Course Name: Design of Electric Motors

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Title: Force Equations in Electromechanical Systems-2

Greetings to all, in the last lecture we have discussed regarding the force equations with respect to the magnetic circuits. In this lecture we will discuss force equations only in terms of air gap magnetic fields and energy stored at the air gap, okay. That analysis we will see and then multiple winding excitation what is the force equation we will see. So, first I will start with energy stored in the air gap or force equations in terms of air gap fields. Consider the same magnetic circuit, I will redraw the same magnetic circuit. Consider a I core, sorry C core and a I core which is moving and permeability is mu and area of cross section is Ac, okay and C type core has a winding of number of turns N and current is I, okay and excitation voltage is V and as per the Orsted's magnetic principles we can see here magnetic fields are in this direction, okay.

Now, we will apply the Ampere's law, okay. The MMF is N into I is equals to Hc into lc plus Hg into lg. How we have concluded this equation in the last lectures? MMF is equals to integral of H dot dl, right. So, if we will apply a integral over the magnetic lines or magnetic loop, then we will end up with this equation.

From here number of turns N equals to Hc lc plus Hg into lg divided by I, okay total MMF divided by current. This is equation 1.

Now, in the last lecture, we have discussed the field energy, right. The field energy expression is integral I d psi, right, okay. So here, substitute I value from equation 1, okay.

For easy understanding, we can replace I and N, okay. Substitute equation 1 here and flux linkage is equals to what? L into I. Here, inductance is nothing but N into flux divided by I into I. So, it will result in N phi that is equals to NBA.

So, I am substituting equation 1 as well as equation 2 in this equation, okay. So, the equation will look like in this manner. Hc lc plus Hg lg by N into derivative of N into B into A. So here, number of turns is a constant and area cross sectional area of the core also a constant. So, if I will bring the N into A outside, then this integration will result in this manner Hc lc into N into area of cross sectional area of the core divided by N into d Bc plus integral of Hg lg into N into Ag divided by number of turns N into d into Bg, okay. Here Ac is the cross sectional area of the core and Agis the cross sectional area of the air gap, okay.

If I will neglect the fringing effect and if I will consider the Ag equals to Ac, okay. So, the fringing flux is nothing but the flux which is going outside the area of Ac, okay. This is nothing but fringing flux which is coming outside the core cross sectional area. Here flux equals to B into A, right, flux magnetic field lines over an area. So, phi G equals to BG into AG and it is equals to phi C that is nothing but BC into AC because we are considering the uniform flux over a core.

Now we can conclude that AC equals to AG, then BG equals to BC, okay. In the earlier equation, field energy equals to integral of HC LC into AC into derivative with respect to the flux density BC plus integral of HG LG AG magnetic field intensity at the air gap, length of the air gap, mean length of the air gap and AG is the cross sectional area of the air gap. Here air gap is LG by 2 into derivative with respect to the DBG. Anyhow magnetic fields at air gap and magnetic fields at the core both are equal. So, we can consider same also.

So, here Hc value and Hg value are both are same, it is both are not same. HC is the intensity in the core is much smaller than HG because BC and BG both are same, but the permeability with respect to core is mu naught into mu r. Here BG is nothing but mu naught. So, this value is larger. BG by mu naught value is higher as compared to the core flux intensity BC by mu naught mu r.

So, the first term, the magnetic field energy with respect to the core, this term we can neglect it and second term we can consider it. So, the first term with respect to the field energy in the core and second term field energy with respect to the air gap. So, if I will consider the first term, I am neglecting it, but if I will consider the field energy with respect to the core is equals to HC LC AC into D derivative of magnetic field, magnetic fields. So, it is equals to here LC and AC, I can replace with volume, HC into volume into magnetic fields derivative. So, the magnetic field of core with respect to the volume is nothing but energy density of the core that is equals to integral of HC into D BC.

So, this term completely we are neglecting in the above equation. This is equation 3. In equation 3, I am neglecting the first term. So, the equation 3 will result in field energy with respect to the air gap or with respect to the complete system because the field energy with respect to the core is small or negligible. So, integral of HG LG into AG into DBG. So, this integral HG is nothing but BG by mu naught LG AG into derivative of magnetic fields. So, this equation finally results in BG square by 2 mu naught into LG into AG. So, this is field energy at the air gap. Now, if we will find the force equation, force equals to what partial derivative of field energy with respect to the air gap length LG I am considering. Instead of displacement, I am considering here LG.

For a linear system, both field energy and core energy are equal. We can do the partial derivative and finally, force is nothing but B square by 2 mu naught into AG will come. So, this is the final force equation in terms of B and mu naught. Torque equation if you want, we have to partial derivative the field energy with angular displacement. From the equation 6, force per unit area results in magnetic pressure.

We can call this term magnetic pressure that is B square by 2 mu naught.

Next we will discuss about the multiple winding excitations. Here the units for force are Newton. Here the force per magnetic pressure or units are Newton per meter square. Now, we will see the force equations with respect to the multiple winding excitation.

Here a magnetic circuit, I am taking the same type of magnetic circuit, one side the stationary C type core with a permeability mu and cross sectional area AC and other side moving I core which has a permeability mu and area is again AC. Here the air gap length is LG by 2, this distance LG by 2 and consider the coils at the primary side or stationary C core is the first coil, current flowing is I 1 and number of turns are N 1 and the moving I core also consists of a coil with a number of turns N 2 and current is I 2. As per the magnetic Orsted's principle, we can find the flux directions or apply a thumb rule with respect to the first coil. We can see this side with respect to the second coil, this side. So, both fluxes are in the same direction that is clockwise manner, so green color one.

The analysis and everything will be same as the single winding excitation. Next, I will start with field energy. The field energy expression is nothing but field energy is equals to I d psi, integral of I d psi. With respect to these two coils, what is the change in electrical current? Change in electrical energy is nothing but V 1 I 1 plus V 2 I 2 into dt, change in electrical energy. Now, the change in magnetic field energy also same.

It is equals to change in magnetic field energy under the condition losses equals to 0 and mechanical output is equals to 0. There is no mechanical output and losses in the system are 0. So, if I bring this equation here, the field energy is equals to integral of V 1 I 1 plus V 2 I 2 into dt. So, here V is nothing but what? State of change of flux linkages d psi 1 by dt into I 1 plus d psi 2 divided by dt into I 2 into dt integration. So, this will result in integral of I 1 d psi 1 plus I 2 d psi 2.

Directly also from this equation, we can write the field energy equation, integral of I 1 d psi 1 plus I 2 d psi 2. So, this is the field energy expression.

Once we know the field energy, then we can find the force equation, but here what is the flux linkages? In the last lecture, we have derived that flux linkages phi 1 equals to L11 I 1 plus L 12 I 2. L 12 is the mutual inductance and L 11 is the self inductance. So, we can write this equation like L 11 into I 1 plus M into I 2.

Similarly, flux linkages with respect to the coil 2 is nothing but psi 2 that is equals to L 22 into I 2 plus mutual inductance into I 1.

So, if we will substitute this psi 1 and psi 2 in this equation 2, earlier one I am taking equation 1. If I will substitute these two equations in equation 2, then field energy is equals to integral of I 1 d into derivative of L 11 into I 1 plus mutual inductance into I 2 plus integral of I 2 into derivative of L 22 into I 2 plus mutual inductance into I 1. So, from this equation, we can conclude that I 1 into d I 1 L 11 is a constant plus M into integral of I 1 d I 2 plus L 22 integral of I 2 d I 2 plus mutual inductance integral of I 2 derivative of I 1. So, this term and this term, we can write it with respect to the UV principle like derivative of U and V is nothing but what? U into dV plus V into dU.

So, if we will consider I 1 and I 2 equals to x, then dx equals to what? Same thing, I 1 into d I 2 plus I 2 into d I 1.

So, as per this analysis, we can replace these two terms with M into integral of derivative of I 1 I 2 or I can say that derivative of x, I am assuming I 1 I 2 product as a variable x. So, dx M into x. So, here x is nothing but mutual inductance into I 1 I 2. So, this term in the above equation that will result in L 11 into I 1 square by 2 plus mutual inductance into I 1 and I 2 both terms and L 22 into I 2 square by 2.

This is the final field energy expression equation 3. So, this is with respect to the self inductance of the stator side coil and this is the self inductance term with respect to the moving I bar and this is the mutual. Now, the field energy equation is derived. So, from this equation, the force equation can be derived. F equals to what? Derivative of partial derivative of field energy with respect to the displacement where psi is constant and minus term and in a linear system partial derivative of co-energy with respect to the x where I is constant.

Any equation we can utilize it. Now, if we will do the partial derivative of above equation, then the force equation is nothing but half I 1 square into d L 11 by dx plus half I 2 square d L 22 divided by dx plus I 1 I 2 into d M by dx.

In this equation, we can see that this term and this term are the forces with respect to the reluctance terms. Based on the reluctance, we are getting the force here. In this term, we are seeing the force with respect to the mutual interaction between the two fields. Mutual interaction between either two currents or two fields here, the interaction with respect to the one source and reluctance.

If we will find the torque equation in terms of angular displacement for a rotating systems, then the torque equation is nothing but half I 1 square d L 11 divided by d theta. That is change in self inductance with respect to the angular displacement plus half I 2 square change in self inductance with respect to the rotating body over a angular displacement d theta plus mutual inductance term d M by d theta.

How mutual inductance is varying with respect to the angular displacement? This is the final torque equation. Based on this torque equation in the coming lectures, we will see the different machines. What kind of torque components can be available? Like this kind of reluctance torques, we can observe in switched reluctance machine or seen reluctance machines.

Mutual torques are interaction of two fields attraction and repulsions of two fields. We will see in DC machines, synchronous machines and induction machines. Most of the machines, we will see the interaction of two fields that is the mutual torque. We will see in the reluctance machine where the torque will be developed based on the variable reluctance or variable inductance thing is nothing but the reluctance type of torque components.

With this, I am concluding this lecture. In this lecture, we have discussed the force equations in terms of magnetic fields and energy stored at the air gap. Also we have seen and the force equation for a multiple excitation systems we have discussed. Thank you.