Course Name: Design of Electric Motors

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Week: 06

Lecture: 30

Title: Electric Machine Sizing Equations-Output Power Equation in terms of D^3L Product - 1

Greetings to all, in the last lectures we have discussed the design equations in terms of volume product right. In that equation we have not considered the stator geometry like what is the stator outer diameter, what is the rotor outer diameter, rotor inner diameter as well as slot ah geometry or slot area and then flux densities at the back iron and flux densities in the teeth we have not considered. Only we considered at the air gap what is the flux density, what is the current density and what is the inner diameter of the stator based on those three variables we have defined the power equation right. So, in order to ah include the flux densities at the different parts of the stator and rotor and slot areas of the stator as well as rotor and ah stator outer diameter as well as rotor outer diameter and rotor inner diameter all these geometries and along with that actual current densities like if you will see the actual current density is happening inside the slot right more current density. So, with respect to the actual current density we will derive the power equation. So, from that power equation we can find the physical dimensions of a machine like outer diameter as well as inner diameter of the stator and length of a core ok.

Let us consider a three phase machine we can see here three phase machine having a outer diameter of D naught s and inner diameter D i s and inner diameter of the rotor is nothing, but D i r and D naught r are the outer diameter of the rotor and lg is the air gap diameter sorry air gap length lg is equals to air gap length. In the previous analysis we have not considered the slot geometry and we have not considered the outer diameter of the rotor right. In this analysis to find the accurate power equation we will consider the flux densities at the back iron that is B c and flux densities at the teeth that is B t and flux density at the air gap that is B g value. So, we will see the all possible magnetic flux densities at different parts of the iron.

Here we can see that the maximum flux density will happen in this region right where the area is less. In this region we will see the maximum flux density right as compared to the air gap as compared to the core this portion. This portion is the core portion and this portion is the air gap this is the core portion. So, flux densities at the different parts we will consider and along with that thing we will see the actual area of the slot. We will consider the actual slot dimensions like slot area if you want to represent then this is the slot area.

So, by considering this slot area we will derive the power equation in terms of d cube l product this is the slot area As. And the slot dimensions we can see here back iron length is dc and the radius from this portion to this portion from this point to this point the radius is r 2 and this portion is d s. And similarly this one is r 1 and between this gap this gap is nothing but d naught and air gap length is l g and the width of the teeth is nothing but t small t and the inner diameter is D i s and outer diameter is D naught s these are the different dimensions of the stator core. So, by considering all these dimensions we will find the power equation in terms of B values and then electric loading values and then slot dimensions. So, B g B t and B c different flux densities we will consider for deriving the power equation.

So, first we will see the flux densities at the different portions of a stator core. We will see the flux lines how the flux lines are flowing. So, here flux lines are coming in this manner and in the core back iron the flux lines are in this manner. So, our task first task is to identify what is B c what is B t and then what is B g flux at the core and flux in the teeth and flux in the air gap and along with that find the slot area. First step I will start with the flux density at the air gap peak flux density that is B g peak we have to find in terms of flux.

In the average flux in the core average flux in the teeth average flux in the air gap it will be same. The flux inside the core will be same if we neglect the fringing and everything then flux in the core also is same. Now, in order to find the flux density maximum at the air gap consider the flux average is equals to 2 by pi flux peak value. Here flux in terms of B and A we can write it B g peak into A. Area A is nothing but area of cross section with respect to that particular flux lines.

Here area is nothing but B g peak value into pi D I s into le by p. This part is with respect to the x and this part is with respect to the y. x into y we can get the area with respect to the cross sectional area for air gap flux. Just find the area here with respect to this flux lines perpendicular cross sectional area that is pi D by P with respect to the one particular pole and le is the length of a core. Then B g value is equals to B g peak equals to flux average into P divided by 2 into D I s into L e.

This is equation number 1. Next, we will find the flux density at the teeth. So, generally the teeth area will be smaller. We can observe in this figure. So, that the teeth area is nothing but this one.

The teeth area is smaller. So, because of that reason the flux density will be higher in the teeth portions. We will find that value exactly what is flux density at the teeth peak value. So, again flux at the teeth is nothing but average is equals to 2 by pi phi t peak value. So, here 2 by pi is constant flux is nothing but B into A, B t peak into area is nothing but cross sectional area with respect to this portion we have to calculate.

So, here cross sectional area with respect to this particular point is nothing but y axis is length into some stamping factor k s. Here k s is the steel stamping factor or steel stacking factor and le is the effective length of a core including the air ducts and y axis length is what t is width of a single teeth like that n number of teeth are there in the on the stator circumference. We can see here t is the width of one particular teeth, one particular teeth width is t. So, like that how many teeth are there under one pole or how many teeth are there under hover a circumference is nothing but Qs is nothing but number of stator slots into t. Then teeth length we have identified in a complete stator circumference per pole if you want then by P we have to add this is the total area.

Then 2 by pi Bt peak Qs into t le k s divided by P this is the average flux at the teeth. So, maximum flux density at the teeth is nothing but pi by 2 into flux average into p divided by Qs t into le k s. So, the flux average I am considering same. So, phi t average or phi c average or phi g average all things I am representing as a phi average. So, this is equation 2.

Next what is the flux density at the core or back iron? So, look at here the lamination of the stator core this is the back iron this one above the slots this portion from here to here is the back iron. So, how to find the flux density at that particular point? We will see the area where the flux lines are coming this is the place where the flux maximum flux lines are entering into the teeth. So, we have to find the area with respect to that particular point. Now, the flux core average value is nothing but 2 by pi flux at the core peak value that is 2 by pi Bc peak into a here A is the area of cross section in order to find the area of cross section y axis length is nothing but le into k s le is the effective length of the core and k s is the steel stacking factor and x axis direction we are finding with respect to this manner perpendicular cross sectional area x axis length will be pi into D naught s minus pi into D naught s minus 2 d c this value is d c the back iron width by 2 mean we have to take then this will results in d c value pi into d c into le into k s is the area. Then the final equation with respect to the phi c average is nothing but 2 by pi into pi d c back iron width length of a stator core and steel stacking factor into Bc peak that results in B c peak equals to phi average divided by 2 into d c into le into k s this is equation 3.

Now, we have calculated the flux densities at the different parts Bg at the air gap Bt at the teeth Bc at the core since the area with respect to the teeth is higher. So, Bt value is greater than the Bg and next Bc and Bc is greater than Bg value with respect to the flux is same and area is different for different portions. So, based on that relation we can conclude that the maximum flux density will happen at the teeth the maximum next flux density at the core and the next case will be at the air gap. Now, we will see the ratios like Bt max flux densities at the different portion what is the ratio Bg by Bt is nothing but equation 1 divided by equation 2 it will results in B into phi average divided by 2 Dis into le divided by pi by 2 flux average into P divided by Qs into t into le into k s. Then this ratio will be Qs into t k s divided by pi into Dis just rearrange the terms finally, the teeth width we have to solve.

So, the teeth width is equals to Bg by Bt constant value into pi into Dis divided by Qs into k s this is equation number 4 we have find the teeth width. Similarly, find the back iron length or back iron width d c by taking the ratio of B g by B c that is equals to equation 1 divided by equation 3. So, finally, after solving this expression we will see d c is equals to back iron length Bg by Bc constant into inner diameter of the stator divided by number of poles into steel stacking factor this is equation number 5.

Now, consider the slot dimensions we will take the we will go back the same image here analyze the slot area as well as different diameters. See here we have to analyze the dimensions with respect to the rotor as well as stator and slot dimensions.

So, the slot dimensions we can see here with respect to the different parameters how the slot dimensions are there and with respect to the stator outer diameter as well as inner diameter we can write the equations in terms of Dis and D naught r and slot dimensions. So, that is outer diameter of the stator is equals to inner diameter of the stator plus 2 into D naught is the slot opening height plus r 1 plus d s plus r 2 plus d c and D naught r outer diameter of the rotor is nothing, but D i s minus 2 l g this is equation number 6.

These equations I have derived from the figure the from the slot image figure we can see here. So, this value is d c r 2 and then d s and then r 1 from here to here and slot opening is d naught and remaining one is the l g value. Now, perimeter of the slot with respect to the w 1.

So, if I want to calculate the perimeter in this line where w 1 and t we know width of the teeth as well as width of a slot at the bottom side we know. That means, the perimeter of the stator in terms of w 1 Qs into t plus w 1 is equals to d I s plus 2 into d naught plus r 1 into pi right. So, here Q s is the number of slots into t plus w 1 is nothing, but this width. So, like this how many widths are there Qs. So, Q s into t t plus w 1 is nothing, but the perimeter actual perimeter we know that d I s plus 2 d naught plus r 1 this length we know up to this point length we know right up to this point that is d I s the remaining one is nothing, but 2 into d naught plus r 1 add that thing into pi is nothing, but circumference this is equation 7.

Similarly, the perimeter with respect to the w 2 at the slot top side at this particular point where w 2 we know and t also we know q s into t plus w 2 is equals to d naught s in terms of D naught s we can write it right D naught s minus 2 into d naught d c right back iron length d c plus radius at the bottom side of the slot into pi is equation number 8 or we can write in terms of D I s also D I s means plus the remaining terms d naught plus r 1 this is d naught and d s r 1 r r 1 up to r 1 we have to write d s into pi this 2 comes under same equation number 8 8 a and 8 b.

We have find the perimeter or circumference with respect to the w 1 and w 2 this equations involves in inner diameter of the stator as well as outer diameter of the stator D naught s and slot geometry slot geometry we can see the slot opening and the radius at the bottom side of the slot that is this one this radius is nothing but r 1 and this radius here is nothing but r 2 we can see here this radius is nothing but r 2 this radius and this radius is nothing but r 1 next task is to find the area of the slot. Now, we have calculated the flux densities at the different parts and the slot information now area of the slot we will find. So, the slot we have considered in this manner this is the slot dimensions from here to here it is d c and from here to here it is r 2 this is d s and from here to here r 1 and this is d naught we have to find the slot area this area we have to find. So, for easy analysis we can split this area into 3 parts the first part will be in this manner and second part will be in this manner and third part will be in this manner.

So, it is a trapezoidal kind of thing. So, calculate the area for this thing this height is d s and it is w 2 and it is w 1. So, w 2 into d s minus the total square area minus the 2 times the half b h area the triangle area is nothing but this one this we have to remove from the square area. So, 2 into half b h that is nothing but w 2 minus w 1 by 2 into d s here 1 more by 2 will come with respect to the half b h then it will results in w 2 plus w 1 by 2 into d s and the second case here the radius is r 2 and here also r 2 and this is w 2 minus 2 r 2 this width whatever the width is there w 2 minus 2 r 2 calculate the area of this portion as a rectangle and this portion again arc. So, this area you just split into 3 parts this is part 1 and this is part 2 and part 3.

So, this we can write it as pi r 2 square by 4 plus again the third point is pi r 2 square by 4 and the second part is nothing but rectangle w 2 minus 2 r 2 into r 2 height again. Similarly, third case also from here to here r 1 and here also r 1 this region consider it as a rectangle and calculate the area. So, if we club all these 3 terms 1 2 3 4 and 1 2 and 3 then area of the slot is nothing but final equation w 1 plus w 2 by 2 into d s plus w 2 minus r 2 into c 1 into r 2 plus w 1 minus r 1 c 1 into r 1 this is equation number 9 here c 1 is a constant which is equals to 2 minus pi by 2 solve this term 1 and term 2 and term 3 with respect to the different areas then final area of the slot we will get in this manner this is the area of slot. From equation 9 and equation 4 to 8 we have to find now what is w 1 and w 2 and what is t and what is d s all these things we have to substitute in area of the slot equation that is equation number 9. So, equation 9 involves the w 1 and w 2 and d s and these 3 are the unknowns.

So, w 1 and w 2 and d s we have to find equations and then we have to substitute there first we will start with w 1 that is the width of the stator slot at the bottom side. So, from the equation 7 and equation 8 we can find w 1 and w 2 just rearrange the terms from equation 7 and 8 then we can get the w 1 is equals to pi by Q s into d i s inner diameter of the stator into 2 into d naught plus r 1 minus t. In this equation we know the value of t from the flux density ratios in equation 2 and 3 and 4 we have discussed the different flux densities and we have taken the ratios the t value is nothing but this one equation 4 here we can see that t value and d c value also back iron value also that is equation 5. So, from equation 4 and 5 we can substitute the d c value and final w 1 is nothing but pi by Q s into d i s plus 2 into d naught plus r 1 minus t is nothing but d i s into pi divided by Q s into k s steel stacking factor B g by B t this is the t value. Now, define B g by B t into 1 by k s as a constant of Gt then w 1 is equals to pi by Q s into d i s into 1 minus g t plus 2 into d naught plus r 1 this is equation number 10.

Similarly, w 2 we have to find in terms of d naught s and D i s w 2 is equals to from the equation number 5 and 8 and 7 then pi by Q s into D naught s minus 2 into d c plus r 2 minus t equation number 5, 7 and 8. Yeah, 8th equation is this one and 5th equation is with respect to the d c and 7th equation is this one. So, from these three equations we have rewritten the w 2 terms and then substitute the t value as well as d c value here. Here 2 d c is nothing but 2 into D i s into B g by B t from the equation 4 or 5 I think this B g by B c value we have derived right divided by k s into P. Here also this multiple values we can define it as a G c some constant and already t has some constant that is t equals to D i s into pi by Q s into k s B g by B t.

Here also the constant we have assumed this is G t substitute this values in this equation and finally, we will end up the w 2 equation pi by Q s into D naught s minus D i s into G t plus G c minus 2 r 2 this is equation number 11 with respect to w 2. So, from equation 7 and 8 we have find the parameters right with respect to w 1 and t and D naught s and other things find the w 2 minus w 1 that is equals to pi by Q s or we can say directly equation 7 equation 8 minus equation 7. Then we can see the final outcome pi by Q s into D naught s minus 2 into d c plus r 2 minus d i s minus 2 into d naught plus r 1. So, these terms with respect to the equation 8 and these terms with respect to the equation 7 finally, pi by q s d naught s minus 2 into d naught plus r 1 plus r 2 plus d c minus d i s right. So, this entire term we can replace with 2 into d s right from the slot diagram if we will see.

So, here D i s plus d naught plus r 1 plus d s plus r 2 plus d c all these terms if we will add 2 times to the inner diameter of the stator then we will get the outer diameter right. So, in with respect to that analysis we can get here the total term is nothing, but d s then finally, d s is equals to w 2 minus w 1 by 2 pi into Q s this is equation number 12. Now, we have to substitute all these values in the equation 9 that is the slot area. So, slot area equation is this one substitute w 1 w 2 and then d s value. So, slot area equation is this one here w 1 we know w 2 we know w 1 is nothing, but equation number equation number 10 and w 2 is nothing, but equation number 12 sorry 11 and double d s is nothing, but equation number 12.

these three equations in 9 then area of the slot is equals to w 1 plus w 2 by 2 d s is nothing, but w 2 minus w 1 into Q s by 2 pi from the equation 12 plus remaining term I am rewriting as it is.

Let us consider this is term 1 and term 2 solve these two terms by substituting w 1 is equals to pi by Q s into d s D i s is nothing, but inner diameter of the stator 1 minus G t is a constant plus 2 into d naught plus r 1. This is the equation for w 1 and equation for w 2 is pi by Q s D naught s minus D i s into G t plus G c minus 2 into r 2. This is with respect to the equation 10 and this is with respect to the 11 those two equations we are substituting in the area of the slot equation. At the end the term 1 in the slot area equation is nothing, but pi by 4 q s into D naught s that is outer diameter of the stator square plus D i s inner diameter of the stator square plus G t plus G c whole square minus 2 into D naught s and D i s into G t plus G c. Here G t and G c are the constants plus 4 r 2 square minus 4 r 2 into D naught s minus D i s into G t plus G c minus D i s square into 1 minus G t square minus 4 into d naught plus r 1 square minus 4 capital D i s into 1 minus G t into 1 d naught plus r 1. This is the total equation with respect to the term 1.

Then term 2 this final equation will be equals to 4 pi by pi by 4 Q s into D i s square into G t plus G c square minus 1 minus G c square minus 2 D i s and D naught s into G t plus G c plus D naught s square plus r 2 square pi by Qs minus pi by Q s into r 2 D naught s plus pi by Q s r 2 d i s into G t plus G c minus pi by Q s into D naught square plus r 1 square plus 2 d naught r 1. I am expanding the above equation. So, final equation of term 1 with respect to all terms we can see here and then term 2 is with respect to the slot dimensions that is w 1 and into r 1 minus 2 by 2 minus pi by 2 into r 1 square plus w 2 into r 2 minus 2 minus pi by 2 into r 2 square.

Here 2 minus pi by 2 is nothing, but c 1 we have considered while discussing the slot area. Here we can see the c 1 is a constant that is 2 by 2 minus pi by 2. The same thing I considered here just substitute w 1 and w 2 add term 1 that is equation number 12 and 13. So, these two equations will be 13 and 14.

Substitute 13 and 14 in slot area a s equals to the final equation pi by 4 Q s into D i s square that is inner diameter of the stator into G t plus G c square minus 1 minus G t square minus 2 into inner diameter of the stator and then outer diameter of the stator into G t plus G c constants plus D naught s square.

This is the equation with respect to the first term and then remaining terms with respect to the slot dimensions r 2 square into minus pi by Q s plus pi by 2 minus 2 minus pi by Q s into d naught square minus r 1 square into minus pi by Q s plus 2 minus pi by 2 minus pi by Q s into D i s into d naught into 1 minus g t 4 Q s by pi. I am multiplying to make the common bracket with respect to the total equation. Just 4 Q s by pi and outside pi by 4 Q s I am taking it common. Then final equation we can see A s that is slot area is equals to in the form of pi by 4 Q s into f of lambda plus some constant term. The second term from this point to this point, this complete term I am representing as x and from here to here I am representing as a function f of lambda second order equation.

Then f of lambda is equals to a s into 4 Q s by pi minus x will come. So, minus x term I am replacing with plus delta a s into 4 Q s, Q s is the number of slots by pi minus x I am replacing with plus delta. Here delta is equals to 4 r 2 square minus 2 q s r 2 square plus 8 Q s by pi into r 2 square plus 4 d naught square minus 4 r 1 square plus 8 Q s by pi r 1 square minus 2 Q s r 1 square plus 4 into D i s into d naught into 1 minus G t. This is the total equation for delta it is a constant this delta is also equals to minus x. So, this is the final equation with respect to the function f of lambda.

The equation number 14 or 15 this is the equation number 15 this function represents the copper function. How much amount of copper we are utilizing in the electrical machine? Because this f of lambda function depends upon the slot area number of slots and slot geometry. Slot area we have considered a number of slots and slot geometry and Bg value, Bt value and Bc value. We can see here with respect to the stator slot this is the stator lamination where we have considered the flux densities at the different parts of the core. f of lambda is nothing but it is a second order equation I am representing with a lambda square minus 2 b lambda plus 1.

Here a is nothing but G c plus G t square minus 1 minus G t square b is equals to G t plus G c and then lambda is equals to D i s by D naught s. And one more thing here we have to take it D naught s common in the this equation and f of lambda into D naught s will come here D naught s square. So, this side it will be D naught s square and f of lambda equals to pi into D naught s square and delta divided by D naught s square this is the final equation. f of lambda final equation is nothing but A s into Q s divided by pi D naught s square by 4 plus delta divided by D naught s square.

This is the equation number 15. I have rewritten the equation. This equation involves the all slot geometry as well as slot area and different flux densities at different parts of the core. With this I am concluding this class. In this lecture we have discussed the D cube l sizing equations where we have considered the slot geometry as well as different flux densities of the iron core like we can see here the flux densities at the back iron, flux densities at the teeth and the slot area everything we have considered to realize the sizing equations. Thank you.