Course Name: Design of Electric Motors

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Title: Carter's Coefficient of Electrical Machines

Greetings to all, with respect to the generalized procedure to design the any type of electrical machine, we have discussed as of now, the initial design equations and then sizing equations with respect to the D square l and D cube l products and then, stator core design and rotor core design. First three steps we have completed. The next thing will be performance parameters. So, how to analyze the performance parameters to see the machine behavior with respect to the magnetizing currents? You can see here, the magnetizing currents and no load currents and full load currents, how to calculate it and MMF's, fluxes and B values, flux densities and flux intensity values at different parts of irons and different inductance values and torque components, what is the starting torque, peak torque and rated torque and etcetera and efficiency things with respect to the all type of losses, iron loss and copper loss and frictional and windage losses and etcetera. How to calculate the efficiency also, we will discuss with respect to the performance parameters. These performance parameters are used to analyze the machine behavior.

So, first we will discuss about the magnetizing current. In this lecture, we will discuss how to analyze the magnetizing current. In general, all type of rotating electrical machines consist of air gaps. In order to establish the flux at the air gap, we require the current.

That current is nothing but magnetizing current. Generally, MMF N into I is equals to flux into reluctance and which is equals to H into l. So, in order to find the current, we have to find the flux intensity over the length of the magnetic path divided by number of tons or flux into reluctance divided by number of tons. In order to find the magnetizing current, we have to find the H fields over the different parts of iron and fluxes or reluctance with respect to the different parts of iron, we have to analyze. If we will excite the stator winding, let us consider this is the 3 phase machine, 4 pole and 36 slot stator core, I have considered and we are exciting with 3 phase supply and the flux lines will form in this manner.

The flux loops we can see, observe in this figure. Because of the excitation, the flux is flowing through the stator teeth. We can see here, the stator teeth and air gap, rotor teeth and rotor back iron and again back to the rotor teeth in this portion and air gap, then back to the stator teeth and then stator back iron or stator core. So, the flux is flowing through the following path like 2 rotor teeth and 2 stator teeth and 2 air gaps, 1 stator core back iron and 1 portion of rotor core back iron. We can see here, red color line.

So, it is flowing through 1 portion of stator core and 1 portion of stator teeth and then air gap, rotor teeth and rotor back iron and like that. So, like this flux is flowing through the different parts of iron. There is a MMF drops also. We have to find all these MMF drops and with respect to that particular MMF and number of tons with respect to the stator winding, then we can find the magnetizing current. The first step, we have to analyze the fluxes and reluctance as well as MMFs in different parts of iron.

Once we have calculated all these things, then we can find the magnetizing current. So, first I will start with flux. How to visualize the flux in the different parts? The flux lines as well as B fields in different parts of iron depends upon what? Area of that particular part as well as reluctance. The flux lines as well as B fields depends upon the area as well as reluctance of that particular part. For example, let us consider the air gap.

If there is no slotting, the magnetic fields will flow in this manner. Depends upon the slotting or un-sloting. If there is a slotting like this manner, this is the air gap. So, the flux lines here will flow in this manner and there is a chance to flow in this way also. Depends upon the slotting effects, the flux lines at the air gap are flowing.

If we will consider the core or teeth portion, it depends upon the slot shape and teeth width and back iron width. So, here flux is depending upon the indirectly, the width and slot shapes are nothing but length and area. It is depending upon the reluctance values as well as length and area of that particular part. So, we can see here the flux in the rotor core depends upon the back iron width and teeth width and slotting effects, whether uniform air gap is there or not. Based on that, the flux lines will vary.

So, first with respect to the effective air gap, how the flux lines are varying, we will analyze after that fluxes and reluctance values with respect to different parts of iron, we will discuss. So, let us consider the two cases. Case one, where the stator as well as rotor does not have any slotting effects. Both are cylindrical type of things. Otherwise, one side slotting effect is there, other side cylindrical rotor is there.

There is no slots at the rotor side. For a slot pitch tau s, I am considering stator side slots are there and this is the air gap length l g. Here the flux lines will flow in this manner in an air gap. The slot pitch with respect to the stator side is tau s. Here reluctance will be some value will be there and permeance some value will be there.

Case two, if we will consider the slotting one. Let us consider either stator side or rotor side, we have the slots. Now, we will see the slotting effect. Here the flux lines will flow in this fashion, straight lines. The length of this straight line will be l g in the air gap.

The width of stator teeth will be t naught and this one will be b naught and slot pitch will be from this point to this point is slot pitch that is tau s. If we will see the flux lines in this region, where at the side portion of the slot or teeth, here the flux lines will flow in this fashion. Other side of the slot will be in this manner. So, half of the portion will diverge or with the flux lines will flow towards the stator teeth like bulging effects of the flux lines or in a magnetic circuits, we have discussed the fringing type of flux. Same fashion here, instead of flowing the flux lines from this portion, it will flow through the sides of the stator teeth or rotor teeth.

Here the length will be l g plus some radius portion. This is the radius r. The length of the flux lines now is equals to l g plus pi into r divided by 2. How this pi r by 2 is arrived means the total perimeter will be 2 pi r and quarter portion only we are seeing that is by 4. So, we can get the effective length of the flux lines with respect to the sides of the stator teeth in this portion is l g plus pi r by 2.

Now, we will see the reluctance with respect to these two cases. Definitely the reluctance with respect to the case 1 and reluctance with respect to case 2 will be different and permeance which is inversely proportional to the reluctance, permeance also will change. So, we will do the analysis with respect to the permeances. Let us consider the case 1. For case 1, the permeance is equals to that is unslotted portion, permeance is equals to 1 by reluctance that is mu a by l.

Here l is nothing, but effective length of the air gap that is l g e without any slotting where the flux lines are flowing in a straight line manner that is l g e and slot pitch will be tau s. So, the cross sectional area we will calculate by considering the slot pitch tau s in mm into length of the core will be l e. This is the permeance with respect to the unslotted core.

This is equation number 1. Now, slotted core with respect to the case 2, whatever we have discussed here with respect to this one we will discuss.

Here the permeance is a combination of two terms. The permeance with respect to slotted portion is equals to permeance with respect to the teeth portion that is P1 plus permeance with respect to the dip portion where the slot depth is there, depth of the slot where we are placing the conductor that is flux lines are entering through the side walls of the stator teeth. Two portions are there because of two I am considering here. I will redraw that thing here. So, we can see in this figure.

So, the permeance with respect to this portion is P1. The permeance with respect to this portion and this portion is nothing but P2. Here also P2 is there and here also P2 is there because of that reason. The effective permeance with respect to the slotted type of structure is nothing but ps is equals to P1 plus 2 P2. At the end, we will equate this one and this one to find the effective air gap length.

So, the effective air gap length lge in this type of case 1 and effective type of effective length of air gap in the slotting portion, what is the difference? As of now, we have considered lge is constant length of air gap because of slotting effect. This length of air gap may not be same. We can see here in a region 1, the length will be lge. In a region 2, the length will be lge plus pi r by 2. That is what we have discussed here.

So, with respect to this different lengths, we will calculate the permeances. So, the first term, the permeance with respect to the top of the stator teeth or teeth portion is nothing but 1 by reluctance. That is equals to mu A by l. So, here mu is nothing but mu naught. Here also it is mu naught because it is an air gap.

mu naught into slot pitch is tau s minus b naught because we have to calculate this length. This length I am calculating that is tau s minus b naught into le divided by lg. le is the length of the core and lg is the area length of the air gap. Now, in order to find the permeance in a slotted portion where the flux lines are entering from the sides of the stator teeth, for that permeance is equals to 1 by mu A by l same equation. Let us consider any small portion of the thickness d r.

Here the flux lines are coming in this fashion. This is the small portion of d r. Any small portion in this interval, we can consider it and this is lg length of the air gap where the flux lines are entering into the stator teeth. In this manner, flux lines are entering into the stator teeth. So, d r is the portion of thickness and length of the core is le over a single stator slot.

If I will draw, if I will write the permeance equation, the change in permeance is equals to mu A by l that is mu naught into area is nothing but small portion d r into le divided by actual length of the flux lines that is lg plus pi into r by 2 or length of the air gap. I can say from this point to this point, length of the air gap is lg plus pi r by 2. The total permeance with respect to one side of the stator teeth P 2 is equals to integral of limits 0 to b naught by 2. This portion where the flux lines are flowing that is b naught by 2, half of the portion is flowing this side and half of the portion is flowing this side. This is b naught by 2 and here also it is b naught by 2. The limits will be 0 to b naught by 2. Then mu naught d r into le by lg plus pi into r by 2.

We have to solve this equation by considering lg plus pi r by 2 equal to x. After this value is equals to x, then d r is equal to x. In order to do the d r value, we will rewrite this equation like 0 to b naught by 2 into mu naught d r into le divided by 2 into lg by pi into plus r. So, in order to make the derivative, I am removing the terms with respect to the r.

Only r will be there and remaining terms I am bringing outside. So, it will be 2 by pi, outside will be there in this equation. So, the final thing 2 into mu naught le by pi integral 0 to b naught by 2 into d r by 2 into lg divided by pi plus r. So, we can consider the denominator term 2 into lg by pi plus r equals to some x.

So, d r equals to d x. So, I can substitute 0 to b naught by 2 d x by x 2 into mu naught le by pi. This is permeance with respect to one side of stator t. By solving this equation, the permeance with respect to one side of stator t will be 2 into mu naught le by pi into log ln of 1 plus pi into b naught divided by 4 lg. In between, I have not derived the steps. I gave the hint like x equals to this 2lg by pi plus r.

We can consider and make the derivative and this term integration will be ln of x plus c. We can do and c value. We can find constant value and substitute it back. Then we can find this expression. Now, the effective permeance with respect to the slotted rotor or stator like case 2 is equal to slotted portion is equal to permeance with respect to the teeth portion plus 2 into permeance with respect to the flux lines entering at the side of the stator teeth.

If I will add these two things like this is equation number 2 and equation number 3 and equation number 4. equation number 3 and 4, we can find the resultant permeance with respect to the slotted rotor is equals to mu naught into tau s minus b naught by lg into le plus 4 into mu naught into le by L pi into log of 1 plus pi into b naught by 4 lg. This is the effective permeance equation. If we will rearrange the terms by taking mu naught and le outside tau s minus b naught by lg plus 4 into 4 by pi into log of 1 plus pi into b naught divided by 4 into length of air gap. This is permeance with respect to the slotted portion equation number 5.

In case 1, we have derived the permeance with respect to the unslotted one and from case 2, permeance with respect to the slotted portion we have derived. In an ideal condition, we are considering the length of air gap will be lg only, but in order to find the actual MMF's and actual fluxes with respect to the different air gaps in a different slot shapes, we have to equate these two things. Then we will see mu naught into le into slot piece divided by effective length of air gap is equals to this term. This term completely I will copy. If I will substitute here, then mu naught term, mu naught le term will be cancelled each other and if we rewrite the terms in the form lgeffective length of air gap is equals to tau s divided by tau s minus b naught into sorry plus 4 into lg by pi into log of 1 plus pi into b naught by 4 lg into length of air gap in a unslot like slotted mission where the difference between the teeth position to the stator outer diameter.

We can see that one here in this image. This length will be from actual air gap length will be lg and effective length with respect to the unslotted position type of machine will be lge. So, we are comparing these two types and we are calculating the effective length of air gap. So, this complete term we can represent it as a some constant that is called as a K c into lg. Here K c is nothing but, carter coefficient. It is used to find the actual air gap length of a machine.

Any type of machine we can utilize the same equation. This is equation number 6 finalequation for the carter coefficient where K c is equals to tau s slot pitch divided by slot pitch minus b naught. Here b naught is nothing but, slot width into 4 into lg divided by pi into ln of 1 plus pi into b naught divided by 4 into lg. Do not confuse this b naught and slot opening b naught. This b naught will be with reference to this slot portion only.

Here b naught will be the slot width I can say. So, this is the carter coefficient. If the double slotted machine we can see here the stator lamination as well as rotor lamination. This is the stator lamination and this is the stator lamination. I am showing here if the both sides we have slotting then it is double slotted machine where the effective length of air gap lge is equals to K c s that is carter coefficient for stator carter coefficient for rotor into lg where K c s is equals to tau s divided by tau s minus b naught same equation. Only we have to replace with slot pitch with respect to the stator slot pitch with respect to the rotor 4 into lg by pi ln 1 plus pi into b naught by 4 into lg.

Similarly, K c r also just replace tau r and tau r plus the remaining term we can substitute it here. So, these are the carter coefficients for stator and carter coefficients for rotor in the bracket same 4 into lg divided by pi ln of log of 1 plus pi into b naught by 4 lg. So, with this we can calculate the effective length of air gap. Once we know the effective length of air gap we can calculate the magnetic fields at the air gap and area of the air gap and MMF's also and flux.

All those things we will discuss in the coming lectures. Here we can see the carter coefficient and some empirical formula this is with respect to the derivation by considering the conventional reluctance and permeance we have derived the carter coefficient will be this equation. Some empirical formula to find the carter coefficient is K c is equals to slot pitch tau s divided by tau s minus b naught square divided by 5 into lg plus b naught. This is the approximate and empirical formula to find the carter coefficient. For stator it will be tau s for rotor in terms of tau r we can represent.

We can utilize either this equation or this equation. This is equation number 7 and this is equation number 8. We can consider equation number 7 or equation number 8 to find the carter coefficient. In general for open type of slots the effective length of air gap will be greater than 70 to 80 percent of actual length of air gap. For semi-open type of slots like for all type of electrical machines we are utilizing semi-open type of slots where the effective length of air gap will be 15 to 25 percent of lg higher than this much like here 1.7 times the actual lg here 1.25 times the actual lg. The effective length of air gap with respect to the different type of slots we can see here lg plus 70 to 80 percent of lg plus 15 to 25 percent of lg. With this I am concluding this lecture. In this lecture we have discussed the carter coefficients to find the effective length of air gap. In the coming lectures we will utilize this effective length of air gap equation to find the magnetic fields at the air gap and MMF's with respect to the air gap. Thank you.