Course Name: Design of Electric Motors

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Title: Electric Machine Sizing Equations-Output Power Equation in terms of D^3L Product - 2

Greetings to all, in the last lecture we have discussed the sizing equations of a electrical machine with respect to the D square L product and then D cube L product equations we have discussed, right. In this lecture we will continue the D cube L sizing equations by considering the different flux densities we can see here. So, different flux densities in the core like back iron is nothing, but Bc and Bt value and Bg value and slot dimensions with respect to the back iron and radius of the slot all dimensions with respect to the stator slots also we have considered to realize the power equation. We will continue the same analysis and we will derive the output power equation in terms of by considering the flux density at the air gap and flux densities at the teeth and flux densities at the core and slot dimensions and stator geometry. By considering all these parameters we will discuss the D cube le sizing equations for a electrical machine. So, we have derived the copper function in the last class in terms of area of the slot and number of slots and outer diameter of the stator and this here delta is a constant and divided by outer diameter of the stator square.

So, I will consider the power equation. Let us consider the system or machine the input we are giving P in and output we are taking P naught. P naught is the mechanical output power that is 2 pi NT by 60, Ns is the synchronous speed and Te is the developed electromagnetic torque and here losses and other energy conversion is happening in a magnetic circuit and the input side we have the power equation for the AC machine m into V phase RMS into I phase RMS into power factor. This is the input power and this is the output power like the last diagram we have discussed in the last class right P in and then power at the air gap here stator losses will be there both I square are loss and iron losses and then power at the air gap are rotor input and then rotor losses both I square are loss as well as iron loss.

Then this rotor output is nothing but in mechanical output and here mechanical losses will be there like friction and windage losses. Finally, we are getting the mechanical output power this is inside the system or inside the machine. So, here efficiency is nothing but output power divided by input power. If the efficiency with respect to the air gap is nothing but output power divided by power at the air gap with respect to this particular point power input at the air gap and power at the output side will give the efficiency for the air gap. From this equation P naught is nothing but in efficiency into power at the air gap.

If we will neglect the losses the stator side losses then efficiency at the air gap into P in we can consider it here that P in input power is nothing but m into V phase into I phase into cos phi. Otherwise we can keep it same thing and neglect the other losses stator losses in this equation P g will be m into V phase rms at the air gap I am considering without losses and then I phase current at the air gap and then power factor.

I will consider the same equation and here the voltage we have to represent in terms of magnetic loading and current we have to represent in terms of actual current density. In the D square l sizing equation we have considered the electric loading which is at the air gap. We can see here this is the mission current density at the air gap we have considered in the D square l sizing equations.

Now, we will see actual current density with respect to the slots that is nothing but J, J is the actual current density. First I will take the voltage V phase peak equals to what? Power equation is nothing but efficiency into m V phase rms I phase rms into cos phi g power factor. If we will replace the power voltage and current terms in terms of peak then we will get the output power is equals to efficiency into power factor into m by 2 V phase peak and I phase peak because V phase rms is equals to V phase peak by root 2. Similarly, for current substitute it here. Now, V phase equation we have derived in the last lectures with respect to the Faraday's law.

Here V phase peak is equals to 2 pi into N phase into phi average into frequency into winding factor. This equation we have discussed in the last lecture where we have derived the D square l sizing equations. This is equation number 16 and then V phase peak is there. Here flux average in terms of magnetic flux density we have to represent flux average is equals to 2 by pi flux peak value that is 2 by pi into B g peak flux density peak value into area. Here area with respect to the air gap flux lines will be 2 by pi into B g peak value area is nothing but pi D I s by P into l e.

At the air gap flux lines are entering in this manner and it is going like this. We have to find the cross section with respect to this flux lines with x axis distance is nothing but pi D I s by P and y axis distance is nothing but length of a core. Just substitute here then we will get the area and final voltage equation is nothing but V phase peak is equals to 2 pi into N phase is number of turns per phase and in place of flux average we have to substitute this equation that is 2 into B g peak into d I s by P number of poles into l e. This is the flux average into frequency into winding factor. This I am considering as a equation number 17.

Next thing we have to find the I phase peak value in terms of current density J. Generally, the current density with respect to the winding copper winding it may be different value aluminum winding it may be different value. So, current density is nothing but I rms divided by area of cross section of that particular conductor. If it is one conductor is there, this is the conductor and it is carrying a current I rms copper wire if I will assume then J equals to here area of cross section is small ac then J equals to I rms divided by ac. Now, in place of electrical machine if we will see the slot diagram will be like this manner.

The lamination with respect to the stator frame we can see here. The lamination for a single stator laminations for a stator core we can see here I am considering here a single lamination slot area we can observe here in this we are placing n number of conductors. We can observe in this one in the small stator core we have placed n number of conductors in a single slot. How to find the current density here with respect to this many number of turns? First we will see the number of turns and then copper area we will calculate it. So, generalized equation we can represent it as I rms per total copper area how much? ac is nothing but total copper area.

Let us consider this is the single slot here n number of conductors are there. These conductors copper area will be ac capital ac. Now, we have to represent this capital ac in terms of total number of conductors. The total number of conductors is nothing but what? This is with respect to single conductor. Now, with respect to the total number of conductors we have to substitute here.

The total number of conductors for a single slot is nothing but total number of conductors Z by total number of slots. There is nothing but conductors per slot. Here total number of conductors Z is nothing but 2 into number of turns per phase into total number of phases by qs. This is nothing but conductors per slot. Substitute this conductors per slot here.

Then this is the total current density or actual surface current density with respect to the conductors placed in a single slot. Then I phase rms divided by total copper area into n phase into 2 into m divided by Qs. This is the current density. Here replace the I phase rms with respect to the I phase peak. Then J equals to I phase peak by root 2 into ac copper area into number of turns into 2 into number of phases by number of slots.

Why by root 2 came in? I am representing the current rms quantity with respect to the current peak quantity ok. So, this equation is equals to now J is equals to I phase peak into root 2 m into n phase divided by ac into qs. This is the actual current density. Now, ac value copper area will replace in terms of As slot area. As is the slot area we have analyzed with respect to the slot dimensions right.

So, we will represent the copper area in terms of slot area. If I will consider a slot like this, here there will be a some liner or separator insulating papers will be there before placing the conductors. After that we are placing the conductors like this and there may be some separators will be there or insulating papers and there will be some closer for the slot. This is the closer for the slot like this the slot actual slot area As is equals to the amount of copper we are placing inside the slot plus insulating materials like paper. Insulating paper used for liners like this red color one and separators for multilayer windings insulating materials plus closers like sticks we will utilize it for closing the slots here.

This is the closer. So, the total area of the slot is nothing but this one. If you want to represent the slot area in terms of ac only, we will add some constant that is kcu copper fill factor. Ac into copper fill factor is nothing but total slot area. Here Ac by kcu we have to consider because copper slot fill factor 0.

62, 0.75 will be there then total area of the slot is equals to Ac by kcu. Slot fill factor is nothing but with respect to all these materials copper insulating materials and other varnish and other things. So, slot fill factor ksf slot fill factor is nothing but the total copper plus the area with respect to the other materials divided by actual slot area is nothing but slot fill factor. So, here we will focus only with respect to the copper slot fill factor will represent actual slot area in terms of copper area. Now, the current density equation will be in this manner I phase peak current peak into root 2 into number of phases and transfer phase divided by in place of copper area substitute slot area into copper fill factor into Qs.

This is equation number 18. Substitute equation number 17 with respect to the V phase peak and 18 with respect to the I phase peak in the power equation. So, final power equation P naught is equals to efficiency into cos phi g into number of phases by 2 into voltage equation I am writing here 2 pi into N phase number of turns into frequency winding factor 2 by p and bg peak value into Dis into length of the core. This is with respect to the V phase peak and with respect to the I phase peak in terms of current density that is Jl into area of the slot and copper fill factor number of slots divided by square root 2 into number of phases into N phase. So, here m and m will be cancelled and N phase and N phase will be cancelled and this 2 will be cancelled. Finally, the power equation is equals to root 2 by efficiency cos phi g that is power factor into frequency winding factor Bg value peak and Jl into area of the slot copper fill factor number of slots Qs D is into le by P.

This is the final power equation. So, here we have to substitute the slot area in terms of copper function. Slot area we have derived right A s is equals to what we have derived in f of lambda that is copper function into first I will write the f of lambda equation. Then we can rewrite the equation with respect to the area of slot f of lambda is equals to As into Qs by pi D naught s square by 4 plus delta divided by D naught s square. We have derived right that is slot area equation in terms of copper function.

From this equation rewrite the equation in terms of area of the slot then f of lambda minus delta divided by D naught s square into pi D naught s square by 4, 1 by Qs. This is nothing, but area of the slot right substitute this equation in the power equation. Then we will see the final power equation. So, the final power equation P naught is equals to square root pi into efficiency power factor D Is into le by P Bg magnetic loading peak into frequency winding factor into electric loading copper fill factor into Qs. The remaining A s term we have to substitute here.

A s term is nothing, but f of lambda minus delta divided by d naught s square into pi D naught s square by 4 into 1 by Qs.

Solve this equation finally, the equation will turn and one more thing here in place of f substitute Ns equals to 120 f by P right synchronous speed from here f is nothing, but N s into P by 120 in place of f substitute the synchronous P term as well as P term then the power equation is nothing, but f of lambda minus delta divided by D naught s square into pi square by 240 root 2 into efficiency cos phi g that is power factor D Is into le into D naught s square into Bg into Jl that is electric loading in terms of actual current density into synchronous speed then copper fill factor and winding factor this is the final power equation. Now, in order to represent this equation in terms of D cube l term we have to multiply and divided by this equation by D naught s then we can rewrite this equation like f of lambda minus delta divided by D naught s square. So, this D naught s at the numerator side and d i d D naught s at the denominator side and D Is at the numerator side we can replace it with lambda then pi square by 240 root 2 efficiency cos phi g into le and D naught s cube Bg actual current density synchronous speed and copper fill factor and winding factor. Here f of lambda is the second order quadratic equation that is a lambda square minus b lambda plus 1 bring lambda inside the bracket this lambda value bring it inside then the final equation of power will be derived.

So, the final equation of the power will be P naught equals to f naught of lambda that is the third quadratic equation now third order delta into lambda divided by D naught s square into pi square by 240 root 2 into efficiency cos phi g into copper fill factor and winding factor magnetic loading actual current density and then D naught s cube into le and then synchronous speed this is the final power equation number 19.

In this equation lambda is equals to D Is by D naught s the ratio of inner diameter of the stator to outer diameter of the stator and f of lambda is a third order equation a lambda cube minus 2 b lambda square plus lambda this is the final output function f of lambda f naught of lambda. Whereas, the copper function what we have derived a lambda square minus 2 b lambda plus 1 this function represents the copper function for a machine and this function is nothing, but output function output power function this is the final power equation in terms of D cube l. So, here we can represent this complete term as a constant term C 1 and this is the volume term along with the one more D and then speed. So, power is equals to C 1 into D cube l product that is volume into another diameter into synchronous speed this is final equation for the power.

So, here we can observe that output power is directly proportional to f of lambda output function this f of lambda function is depending upon the B g value flux density is at the air gap flux density is at the teeth flux density is at the core value back iron and these 3 values will deviate the or will influence the output function and that will eventually results in deviations in output power. In order to increase the output power we have to maximize the f of lambda function by considering different values of lambdas we have to maximize the output power and this f of lambda f naught of lambda function also involves the slot dimensions and slot area and stator geometry also including the teeth slot area slot height depth everything and similar to the D square l product here also power equation is directly related to the magnetic loading, but magnetic loading is limited by saturation limits and then core losses and type of materials what kind of materials we are utilizing and which type of cooling mechanism we are utilizing it and then it is directly proportional to the actual current density actual surface current density with respect to the conductors and then it is proportional to the synchronous speed and also power factor. So, by changing the magnetic flux density or magnetic loading we can increase the power or by electrical current density also we can increase the output power and Ns also by changing Ns also we can vary the output power and by changing the power factor also we can change the output power. Now, we will see how to maximize this function for any function in order to maximize the function means what we have to do partial derivative right derivative of this function with respect to the lambda right equate it to 0 and find the lambda values. If we will substitute the lambda values in f0 of lambda if it is greater than 0 then it will be lambda is maximum value and if it is less than 0 then lambda will be minimum to find the minimum and maximum values for any function.

Now, we will see f0 of lambda we will take and we will find the roots with respect to the maximum and minimum a lambda cube minus 2 b lambda square plus lambda do the derivation with respect to the lambda that is equals to 0 then 3 into a lambda square minus 2 b lambda into 2 plus 1 is equals to 0 right. From here lambda is equals to minus b plus or minus square root of b square minus 4 ac by 2 a right equations in order to find the roots for second order equation here lambda equals to plus 4 b plus or minus square root of 16 b square minus 4 into 3 a divided by 3 into a 3 a into 2 3 a is the actual value of a and then into 2. So, from here we can find the roots for lambda value that is equals to 2 b plus or minus 4 b square minus 3 into a square root divided by 3 a. So, from here lambda maximum value and lambda minimum value we can find by taking the second derivative. These are the roots for this output function equation.

If we will observe the output function as well as copper function where these two equations are depending upon the lambda. Lambda value is nothing but D I s divided by D naught s. If we will see the actual image of the machine here we can see if D naught D I s by D naught s value is very high like lambda value is greater than 1. First condition I will take lambda greater than 1. In this condition D I s is much higher than D naught s.

It is not possible right with respect to this particular machine where the inner diameter we can see here the inner diameter of the machine is higher than the outer diameter of the stator. Here we can see the machine. And next if lambda value is equals to 1 then what it will happen DIs is equals to D naught s that means, this area will not be there. There is no space for winding right no stator that means, if the inner diameter as well as outer diameter both are equal then there is no space for stator that is also not feasible solution. And then we will consider the lambda equals to 0.

Lambda equals to 0 means eventually DIs is equals to 0. D I s is nothing but inner diameter of the stator. Inner diameter of the stator if it is equals to 0 then completely we have the stator only right. This space is not there D I s value is equals to 0 that means, we do not have space for rotor no rotor if lambda equals to 0. Next condition I will take lambda is less than 1.

If lambda is less than 1 means D I s is greater than D naught s. Lambda is less than 1 means D I s is less than D naught s that means, inner diameter of the stator is less than the outer diameter of the stator it is possible feasible solution feasible solution to design a machine. Here next case we will consider lambda is less than 0.

4 to 0.5. Lambda value is less than some value means DIs is less than the D naught s where it is approaching towards the rotor side. There is no space for rotor it is difficult to accommodate the rotor in this condition. To make the feasible solution the lambda value should be in between the 0.4 to 0.5 and 1 with respect to this condition we have to select the lambda value to accommodate the rotor as well as stator.

In this condition what it is happening if we will consider the lambda value is less than some value then D I s is very small. That means, there is no space for rotor this area will come down. If lambda value is less than 0.4 or 0.3 like that then this area with respect to the rotor will come down that is not feasible and if we will increase lambda value then this area stator area will come down that is this one this area will come down if I will increase the lambda.

So, in between that particular point we have to select it that range will be this one. The preferable range to select the lambda value that is D I s inner diameter to outer diameter of the stator ratio is equals to should lie in between the 0.4 to 1. With this I am concluding today lecture. In this lecture we have discussed the sizing equations of an electrical machine with respect to the D cube l equations D cube l product.

In this sizing equations with respect to the D square l sizing equations we have considered the flux densities at the different parts of the stator and slot dimensions and stator geometry everything we have considered and we have derived the D cube l product and sizing equations for a electrical machine. Thank you.