Course Name: Design of Electric Motors

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Week: 08

Lecture: 44

Title: Design of Induction Machine- Rotor Design -5 (Resistance of Rotor Winding)

Greetings to all, in this lecture we will discuss the equations for bar resistance and end ring resistance, because the slip ring induction motor is nothing, but the cage right. So, for cage how to find the resistance value we will discuss in this lecture. Let us take the resistance for a single bar, which we have already discussed. Resistance of the single bar that is R b is equals to resistivity of the bar material, whether it may be aluminum or copper that is resistivity of the bar material, length of a single bar divided by area of the area cross sectional area of the each bar. This is in the ohms equation number 1 to find the bar resistance, single bar we have discussed. Now, in the last lecture we have discussed the skewing of the rotor bar, right.

This is the actual bar length that is le without any skewing. Now, if you will skew the rotor bar, then the length of the bar will definitely will increase, right. If you will consider it as a right angle triangle and skewing angle will be alpha, where alpha is equals to skew angle, which is equals to 2 pi by Qs is nothing, but slot angle in the, in with respect to the mechanical aspect into P by 2 will give the electrical angle into S s q will give the how many slots we are short skewing. So, alpha equals to this one and le we know and length of the actual bar with skewing we have to calculate.

Then as per the trigonometric equations, length of the skewed bar is equals to le divided by cos alpha. Alpha we know that is 2 pi by Qs into P by 2 into S s q by substituting the alpha value and le value, then we can find the length of a bar, skewed bar. After knowing the length of skewed bar, we have to find the end ring resistance also. If we will see the rotor mesh is nothing, but combination of resistance and inductance values like this it will be n number of bars are there and end ring also there, 2 end rings will be there, circular end rings. We can see here this is one end ring, one side and other side one more end ring is there, the rotor structure I am showing here.

This is the resistance of the bar will be R b, inductance of the bar will be lb, these things are the end rings one R e and L e and resistance of the bar with after skewing it will be R b and inductance will be L b. Current flowing through the branch 1, let us consider this is the branch 1 that is I b 1 and branch 2 is I b 2 and current flowing here is I e 1 in the end ring. In this branch also I e 1 and this is I e naught. In order to find the equivalent resistance for the rotor mesh, first we have to know the value of the resistance of the each skewed bar and resistance of the each end ring. So, we will calculate the resistance of the end ring first, that is R e.

Resistance of the end ring R e is equals to resistivity of the end ring material into length of the end ring divided by area of cross section of the end ring. In the previous lectures, we have discussed and we have calculated the cross sectional area of the end ring also we know and length of the end ring we can calculate it by doing the mean length. Let us consider this is the end ring. The depth will be d e and thickness will be t er. In order to find the mean length of the end ring, we have to take the outer diameter of the rotor that is D naught r and inner diameter with respect to the end ring by considering d c.

So, inner diameter by considering d c will be D naught R minus 2 into d c. From here, we can calculate the mean length of the end ring l e R equals to pi into D naught r minus 2 Rc plus pi into outer diameter of the rotor, that is D naught R by 2. It will result in pi into D naught R minus d c, d c R, depth of the back iron. So, this equation will give the length of the mean length of the end ring by substituting mean length here and resistivity as well as cross sectional area of the end ring. Then we can get the end ring resistance.

Already we have calculated the bar resistance and now we have calculated the end ring resistance also by utilizing this equation. By substituting R b value and R e here and from this network, we have to find the effective resistance for the rotor mesh and effective inductance for the rotor mesh. For that, apply a KVL for this loop. This is the loop we can consider. For this loop, apply a KVL equation.

Take the voltage e 1 back EMF with respect to the branch 1, that is, e b 1 minus e b 2, which is equal to R b into I b 1 minus I b 2, other branch resistance current also I am considering plus 2 into, 2 times the end ring resistance are there, right, this side and this side, 2 R e into I e 1 plus L di by dt terms should come, right, L b into di b 1 minus I b 2 divided by dt plus 2 times inductance with respect to the end ring into d I e 1 by dt. This is equation number 4.

Just apply a KVL with respect to the mesh, which I have represented here with blue color. Apply that KVL equation and then, we will see this equation and I b 1 plus I e naught is equals to I e 1 from the above figure. In steady state, the phasor form of the voltages and currents, how we can represent the phasor form of the voltages and currents like e b 1, e b 2 and I b 1 and I b 2 are, ok.

So, the e b 1 is nothing but induced EMF with respect to the bar 2 and e b 2 is nothing but induced EMF with respect to the bar 2 and e b 1 is the induced EMF with respect to the bar 1, ok. e b 2, we can represent in terms of e b 1 that is e b 1 into some phase shift e power j theta, right. Here theta is nothing but phase or bar angle that is equals to 2 pi by Q or with respect to the mechanical displacement into P by 2 will give the electrical angle conversion, ok. So, e power j theta, we can mention here E b 1 into e power j into pi into P by Q r. This is the E b 2 equation, equation number 5 and in the same manner, if we will represent the I b 2 is equals to I b 1 into some phase shift with respect to the phasor form e power j pi into P by q r.

This is equation number 6. Same way the current in N rings I e 1 is equals to I e naught, I e 1 is equals to I e naught into e power j pi into P divided by q r. This is the phase shift. In these three equations, what I have done is I am representing the voltages and currents in a phasor form where the phase shift angle between one value to other value like E b 1 and E b 2 difference is the angle theta in phasor form e power j theta. I am adding for E b 1 to get the E b 2 value.

So, by substituting all these equations in the above KVL equation that is equation number 4, E b 1 minus E b 2, I e 1 minus I e 2, we have to calculate first. First we will calculate the E b 1 minus E b 2 is equals to E b 1 into 1 minus e power j pi into P by Q r. Similarly, I b 1 minus I b 2 is equals to I b 1 into 1 minus e power j into pi into number of poles by Q r.

This is equation number 8, equation number 9 and with respect to the entering currents I e 1 is equals to I e naught plus I b 1. Here I b 1 is equals to I e 1 minus I e naught that is equals to I e 1 current in entering currents I e 1 into 1 minus e power j pi into P by Q r.

So, from this equation we can find I e 1 is equals to I b 1 divided by 1 minus e power j into pi P by Qr.

This is equation number 10. By substituting equation number 8, 9, 10 in the KVL equation, we can get the e b 1 equation like e b 1 is equals to e b 1 into 1 minus e power j pi into P by Q r is equals to I b 1 into 1 minus e power j pi into P by Q r. The term will be there into r plus R b plus j omega l into b plus 2 into I b 1 divided by 1 minus e power minus j into pi P by Q s into r e plus j omega l e. In the entering current waveform, sorry in the entering current equation I e 1 is equals to I b 1 divided by 1 minus e power minus j should come.

Here itself it will be minus because I e 1 minus I e naught we are doing, right because of that thing it will come I e 1 into 1 minus e power minus j into pi into P by Q r. By substituting all equations 8, 9, 10 in the equation number 3, then we will get this equation number 11. After solving this equation by rearranging the terms, then we will get e b 1 is equals to in the form of I b 1 into R b plus j omega l into b plus I b 1 into r e plus j omega l e divided by 2 into sin square pi into P divided by Q r sin square by 2 into sin square pi into P by Q r we will get it. I am not deriving in between steps, you just rearrange the terms here and then find the equation in this fashion.

Then by equating the real and imaginary terms R b effective resistance of the bar is equals to R b plus R e divided by 2 into sin square pi into P by Q r. This is the effective resistance of a bar. Now, to find the equivalent resistance per rotor mesh that is R r is equals to 2 into Rb 2 times the effective resistance of the each bar of the rotor we will consider and we can find the rotor resistance per mesh. This is the final equation to calculate the rotor resistance.

Similarly, inductance terms also we can calculate it. While discussing the inductance equations, we can recall the same equations and we can find the inductance values at that time.

In this lecture, we will focus only with related to the rotor resistance. That resistance equation is the equation number 12. After knowing the rotor resistance, we will find the weight of the rotor cage with respect to the winding. So, weight is equals to density into volume. Here, density with respect to the aluminum we have to calculate or copper we can consider that is density into volume is equals to n or the number of rotor bars into length of each bar into area of cross section of each bar plus 2 into length of end rings into area of cross section of each end ring because 2 end rings are there and n number of rotor bars are there because of that reason N r into l into area of cross section.

Here, 2 end rings are there because of that reason 2 into l e into a e r we are doing. The total thing we can get it the weight of the rotor cage winding that is equation number 13.

Now, in order to calculate the inner diameter of the rotor, D I r is equals to we know the outer diameter minus 2 into the slot opening height that is D naught r plus radius at the slot upper side and slot depth d s r and radius at the bottom side of the rotor slot and the back iron width. This equation will give the rotor inner diameter. This rotor inner diameter we can calculate from the output function equations also where we have to calculate the lambda 2 is equals to D I r divided by D naught r where the function f of copper function f of lambda 2 f r of lambda 2 and f naught of lambda 2 has to be maximized based on this thing.

We can find the lambda 2 value. Once we know the lambda 2, then we can find the d I r value also. Otherwise, we have to follow this equation number 14.

If we will follow the sizing equations, then we have to utilize the copper function with respect to the rotor geometry and output function with respect to the rotor geometry, we can calculate and then associated lambda value take it and calculate the inner diameter of the rotor. Similarly, the area of the rotor slot should be greater than the bar area or cross sectional area of the each bar.

If it is winding like slip ring induction motor is there where the windings are there, A r should be greater than the number of tons per slot into cross sectional area of the each conductor.

With this equation, we can verify the window area with respect to the each slot of the rotor. So, with this, I am concluding this lecture. In this lecture, we have discussed the bar resistance with respect to the skewed rotor and entering resistance and final cage structure of the rotor mesh resistance also equivalent resistance we have calculated. The rotor inner diameter and window check equations we have discussed with this complete rotor geometry with respect to the dimensions with respect to the windings like rotor bars resistance, weight, volume and everything we have calculated. Thank you.