Course Name: Design of Electric Motors

Professor Name: Dr. Prathap Reddy B

Department Name: Electronic Systems Engineering

Institute Name: Indian Institute of Science Bengaluru

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Title: MMF Distribution of AC Machines

Greetings to all. In this lecture, we will discuss the MMF distribution of stator windings with respect to concentrated winding, short pitch windings, distributed windings, and we will also derive an expression for stator MMF and peak magnitude of stator MMF, like fundamental magnitude. Let us consider a magnetic circuit, that is, machine and here, effort variable is MMF n into I and flow variable is rate of change of flux, that is, d phi by d t. The MMF n into I is responsible to magnetize the stator core. Let us consider this is stator core and also to magnetize the rotor core and third task is to establish the required flux density at the air gap. This region is air gap region and air gap length is l g.

In order to establish the higher value of flux density at the air gap, MMF value we have to increase. If we will consider the flux density at the air gap is sinusoidal, then the induced rotor currents I r also sinusoidal. With respect to the sinusoidal induced currents at the rotor, there is no torque pulsations, smooth torque we can achieve, but how to achieve the sinusoidal flux densities at the air gap is a question. To achieve sinusoidal flux densities at the air gap, we have to give the effort variable, that is, MMF N into I also a sinusoidal fashion.

To achieve sinusoidal MMF waveform, we have to distribute the conductors in the stator core, n number of conductors we have to distribute on the stator core. Then only we can attain the sinusoidal MMF waveform. With respect to the practical limitations, the number of slots inside the core and slots per pole per place number also limited. Because of these regions, we will not get the sinusoidal MMF waveform at the stator side. We will get the stepped type of MMF waveform, whether it is 3 step or square waveform or multi stepped waveform with respect to the different winding patterns.

We will discuss in detail and harmonics associated with this kind of MMF waveforms also we will analyze. That harmonics, we will call it as spatial harmonics with respect to MMF waveform. This MMF waveform and MMF value or expression, we will utilize it to analyze the magnetizing inductance as well as magnetizing current LMS and IMS. Both parameters with respect to the equivalent circuit of the machine, we required the expression for stator fundamental MMF magnitude as well as expression. So, we will see that expressions and magnitudes with respect to the different type of windings.

First, let us consider the concentrated winding 2 pole 3 phase machine. All phases are symmetrically distributed mechanically as well as electrically excited with 120 degree supply and it is a single layer winding full pitch. Let us consider a single phase conductors where the slots per pole per phase, I am considering only one. So, this side one conductor and this side one conductor with respect to the one particular phase. For easy analysis, we will derive the expression for one phase first.

Then at the end, we will add the MMF equations with respect to all three phases and we will derive the single equation with respect to stator MMF. Here, I am considering two conductors, one is under north pole, other is under south pole. The current under north pole side conductors is cross and south pole side conductors it is dot and this is the stator inner diameter and some air gap will be there that is L g and the dotted line is representing the rotor outer diameter. The gap is air gap length L g and we have to establish the flux density B g value at the air gap. As per the Astar's magnetic principles, we can see with respect to the cross, the magnetic flux lines are clockwise direction.

Apply the right hand thumb rule and we can find the magnetic flux lines and with respect to the dot, it will be anticlockwise direction. So, anticlockwise means in this direction, the resultant B fields are in this fashion, the flux lines are flowing from north to south. If we will observe this flux loops, the flux lines are flowing from stator to air gap, then rotor, then back rotor to air gap, then stator. We can see this line. So, flux lines are flowing from stator core to air gap.

This portion is the air gap, then again rotor, then here again air gap region, then coming back to the stator core. If we will apply the Ampere's law with respect to this magnetic flux line path, H dot d L is equal to total m m f is n into I. Here, n is nothing but number of turns per phase. Here, only one coil side with respect to one phase is considered. Total 6 slots are there.

Each phase is conductors are placed in 2 slots. So, this H dot d L integral, we have to apply along the flux loop with respect to the stator core H s into L s plus with respect to the air gap H g into L g plus rotor core will be H r into L r. L r is the magnetic length with respect to the rotor core, the flux line path. Then, H g L g here, I am considering symmetrical air gap in both regions. In this region as well as in this region, the air gap I am considering symmetrical.

So, the length of air gap is same and H g also I am considering same value that is equals to n into I. If the stator core as well as rotor core having a permeability value mu r is high in both stator side and rotor side, then the magnetization required or m m f required to magnetize the core material is very small. With that, we can neglect these two terms and the maximum amount of m m f is required to establish the required flux density at the air gap. H s into L s plus H r into L r also value will be there that is very small. Here, H g into L g is equals to n I by 2.

This is the m m f or magnetic potential between the stator and rotor. This is stator side and this is rotor side. If we see with respect to the air gap, this magnetic potential between the stator and rotor at any point along the air gap will be same that is n I by 2 with respect to this expression. Now, we will see the m m f waveform and we will analyze the equation for stator m m f. This is cross and this is dot is under north pole, this is under south pole with respect to this coil side or conductors n into I.

So, the step change will be n into I and here, current is coming out and step change will be again n into I. So, the total step change will be n into I always, but because of the magnetic potential between stator and rotor along the air gap is always n I by 2. If I will consider the symmetrical axis that is this one with respect to the symmetrical axis n I by 2 at the top side n I by 2 at the bottom side and consider the y axis reference will be this thing and theta is in this direction and this is 0 and this value is pi and this is 0. This distance is pole pitch that is tau p and peak magnitude with respect to 0 axis is n I by 2 already mentioned. So, for this kind of square wave, we can derive the expression by utilizing the Fourier series.

It is an odd function a n value and a naught value are equals to 0 or we can consider it has the symmetry with respect to the origin and b n value we have to calculate and final f of theta. We have to find the expression to find the MMF with respect to this kind of square wave MMF that is for concentrated winding where all conductors are placed at one particular place under one pole. So, all related a phase conductors are placed on one particular place at north pole and one particular pole place at south side pole. So, here b n value is equals to fundamental magnitude of this particular MMF waveform because other a n and b a n and a 1 are 0. So, in 2 by pi integral 0 to pi f of theta is nothing but n into I by 2 into sin n theta d theta.

Calculate this, solve this expression, then we can get 4 by pi n into I by 2 into 1 by n. Here n is equals to odd 1, 3, 5 and so on and the expression for MMF wave with respect to the square wave shape f of theta is equals to 4 by pi n I by 2. This is I a, if I will consider it is a phase current n into I, if requires n I a, we can consider into sigma n equals to 1, 3, 5 up to infinity and sin n theta divided by n. This is the expression for MMF with respect to square type of wave. Next, we will discuss with respect to the short pitch winding.

Short pitch winding, consider an example, double layer winding, free phase and slots per pole per phase will be 1 and the number of turns we can see here. Earlier, the number of turns are placed in one slot under north pole and other slot under south pole. This is the concentrated winding we have discussed. The conductors are connected in this fashion. In the fractional pitch or short pitch windings, instead of placing conductors at one particular place, we will distribute into two slots in this fashion.

These two conductors, instead of placing in this slot, we are pushing left wards. Now, the earlier coil pitch is nothing but pole pitch. Let us say from this point to this point is pole pitch. Now, the coil pitch is reduced by one slot that is angle will be alpha. Here, the conductors are connected.

These two conductors will be connected and these two will be conductors will be connected. In a circular fashion, if we will see the coil pitch will be same, these two will be connected and these two will be connected. We have to see that in a circular manner, then this pitch as well as this pitch both will be same and it is short pitched by one slot as compared to the concentrated winding. Let us consider the number of turns are displaced in this fashion.

The slot angle is alpha. This side also, the conductors are displaced in this fashion. This is alpha and current is entering at this point as well as at this point and leaving here at the opposite pole. This is under north pole and these two conductors are under south pole. Now, with respect to the MMF waveform, the turns, ampere turns are splitted into two parts. Now, earlier here, N i ampere turns placed at one particular place here, N i by 2 and Ni by 2.

So, the step here is initially N i by 2. Then, again one more step we will see N i by 2. Then, the peak step will be N i only, the combination with respect to these two conductors. At this particular point, N i by 2 ampere turns will come down and then at this particular point, it will come down to this one. Here, the peak value from this point will be N i.

So, if we will consider the reference axis with respect to the magnetic potential difference between the stator and rotor, it is always N i by 2 along the air gap length. I am considering this is the x axis reference, where top side N i by 2, bottom side also N i by 2 and the angle between these two coils is alpha, that is, short pitch angle. I am considering the 0 axis with respect to the y axis is this one reference. This is 0 and this angle is alpha by 2 and this angle is pi minus alpha by 2 and this is pi. Now, we will apply the Fourier series for this kind of waveform and we will analyze the expression for stator MMF.

Here, also seen the magnitude of MMF f phase is equal to an value and a value in the Fourier series are equal to 0, because it is having a symmetry with respect to origin and it is a odd function. Only b n value will be there, f phase fundamental magnitude will be b n and it will be 2 by pi integral 0 to pi f of theta into sin n theta d theta. So, here 2 by pi integral 0 to alpha by 2, the MMF from 0 to alpha by 2 is 0 into sin n theta d theta plus integral alpha by 2 to pi minus alpha by 2 into here, MMF is N i by 2 into sin n theta d theta plus integral pi minus alpha by 2 to pi, MMF is 0 here, sin n theta d theta whole bracket and we will once if we will solve this equation, finally, we will get the expression 4 by pi n into i divided by 2 cos n alpha by 2 divided by n. Here, alpha is equal to short pitch angle. If we will substitute this magnitude of MMF in the MMF expression, that is f p of theta is equals to f phase into sin n theta 4 by pi Ni by 2 sigma n equals to 1, 3, 5 and so on infinity sin n theta by n into cos n alpha by 2.

This term representing the pitch factor, we can calculate by EMF calculation method also. For alternators, we can calculate the EMF expressions with respect to full pitch winding and short pitch winding. From there also, we can derive the pitch factor cos n alpha by 2. Here, k p is equals to k p n cos n alpha by 2. This is the final expression with respect to the short pitch winding equation number 2.

Next, we will discuss with respect to the distributed windings, MMF waveform as well as expression. Let us consider, instead of placing the conductors in one particular slot with respect to north pole and south pole, the phase spread, we will consider q equals to 3. So, that means, the conductors we will place in 3 slots. To get the multi stepped wave MMF, we will split these 3 slot conductors placed in these 3 slots into 6 slots in this fashion. Similarly, this side also, we are distributing the same number of conductors in 6 slots.

Consider, it is a 2 pole machine and double layer and q value is equals to 3 and effective coil span is equals to 2 q and slot angle will be beta. So, let us consider the conductors in this fashion. 6 conductors will be there and this side also, 6 conductors and the slot angle will be beta by 2 from 1 slot to 1 slot.

It is beta. Here, also beta like this. The slot angle between one coil side to other coil side is beta and we will analyze the MMF waveform now. The actual n into i conductors are splitted into n i by 2 q. Instead of placing at one particular place, we are distributing into 6 place that is n i by 6. So, the step will be with respect to the MMF n i by 2 q, but the total MMF step change will be always n into i. So, here n i by 2 q with respect to the second conductors or coil sides n i by 2 q again, then another n i by 2 q, then like this fashion, it will go from this point to this point.

It will be n i by n i peak value 0 to n i. It is varying, but if you will take the reference axis, then this peak value will come n i by 2 with respect to the reference from this point to this point. If you will consider the change here is n into i. At this particular point, n i by 2 q step change will be there and again n i by 2 q, here again n i by 2 q will come. So, this is the bottom line we can see here.

So, the step change value is n i by 2 q. If you will consider the reference with respect to the x axis that is this line, this one is the reference x axis theta and reference y axis. We can select at any point. For now, we can consider at this particular point, here this angle will be beta by 2 and up to this point, it will be pi 1 pole pitch. So, with respect to the slot angles, we can calculate the fundamental magnitude of this waveform by utilizing the Fourier series. For simplicity, we will utilize the concentrated and short pitch winding MMF equations to find the MMF equation for the distributed winding.

Otherwise, there are two ways to calculate the fundamental magnitude of MMF with respect to the distributed winding. A fundamental magnitude is equal to b n value, because a n and a naught are 0, 2 by pi integral 0 to pi f of theta sin n theta d theta, we can apply and we can calculate the fundamental magnitude. Second method, we know fundamental magnitude with respect to the coil 1. Let us consider this is coil 1, coil 2, coil 3, coil 4, coil 5, coil 6 and so on. So, with respect to the coil 1, by considering this expression, this is the fundamental magnitude with respect to one particular coil.

So, I will copy this thing. Here for c 1, we can take the same value of the fundamental magnitude. Alpha should be replaced with beta. Here, slot angle we are considering, that is beta by 2 and fundamental magnitude with respect to the coil 1 is this one and with respect to the coil 2, fundamental magnitude f s is equal to 4 by pi n into I by 2 into cos n into beta by 2 plus beta. Second coil displacement is beta angle divided by n. Similarly, third coil plus 2 beta, this will be same 4 by pi n by 2 cos n into beta by 2 plus 2 into beta by n and so on up to nth coil, we can calculate it.

Finally, in order to find the magnitude with respect to all coils, we have to add the fundamental with respect to the coil 1, the m m f magnitude with respect to the coil 2, m m f magnitude with respect to the coil 3 and so on, f c q minus 1, 2 into q minus 1. So, here, 2 q is the phase spread. So, up to that coil here, it is 6 number of coils are there. 6 terms will come. Add those 6 terms, then we will get the final expression for the fundamental m m f magnitude 4 by pi n into I by 2 q into n cos n into beta by 2 plus cos n into beta by 2 plus beta plus and so on cos n into beta by 2 plus 2 q minus 1 into beta, because first coil, the coefficient is 0 into beta.

Sixth coil, it will come 2 q minus 1 into beta. Solve this equation and finally, we will get the fundamental magnitude 4 by pi n into I by 2 into n sin n into q into beta by 2 divided by q into sin n into beta by 2 into cos n beta by 2. So, this factor is nothing but distribution factor and this factor is nothing but pitch factor. So, here, if we will consider beta is equal to short pitch angle, this term represents the pitch factor and this term represents the distribution factor. Substitute this f phase in the Fourier series equation, f of theta is equal to f phase into sin n theta sigma n equals to 1 to infinity sin n theta. So, this will give the final expression for m m f with respect to the distributed winding.

This is equation number 3. Next, as of now, we have discussed the m m f with respect to 2 poles. If the m m f with respect to the n number of poles or p number of poles, what is the fundamental m m f expression for p poles? So, here, f phase is equal to magnitude. f phase is nothing but magnitude of m m f 4 by pi into N into I divided by 2 into, this is the magnitude we have derived, right. This is k d into k p, k d into k d n and k p n with respect to all harmonics and we will consider the skewing factor k s q n and slot opening factor k phi n. These two terms we will discuss later and this is the total winding factor k n.

We can represent with k n and divided by n. This is the fundamental magnitude with respect to one particular phase for p number of poles. As of now, for 2 poles, 2 we have discussed, right. Now, for p number of poles, it will be N I by p. Only this change will come with respect to this equation, k d into k p divided by n and instead of 2, just replace with p. Now, finally, f a, f b, f c, the m m f expressions with respect to all three phases in a balanced three phase machine, we have to calculate.

This is the magnitude with respect to one particular phase. For a phase, it will be n into I a, for b phase, n into I b and sign terms will change. Those things we will see now. So, f a is nothing, but m m f expression with respect to a phase, that is, 4 by pi n into I a divided by number of poles into sigma n equals to 1, 3, 5 up to infinity, k n is the winding factor divided by n into sign n into theta. So, here theta I am representing in electrical angle, that is, p into theta by 2.

This is for a phase. Then, for b phase, same expression, n into I b divided by p sigma n equals to 1, 3, 5 up to infinity, k n divided by n, sign n into p by 2 into theta minus 2 pi by 3 into 2 by p. If I will consider, this is the total angle, this is for b phase. Same thing for c phase also, we can write 4 by pi n into I c divided by p sigma n equals to 1, 3, 5 and so on up to infinity, k n winding factor with respect to all harmonics divided by n, sign n into p by 2 theta minus 4 pi by 3 into 2 pi, 2 by p. This is the f c equation. Now, in a balanced three phase current excitations, I a is equals to I s cos omega e t, I b is equals to I s cos omega e t minus 2 pi by 3 and I c is equals to I s, the peak magnitude into cos omega e t minus 2 pi minus 4 pi by 3 or plus 2 pi by 3, anything is fine.

So, substitute these three current relations in the above expressions and add all those things, like the total stator m m f f of theta is equals to f a plus f b plus f c. In these equations, we have to substitute all current expressions also. Then, the final expression will come 3 by 2 into 4 by pi. I am not solving here. We can solve offline these things 3 by 2 4 by pi into N phase into I s is the peak magnitude.

I mentioned, I think, capital I s. This is N into capital I s divided by p into sigma n is equals to 1, 7, 13 and so on up to infinity k n by n sin n into p by 2 into theta minus omega into t plus sigma n equals to 5 and so on up to infinity k n by n sin n p theta by 2 plus omega e t, like this will come the final expression. Solve in between in this place and final fundamental stator m m f required to establish the flux density at the air gap, that is, f s 1 with respect to the fundamental. So, 3 by 2 into 4 by pi into n into I s. Here, n is nothing, but number of turns per phase into I s is the peak value of the stator current divided by p into winding factor. So, this expression, we will utilize it in most of the places to calculate the magnetizing inductance as well as magnetizing currents and other thing.

This is the peak fundamental stator m m f required to establish the required flux density at the air gap. So, with this, I am concluding this lecture. In this lecture, we have discussed the m m f expressions with respect to the concentrated winding, short pitch windings and distributed windings and the relation for stator fundamental m m f with respect to one particular phase and all three phases also, we have discussed. Thank you.