It may be 4 because 3 into flux intensity at 30 degrees divided by 12. So, it will be 1 by 4 H c at 30 degrees. So, equation number 39 will give the flux intensity value of the core with respect to the multiple points. Now, in order to calculate the MMF drop at the core, we have to find the mean length of a core. Let us consider the mean length of a core over one pole pitch is assumed as two-third of the actual length.

If we will consider the actual length of a core with respect to one pole pitch will be pi ds by P plus 2 ds plus 2 dc by P, we are considering. These two terms are coming with respect to, we are calculating the mean length of a core with respect to the midpoint of the back iron. That midpoint of the back iron will come. This is dc and ds will be the slot depth and ds will be the inner diameter of the stator. By considering the mean value, we will get here length of the stator core is equals to two-third of pi into ds plus 2 into slot depth plus 2 into back iron depth that is with respect to the stator and length of the back iron with respect to the rotor core will be two-third of pi into d naught r outer diameter of the rotor minus 2 into ds r minus 2 into dc r by P.

So, by utilizing equation number 40 and 41, we can calculate the length path through the stator and rotor core over one pole pitch. Here, the assumption factor we have considered two-third and outer diameter of the rotor core, we know that is inner diameter of the stator minus 2 into air gap length that will give the outer diameter of the rotor. So, after knowing the flux intensity values at the core and length of a core, we can calculate the MMF. MMF is equals to H into l and Hcs average we have calculated based upon the Simpson rule and then this term is nothing but length of the stator core. Similarly, flux intensity average at the rotor core and length of the rotor core, we have calculated already based upon that thing, we can find the MMF drop with respect to the rotor core.

Once we know the MMF's and flux values, we will calculate it based upon the MMF's and flux values, we can find the reluctance of the stator core as well as rotor core. Reluctance of the stator core is equals to MMF drop with respect to the stator core divided by flux in the stator core back iron thing. And then reluctance at the rotor core back iron is equals to MMF drop with respect to the rotor core divided by MMF drop, sorry flux with respect to the rotor core. Here, flux we can calculate it based upon the equation b into a. Here, B value is considered as a peak that is why 2 by pi is there, phi c at 90 degrees equals to this one.

So, with this we have calculated the flux MMF drop at the air gap and MMF drop at the stator teeth, MMF drop at the rotor teeth, MMF drop at the stator core and MMF drop at the rotor core. All these 5 variables are known thing. Based upon that thing, the MMF with respect to the stator side per pole at 30 degrees in order to establish the air gap fields is nothing but Fs1 of 30 degrees. In order to find the peak value of the MMF per pole required to establish the air gap flux density Bg1 is nothing but Fs1 is equals to Fs1 of theta is a function that is equals to Fs1 into cos theta electrical.

So, theta electrical is equals to P into theta by 2. So, based on that thing, the Fs1 at an angle 0 is nothing but MMF required to establish the maximum value of the air gap flux density. So, after knowing the MMF value that is Fs1 at an angle 0 to establish the maximum flux density value at the air gap, we know the MMF value with respect to the number of turns and currents by equating this equation A and equation B. Then we can calculate the magnetizing current. The magnetizing current is nothing but MMF by number of turns. MMF we have calculated based upon the all drops at various parts of the machine and then divided by number of turns with respect to the winding type that is 3 by 2 4 by pi into 4 by pi number of turns per pole into winding factors.

These winding factors pitch factor, distribution factor and skewing factor and slot opening factors we can calculate it based upon these equations. Here, the slot opening factor k chi 1 is equals to sin chi by 2 divided by chi by 2. So, chi is equals to slot opening B0 s divided by slot tau p it is a pole pitch into 180 degrees. So, with this I am concluding this lecture.

In this lecture, we have discussed the equations to find the MMF drops at the various parts of the machine based upon the MMF drops, how to calculate the magnetizing current we have discussed. Thank you.