

**Course Name: Design of Electric Motors**

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**Lecture: 43**

**Title: Design of Induction Machine- Rotor Design -4 (Skewing of Rotor)**

Greetings to all of you. In the last lecture, we have discussed the rotor slot geometry equations. In this lecture, we will discuss the skewing effects of rotor, why we have to do the skewing of rotor and what are the equations for skewing factor, we will discuss. Like the to select the number of rotor slots, the empirical formulas what we have discussed in the earlier lectures, I have summarized here. To minimize the crawling and cogging effects, we have to select the number of rotor slots, where the difference should not be equal to plus or minus 0 and plus or minus  $nP$ . Similarly, for avoiding the noise and vibrations and synchronous curves, the empirical formulas are like this.'

The generalized equation we can consider  $Q_r$  equals to  $Q_s$  plus or minus 20 to 30 percent of stator number of slots we can consider to minimize all these kinds of effects. The next method like we can follow this empirical formulas and we can select the rotor number of slots. The other way to minimize these effects is nothing but skewing of the rotor, whatever it may be the number of rotor slots, if we will skew the rotor such that the effect effects with respect to the crawling, cogging and slot harmonics and noise vibrations and synchronous curves everything will be minimized. The objective of skewing is to minimize all these type of effects.

Similarly in all electrical machines, we have the rotor as well as stator having the slots. These slots having a variable reluctance and variable air gaps will result in harmonics with respect to the air gap MMF. These harmonics will interact with the rotor conductors and produce the harmonic fields and the interaction of harmonic fields with respect to the main fields results in harmonic torques. So, to minimize the harmonic torques, we have to skew the either rotor or stator to minimize the slotting effects. Generally in an induction machine, we can see here this is an induction machine.

Instead of skewing the rotor or stator, stator side we have the windings and rotor side there is no windings only bars are there. We can see here the some skewed lines kind of thing, skewed bars we can identify here. Depends upon the type of machine, the skewing

with respect to rotor or stator we can select it. For example, for squirrel cage induction machine, skewing the rotor is easy as compared to the skewing the stator. Because stator side we have windings and skewing the windings as well as design with respect to the laminations like stampings for making the skewing at the stator side, it will be a challenging task or difficult as compared to the rotor side.

Because of that reason we can go ahead with the skewing of rotor. Whereas, in the slip ring induction machine or in other machines where the windings are present at the both sides, we can go ahead with the either stator side or rotor side. With respect to the volume and with respect to the number of windings and the stator or rotor, we can decide which one we have to skew. Now, how to derive the relation or a skewing factor equation? We will consider the equation with respect to the stator distribution factor based on that we will derive the skewing factor. Let us consider the stator having a distributed winding like this stator conductors where slots per pole per phase is nothing but  $q$  and slot angle is nothing but  $\beta$  and distribution factor  $K_d$  is equals to for  $n$ th harmonic  $\sin n$  into  $q$  into  $\beta$  by 2 divided by  $q$  into  $\sin n$ th harmonic into  $\beta$  by 2.

This is the equation for the distribution factor for a distributed winding on the rotor side. This equation already we have derived. Based on this equation, we will derive the skewing factor for the rotor. This is equation number 1 I am considering with respect to the distribution factor. Now, let us consider the machine without skewing.

This is the rotor and shaft. Here without skewing means there is no skewing of bars. We can see the straight bars on the rotor. Here in order to minimize the crawling, cogging and other effects, we have to select the number of rotor slots appropriately. The skewed rotor structure we will see now.

Rotor structure only I am considering. The rotor which bars does not have any skewing, we can see the straight bars and with skewing, we can see the bar will be like this manner. This is the skewed rotor bar where all bars are placed in this manner. Here skewing is there. Here no skew at the rotor bars.

This is with respect to the non-skewed rotor, both straighter as well as rotor. Now, let us consider the skewing angle  $\alpha$ . Same rotor structure I am considering of a length  $L$  e. Without skewing, the bar will be in this manner. With skewing, the bar will be like this.

The skewing angle is nothing but the angle from the starting point of the bar to the angle which is going to the other end of the bar. This is the skewing angle  $\alpha$  is nothing but skew angle and which is represented in terms of stator slot pitch or angle. Stator slot pitch in mm means  $\tau_s$  that is  $2\pi$  by  $Q_s$  and angle means  $2\pi$  by  $Q_s$ . This is with respect to the mechanical angle and to convert this angle into electrical into  $P$  by 2 we have to make and how many number of slots we are short pitching that is  $S_{sq}$  I am adding. If  $S_{sq}$  is 1, then slot angle for 1 slot is nothing but this one.

For  $n$  slots, it will be into  $S_s q$  I have to add.  $S_s q$  representing how many slot angles we are considering for the skew angle. Now, the distribution factor or skewing factor for the rotor side  $K_{dn}$  is equals to with respect to the equation  $\frac{1}{q} \sin n \text{ into } \beta \text{ into } \sin n \text{ into } \frac{\alpha}{2}$ . Instead of slot angle, I am representing the skew angle, skewing angle divided by  $q \text{ into } \sin n \text{ into } \frac{\alpha}{2}$ . Here  $\alpha$  is nothing but skew angle.

Now, we have to substitute the value of slots per pole per phase that is  $q$  and  $\alpha$  values with respect to the rotor bar. Now, consider the same skewed rotor bar is a combination of  $n$  number of small stride bars like let us consider in this fashion. This is 1 bar and then like this stride bars we can consider. See here, this is the skewed bar which is a combination of small stride bars. This is 1 bar and this is second bar, third bar, fourth, fifth like that.

It is a combination of small stride bars that length is  $\Delta l$ . The skewed bar I am considering small stride elements in this fashion. This is the small portion of the stride bar  $\Delta l$  like  $\Delta l$ ,  $\Delta l$  and so on. Together, it will give the  $l$  length of the total length of the core  $l_e$  and angle is with respect to this particular point. Skewing angle from this point to this point is  $\alpha$ .

From this point to this point is nothing but  $\Delta \alpha$  with respect to the small portion. From starting point to the ending point of the each bar, small Straight bar I can consider it as a  $\Delta \alpha$ . Now, we will find how many small Straight bars are there in a skewed rotor bar that is  $z$ . Each skewed bar is made up of small Straight bars. Each small bar length is  $\Delta l$  and total length of a core is  $\Delta l_e$ .

Then we can find the skewed number of Straight bars is equals to  $\frac{l_e}{\Delta l}$  and  $l$  terms that is equals to  $l_e$  divided by  $\Delta l$ .  $l_e$  represents the total length of the core and  $\Delta l$  represents the small Straight bar length. Then it will give the total number of Straight bars and in terms of angle, if you want to represent then  $\alpha$  divided by  $\Delta \alpha$ . This also will give the number of Straight bars. With respect to the  $q$  slots per pole per phase definition, here the slots is nothing but number of small bars.

Number of bars per pole per phase, each bar is nothing but one phase. Then phase spread is nothing but  $\frac{\alpha}{\Delta \alpha}$ . Here  $q$  represents the slots per pole per phase or I can say phase spread. Now, phase spread is nothing but from this point to this point, how many conductors are there from this point to this point like how many small Straight bars are there that represents the phase spread that is  $\frac{\alpha}{\Delta \alpha}$  bars per pole per phase. The angle is  $\alpha$ ,  $\alpha$  is equals to skew angle for small portion of a Straight bar that is  $\Delta \alpha$ .

Now, the skewing factor  $K_{dn}$  is equals to  $\sin n \text{ into } \frac{\alpha}{\Delta \alpha}$  instead of  $q$ , we can substitute  $\frac{\alpha}{\Delta \alpha}$  divided by  $\Delta \alpha$  into angle  $\beta$  or  $\alpha$  is equals to  $\Delta \alpha$  with respect to the small Straight bar divided by  $\alpha$  by  $\Delta \alpha$  that is  $q \text{ into } \sin n \text{ into } \frac{\alpha}{\Delta \alpha}$

2. This is the skewing factor. Here what we have substituted is  $q$  is equals to  $\alpha$  by  $\Delta\alpha$  that is phase spread with respect to the single bar combination of small Straight bars and  $\beta$  is equals to  $\Delta\alpha$  or  $\alpha$  skewing angle with respect to one particular slot. Here in order to find the behavior of this function, where  $\Delta\alpha$  tends to 0, here  $\Delta\alpha$  is nothing but angle between two Straight bars, small Straight bars. If  $\Delta\alpha$  tends to 0, what is the behavior of this skewing factor? We have to analyze for that we have to apply the limits  $K_d n$  with respect to the skewing factor  $K_{sq}$ .

I am representing that one limit  $\Delta\alpha$  tends to 0  $\sin n$  into  $\alpha$  by  $\Delta\alpha$  into  $\Delta\alpha$  by 2 divided by  $\alpha$  by  $\Delta\alpha$  into  $\sin n$  into  $\Delta\alpha$  by 2. Here  $\Delta\alpha$  and  $\Delta\alpha$  will be cancelled and the bottom function limit  $\Delta\alpha$  tends to 0  $\alpha$  by  $\Delta\alpha$  into  $\sin n$  into  $\Delta\alpha$  by 2. This function I will solve first bottom one. This function we have to write in terms of limit  $x$  tends to 0  $\sin x$  by  $x$  form if you will write it, then it will result into 1. If we will rearrange these terms of this function in this manner, then  $\sin n$  into  $\Delta\alpha$  by 2 is nothing but  $x$  divided by  $n$  into  $\Delta\alpha$  by 2.

If we will add, then it will result in  $n$  by 2 into  $\alpha$  the extra term  $n$  by 2. I am multiplying and dividing here limit  $\Delta\alpha$  tends to 0. This one results in  $n$   $\alpha$  by 2 directly. So, the above function the skewing factor results in  $\sin n$  into  $\alpha$  by 2 divided by denominator part already resulted in  $\sin n$   $\alpha$  by 2. This is the resultant skewing factor equation.

Here  $\alpha$  is equals to skew angle. Skew angle we have represented  $\pi$  by 2  $\pi$  by  $Q_s$  into  $p$  by 2 into  $S_{sq}$ .  $S_{sq}$  is how many number of slots we are short pitching or how many number of slots we are skewing. If it is one slot skewing, then it will be one. If two slots skewing, it will be two like that.

Then final skewing factor  $K_{sq}$  is equals to  $\sin$  order of harmonic  $n$  into  $\pi$  into  $P$  divided by  $Q_s$  into  $S_{sq}$ . I am substituting the angle  $\alpha$  by 2, then  $n$  into  $\pi$  into  $p$  by 2  $Q_s$  into  $S_{sq}$ . This is the final equation for the skewing angle or skewing factor. We can say this is the skewing factor for the rotor to minimize the slot harmonic effects and other effects with respect to crawling, cogging and etcetera. If  $S_{sq}$  is equals to 1, then rotor side we are skewing one stator slot pitch.

If  $S_{sq}$  is equals to 2 means, we are skewing the rotor by 2 stator slot pitches. For this one, this length will be 1 stator slot pitch. We will see how to select the number of rotor slots, the standard number of rotor slots. We will see now. So, these are the recommended stator by stator and rotor slot combinations with respect to the reference.

We can see here for 2 pole, 4 pole, 6 pole and 8 pole and 10 pole, 12 pole type of machine, how many stator slots means 36 slot frame is there. Then we can consider the 26 rotor slots or 28 or 44 rotor slots we can consider like 36 plus or minus 20 to 30

percent of  $Q_s$  that is 36 again. So, if we will calculate that thing, then we can get either 26 or 28 or 44 slots. With respect to the standard stator slot numbers, we can find the rotor slot numbers. If requires the standard slot combinations, we can get it from this stator and rotor slot combinations.

Other than this rotor slot combination, we can design the rotor with any slot number, and we have to make the skewing of the rotor to minimize the slot harmonics and other slotting, other crawling and cogging and noise vibration effects. Next, we will discuss with some example, how the skewing of the rotor is helpful to minimize the slot harmonics and other type of harmonics we will see now. Let us consider an example for calculating the skewing factors for all dominant lower order harmonics. Dominant lower order harmonics means  $6n$  plus or minus 1 and first order slot harmonics. Slot harmonics are nothing but  $2$  into  $n$   $Q_s$  by  $P$  plus or minus 1, here  $n$  is an integer.

So, here dominant harmonics are what? 5, 7, 11, 13 and then 17, 19, 23, 25 and so on. For the first order and second order slot harmonics are 17 and 19 and second order slot harmonics are 35 comma 37. How I have arrived these numbers means  $2$  into order means say  $2$  into  $Q_s$ , we have to select. First we will see the problem statement. The problem statement  $Q_s$  is equals to number of stator slots is equals to 36 and 4 pole machine and 3 phase system squirrel cage and rotor is skewed by 1 stator slot pitch.

This is the problem statement. For a given data number of slots equals to 36, 4 pole machine, 3 phase type squirrel cage induction machine where the rotor is skewed by 1 stator slot pitch. For that find the skewing factors for all dominant lower order harmonics as well as first and second order of slot harmonics. So, here substitute the  $Q_s$  value 36 by number of poles 4 plus or minus 1, then it will result in 35 and 37. If I will substitute  $n$  equals to 1, then it will result in  $2$  into 1, 36 by 4 plus or minus 1. The first order dominant slot harmonics are 17 and 19, second order dominant slot harmonics are 35 and 37 and all lower order spatial harmonics are  $6n$  plus or minus 1, 5, 7, 11, 13, 17, 19 and so on.

All  $6n$  plus or minus 1 we can see here. Now, we will find the skewing factors for all these harmonics. The skewing factor equation  $K_{sq}$  is equals to  $\sin \pi$  into  $P$  divided by  $Q_s$  and  $h$  of harmonics  $n$  and  $S_{sq}$  by 2. This is the equation we have derived divided by  $n$  into  $\pi$  into number of poles by slots into slot pitches by 2. This is the skewing factor equation. Here we will find the skewing factor for first harmonic or fundamental one  $K_{sq1}$  is equals to  $\sin 1$  into  $n$  equals to 1  $\pi$  into number of poles are 4 and slots equals to 36 and how many slots we are skewing means  $S_{sq}$  equals to 1 divided by 2 divided by same value  $n$  alpha by 2 that is  $\pi$  into 4 divided by 36 into 1 by 2, 1 will be there.

This will give 0.995 Ksq skewing factor and Ksq 5 fifth harmonic one. Same thing we have to substitute n equals to 5 sin of 5 into pi into 4 divided by 36 into 1 by 2 divided by 5 into pi into 4 divided by 36 into 1 by 2. Just n alpha by 2 I am substituting here, then we will get 0.878 for fifth harmonic.

Similarly, if we will do it for all other harmonics, let us say skewing factor for seventeenth harmonic that is slot harmonic thing sin of 17 into pi into 4 divided by 36 into 1 by 2 divided by slot harmonic number 17 into pi into 4 by 36 into 1 by 2.

This will give 0.0585 approximately 0.06. This is the skewing factor value for seventeenth harmonic. Similarly, if we will summarize for the harmonic order V, spatial harmonic and skewing factor Ksq for the harmonic first 5, 7, 11, 13 and 17, 19, 23, 25, 29, 31 and 35 and finally, 37. Then, the Ksq values if we will substitute in the same skewing factor equation and if we will find, then for fundamental it will be 0.995, for fifth harmonic it will be 0.878, for seventh harmonic it is 0.769, for eleventh harmonic it is 0.49, for thirteenth harmonic 0.338 and for seventeenth it is 0.06, for nineteenth minus 0.05, for twenty three 0.19 minus 0.22. Same way for all other harmonics I have calculated already that is why I am noting down as it is 0.19, for thirteenth harmonic it is 0.03. These are the harmonic order and skewing factors. From these numbers, we can visualize that for slot harmonic, seventeenth slot harmonic and nineteenth slot harmonic values and thirty five and thirty seven are the second order slot harmonics. These are the four first order and second order slot harmonics. Skewing values are very less that means, the slotting effects are minimized because of the skewing of one slot pitch. Because of the skewing, the fundamental component also will come down slightly because the interaction of fluxes with respect to the straight rotor bar and interaction of fluxes with respect to the skewed rotor bar will definitely come down.

That thing we can identify here. The KS queue fundamental value is nothing but 0.995 that means, it is coming down by 0.05 into 100 percent. So, 0.5 percent the fundamental value is coming down because of this skewing. Whereas, the slot harmonic seventeenth, nineteenth and other harmonics came down. In order to minimize the other harmonics also, we can select appropriately the skewing angle alpha. That alpha depends upon how many number of slots pitches we are doing the skewing Ssq equals to 1 or Ssq equals to 2 like that we can consider. If you will increase the angle, then automatically the length of the rotor bar also will increase. If the rotor bar length is increasing, then resistance of the rotor side will also increase and losses will increase such that efficiency will come down.

So, with this I am concluding this lecture. In this lecture, we have discussed the skewing effects like why we require the skewing and what is the equation for the skewing factor. We have discussed the objective of skewing is to minimize the effects with respect to the

slotting harmonics and cogging, crawling, synchronous cusp and noise and vibration effects with the skewing we can do. Thank you.