Course Name: Design of Electric Motors

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Title: The Figure of Merits for Electric Motors and Aspect Ratio to Decouple the D^2L Product

In the last lecture, we have discussed the power equation in terms of volume right, how to decide the dimensions of a machine with respect to the given power rating or a given torque rating. In this lecture, we will continue the same discussion on the power equations with respect to the volume and dimensions of a machine. So, the output power what we have derived in the last lecture is P naught equals to pi square by 120 efficiency power factor and magnetic loading, electric loading and then D square l product that is volume product into synchronous speed into winding factor. This is the power equation we have derived in the last lecture right.

Now in order to compare with respect to different machines or this is the machine you consider it for a 100 kilowatt or 1 HP machine I am designing this one, smaller power rating one. So, 1 HP we can consider it as a example.

So, the manufacturer 1 will design the 1 HP machine, manufacturer 2 also will design the 1 HP machine and manufacturer 3 also will design the same machine with different dimensions and different materials and etcetera. So, how to compare it? So some figure of merit or some kind of comparison parameters we have to take right. So, that figure of merit points are with respect to the power density, we can see here power density that is P naught output power with respect to the density is nothing but power per unit volume or power per unit area. We can represent the power density in terms of volume or in terms of area.

The second parameter torque density, torque per unit volume or torque per unit area. So, we know the output power right for a given dimensions. So, find the volume and try to calculate the what is power density and torque density such that manufacturer 1 what is the power density and manufacturer 2 what is the power density or torque density with respect to the volume or with respect to the area. So, we can compare it. So, from these two motors we can pick which one have the better density with respect to the power or better torque density we can pick one machine.

And similarly, other figure of merits or performance parameters are torque to weight ratio. Torque to weight ratio is nothing but the output torque per weight torque per kg and similarly power to weight ratio output power per kg like for example, this machine we are talking with respect to 1 HP right that is 0.7 kilowatt. So, 0.7 kilowatt per how much weight for a kg if you will consider.

So, 0.7 is the power to weight ratio that is 0.7 kilowatt per kg like this we can represent it. So, we can generate 10 Newton meter it is capable to generate it then 10 Newton meter per kg for 1 kg of a weight we can generate 10 Newton meter for 1 kg of weight we can get output power of 0.7 kilowatt like that we can get the figure of merits.

And finally, magnetic shear stress. So, the output power as well as torque depends upon the magnetic loading as well as electric loading right in this equation we can see here magnetic loading as well as electric loading right. The shear stress is nothing but product of these two terms magnetic loading as well as electric loading in terms of RMS quantities Bg RMS El RMS here these two quantities are peak values right in the output equation whatever we have derived. In order to compare the magnetic shear stress for different machines we have to take the RMS quantities. If we will take the RMS quantities then magnetic shear stress is represented with sigma m then electric loading RMS Bg and electric loading RMS.

To convert RMS to peak values Bg into El by 2 right 1 by root 2 into 1 by root 2 will results this one this is the shear stress magnetic shear stress. The units will be Weber per meter square into ampere tons or Tesla into ampere tons this is the shear stress.

Now, we will represent the power equation and torque equation in terms of magnetic shear stress. The output power equation will come in this way if we will replace the magnetic shear stress in the output equation efficiency cos phi g only I am replacing Bg and El terms with 2 sigma m shear stress into winding factor Ns synchronous speed into D square Le that is the volume product. Similarly, torque equation is nothing but pi by 2 efficiency cos phi g and magnetic shear stress and winding factor and volume.

So, from these two equations in terms of magnetic shear stress we can see the what is the output power and output torque. Next we will see the rotor dimensions. If we will see the structure of a machine here in this image the stator inner diameter is D s stator outer diameter is D naught r, but what is the outer diameter of the rotor D naught r right. So, D naught r plus 2 into Lg is equals to inner diameter of the stator right. So, D naught r is equals to Dis that is inner diameter of the stator minus 2 times the air gap.

Now, we will find the volume of the rotor as of now we have seen the volume with respect to the Dis term right. To find the actual value actual volume of the rotor that is this portion actual volume of the rotor is nothing but this one. Here we can see actual volume of the rotor is nothing but this one. So, this is the rotor right squirrel cage rotor squirrel cage rotor is this one. So, the volume will find how much volume is there in terms of D naught r and l e.

So, here volume of the rotor V rotor is equals to area into length right area is nothing but what pi D naught r square divided by 4 into length that is le. Here D naught r equals to what pi Di s minus 2 into lg divided by 4 into le as compared to the Dis inner diameter of the stator the air gap length is very small right. So, neglect the air gap length then volume of the rotor approximately equals to pi Di s square by 4 into length this is the volume equation. I will represent this equation as 8 this equations may be 8 and this equation is 9. So, in terms of rotor volume we can find the power equations.

So, by changing the rotor volume how the power equations are changing also we can find. So, the power equation P naught is equals to pi square by 60 from the equation 8 I will copy the equation 8. So, this is equation 8. So, from this equation substitute the rotor volume we have to rewrite this equation in terms of rotor volume. Rotor volume equals to what? V rotor equals to pi Dis square by 4 into le this equation we can rewrite it as volume is equals to D square l product equals to 4 into V rotor volume of the rotor by pi right.

So, substitute that thing in the equation 8 then power is equals to pi by 15 into efficiency power factor into magnetic shear stress winding factor synchronous speed and volume of the rotor and similarly, torque equals to 2 into efficiency shear stress power factor winding factor into volume of the rotor this is equation 9.

So, from this equation we can conclude that both P naught and T naught the power output power as well as output torque are directly proportional to the volume of the rotor magnetic shear stress and power factor. Mainly it is directly proportional to the volume of the rotor and magnetic shear stress that is what we can observe from the equation 9. So, we have seen the power and torque equations in terms of volume, in terms of rotor volume, magnetic shear stress and speeds other thing, but the main point now is how to decouple the D square le product. So, at the end we will get the D square l equals to some value.

So, how to decouple the inner diameter of the stator and length of the core? We know only volume product whatever the equations from 1 to 9 we have seen volume product we can get some value, but how to get the individual values like what is the inner diameter of the stator and what is the length of the stator? To decouple these two things, we have to consider the parameter aspect ratio. Aspect ratio is nothing, but length per pole pitch the stator length per pole pitch or length per diameter stator length with respect to the diameter ratio. In some literature they will represent length with respect to the diameter and length with respect to the pole pitch we can see here that is nothing, but the aspect ratio l e by tau p is nothing, but length of a stator core and pole pitch divided by pole pitch. Here pole pitch is nothing, but pi into Dis by p perimeter is nothing, but pi into Dis with respect to the one particular pole by P factor will come. So now, with respect to the type of application we have to select the l value and Dis values.

o, to decouple the l value and D value we have to do in a iterative manner. For example, submersible pumps are there or water pumps where length will be very high and diameter will be very less and for example, fans ceiling fans length will be smaller and diameter will be larger. So, depends upon the application we have to select the length and diameters, but the general guidelines as per the literature as per the standards. First method with respect to the iterative approach l by the aspect ratio l by tau p for different machines will be for induction machine it will vary from 0.

5 to 2. For synchronous machine it will vary 0.5 to 2.5. We have to select a number between this range l by tau p and then we have to do the design with respect to the application it is matching then our design is valid. If it is not matching then we have to consider the l by tau p ratio in a different manner.

For example, in this water pump application the length should be higher and D should be smaller. So, for this thing we have to select the higher value of the L by tau p value and similarly for DC machines the aspect ratio will be 0.5 to 2. These are the some iterative approach some numbers as per the literature as well as the standards. The second approach is the waveform based approach where the waveforms are given in terms of pole pitch and power rating.

So, we can get the output power equation in terms of pole pitch that is le by tau p. We can get the power equation in like this manner just multiply the pole pitch and divided by the pole pitch then we can get the power equation. Then we can get some curves in this manner. So, the power rating is increasing and here pole pitch also increasing based upon this thing we can find for that particular power rating what can be the pole pitch here tau p equals to pi D by P. So, once we know this value then we can get the inner diameter of the stator then le also we can find length also.

The third approach is by approximate equations. The l by tau p we have some approximate equations in the literature based upon that thing we can decide the aspect ratio. So, for the aspect ratio for induction machine and synchronous machine and DC machine we will see. So, for induction machine, cube root of pole pace if it is a 4 pole machine the l by tau ratio will be cube root of 2. If it is a 4 pole machine cube root of 2, if it is a 8 pole machine cube root of 4 like that and for synchronous machine it will be square root of pole pairs by 2.

This equation is valid for pole pairs greater than 1. If pole pairs is equals to 1 then the aspect ratio will be varying from 1 to 3 constant number and for DC machines the aspect ratio will vary from 0.5 to 2. So, in this range we can select the aspect ratio and then we can start doing the design because of this approximate analysis the design will not get the unique solution. If the multiple manufacturers or multiple persons if we do the design then we cannot come up with a unique solution or the single solution there will be a multiple solutions because of the multiple variables as well as multiple assumptions.

Next we will see one example how to decouple the D and l for a given power rating. Let us consider a machine of rating 30 kilo watt and 690 volts, 50 heads, 3 phase machine and 1410 rpm star connected squirrel cage rotor induction machine. So, this is the problem statement find the physical dimensions with respect to the diameter and length. We want the volume of the machine as well as dimensions how much diameter inner diameter of the stator and what is the length of a stator. So, from the power equation P naught is equals to the power equation we can copy it here otherwise I will rewrite it pi square by 120 efficiency cos phi winding factor synchronous speed magnetic loading and electric loading and then volume product.

So, we know the output power rating P naught is nothing, but 30 kilo watt we know the V phase frequency and synchronous speed with respect to the 1410 rpm is what there will be a slip. So, the nearest synchronous speed will be 1500 rpm where 90 rpm is the slip speed. Now, we have to assume the efficiency at the air gap and power factor and then winding factor magnetic loading and electric loading all these variables are assumptions for the initial design to find the volume product. So, these many assumptions are there if I will someone will change some particular digit also automatically the dimensions will change.

Let us consider the efficiency is 0.9 or 90 percent and power factor also 0.9 and winding factor is 0.955 and magnetic loading Bg peak value is 1.57 Weber per meter square I am assuming here and average flux density may be 0.5 we can take and electric loading 30000 ampere per meter or 30 ampere per meter mm.

Now, we have to substitute all these values in this equation the equation should turn as output constant into synchronous speed into volume product. So, power equals to C naught Ns into Dis square into le what is the C naught value just substitute all values then volume product D square le is nothing, but 30 kilo watt is the output power divided by pi square by 120 into magnetic loading then electric loading then power factor efficiency all value substituted synchronous speed and others then we will get the D square l product will be 0.0067 meter cube.

Now, consider the aspect ratio l by tau p equals to 1.5. So, le by pi into Dis by poles is equals to 1.5 that implies Le equals to 1.18 into the inner diameter of the stator. So, substitute this value in the above equation then Dis square into 1.

18 into Dis is equals to 0.067 meter cube. Here substitute the l value in terms of Dis in the volume product equation then Dis equals to cube root of 0.0067 by 1.18 will give 0.178 meter. Now, in terms of mm we will see 178 meter for inner diameter of the stator and length of the stator core is nothing, but 1.18 into Dis that will results in 210 mm. So, for a given rating power rating of 30 kilo watt we have find the volume product from that the main dimensions inner diameter of the stator and core length we have calculated.

Next thing is so, the power equation in terms of aspect ratio we will discuss now. P naught is equals to pi square by 120 into efficiency power factor, magnetic loading and electric loading, synchronous speed, winding factor, D square le volume product. Here just add 1 by tau p and pi D by P multiplying and dividing by pi D by P that is pole pitch. Then finally, it will results in pi cube by 120 efficiency cos phi g and magnetic loading, electric loading, synchronous speed, winding factor and Dis cube into aspect ratio le by le by tau p.

So, from this equation we can see that output power is directly related to the le by tau p ratio aspect ratio. Similarly, if we derive the torque equation torque also le by tau p aspect ratio it is involved directly proportional to the output torque rating as well as power Now, what is the volume of the rotor in terms of aspect ratio we will see this will defines the weight as well as the thermal management and other things. So, volume of the rotor is nothing but pi into Dis square by 4 into le. In order to make the cooling mechanism or thermal thing the air gap will matters right the bore diameter. The bore diameter is nothing but with respect to the air gap we can see here the mission structure.

So, with respect to the bore hollow cylinder we can assume it. So, with respect to that how much air will flow in between the air gap and how we can make the cooling system with respect to the rotor volume we will see. So, the rotor volume we have to represent in terms of area of the hollow cylinder or area of the air gap. So, area of the bore is nothing but pi into Dis into Le it is nothing but simple hollow cylinder area is nothing but 2 pi rh. So, here pi into r is nothing but pi into D into h pi D is nothing but pi into Dis and h is nothing but length from this point to this point.

So, from here pi Dis into le by tau p I am multiplying on both above as well as below numerator and denominator pi Dis by p into pi Dis by P. Then it will results in le by tau p aspect ratio then pi Dis square by p number of poles. This is the area of the bore with respect to the air gap. Now, calculate the volume of the rotor equation in terms of area of the bore. Then it will results in from this equation Dis square is equals to area of the bore into p divided by pi square into 1 by aspect ratio.

So, substitute this one in the above volume equation then volume of the rotor is equivalent to area of the bore into number of poles and length of the core divided by 4 pi into 1 divided by aspect ratio. From here we can see that volume of the rotor is inversely proportional to the aspect ratio. If the volume is inversely proportional to the aspect ratio that implies weight of the rotor also inversely proportional to the aspect ratio right, weight is nothing but volume into density of that particular material then we will get the weight. So, the effect of the aspect ratio with respect to the volume weight and cost we will see now. So, if we will increase the aspect ratio then volume of the rotor will come down and weight of the rotor also will come down and cost also will come down we are unable to meet the required torque as well as power ratings ok.

Even though if we will meet the power rating, but the required torque rating we may not achieve by reducing the rotor volume by reducing the rotor volume is nothing but rotor ah conductors area as well as surface current density everything is coming down and weight also will come down and cost also will come down. The meaning l by tau p is high means the l e value is very high and tau p value is very small right or D is value is very small. Let us consider the electric loading, electric loading is nothing but high phase peak into number of conductors per unit length right pi into D is. So, here if we will decrease the ah inner diameter of the stator then electric loading will increase right that means, surface current density is keep on rising. So, the thermal aspect is difficult to maintain or temperature rise is difficult to maintain.

Let us consider the submersible pumps where the length will be very high and diameter is very small. In this kind of machines the cooling mechanism is a complex task ok. Similarly, D square le product also will come down if we will increase the l as well as if we will decrease the D value for a significant value. If D is value is very small and le value is large then this product may come down ok. And torque also will come down and leakage inductance will increase ok.

These are the limitations or the variations with respect to the aspect ratios ok. The analysis as of now we have seen that the power equations in terms of volume and magnetic loadings and synchronous speeds and winding factors and power factors right. In this power equation there is no involvement of geometry right stator geometry as well as stator geometry. We can see here the stator geometry with respect to the slots and ah magnetic field densities at different parts of the iron also not considered and slot dimensions also not considered in this analysis ok. So, in this power equations what we have not considered is the magnetic fields with respect to the air gap only considered with respect to the core with respect to the teeth and stator and rotor has not considered and slot dimensions also we have not considered and geometry of the stator and rotor we have not considered and pole numbers in the final power equation and then actual current densities ok.

So, actual current densities also we have not considered this many things we have ah not considered then this power equation may not be that much accurate, but this is the standard equations we are following to design a any type of machine. In order to make the accurate design we have to consider these components also in the this analysis we have seen only with respect to the air gap flux and length of the core and inner diameter of the stator. There is no slot geometry like there is no slot information there is no slot area and the what is the flux at this back iron in the flux at the teeth similarly with respect to the rotor we have not considered and stator outer diameter, rotor outer diameter or rotor inner diameter this all things we have not considered in this particular equations. So, in order to make the accurate power equation to find the dimensions we have to use the different type of equations that we will discuss in the next lecture that is nothing, but D cube l equations the volume will be D square l product into one more diameter or we will add that is nothing, but D cube l products where we will consider the flux densities with respect to different parts and slot dimensions and geometries of the stator as well as rotors and the actual current densities also. Why actual current density is more important means the current density in this analysis we are considering at the air gap, but the current density with respect to the slots where the actual conductors are placed we have to consider that we will discuss in the next class.