Course Name: Design of Electric Motors

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Title: Force Equations in Electromechanical Systems-1

Greetings to all, in the last lecture we have discussed magnetic circuit analysis with respect to air gap, without air gap, single excitation and multiple excitations and permanent magnets. So, in this class we will discuss the forces involved in the magnetic circuits or electromechanical systems. If I will consider a system, where the input is electrical energy and output is mechanical energy, generally the system which converts electrical energy to mechanical energy, we will call it as a motor and mechanical energy to electrical energy, we will call it as a generator and electrical energy to electrical energy, we can say the transformers. So, here I am considering the electrical to mechanical energy converting system, where the medium is magnetic circuit. At the electrical circuit side, what we have? Coils and sources like voltage source, current sources and different components with respect to the electrical circuits and magnetic circuits, what we have? Cores like magnetic cores and air gaps. At the mechanical side, mechanical energy side, what we have is like electrical rotors, where in the electrical missions the rotating body is the rotors and electromagnets, all moving systems.

So, we will see how to realize the force equations in the magnetic circuits. Let us consider a circuit, magnetic circuit with C type core at one side that is stationary core, it is the stationary one and relative permeability is mu and area of cross section is AC. And consider the other side, we have the moving core that is I type moving core, area of cross section is AC and relative permeability is mu C or mu, same as the C type core. And this moving I bar is connected to a fixed support through a spring, the spring has a spring constant K and the air gap is LG by 2, here also same LG by 2.

And C type core is excited with a winding, consist of a number of turns N and a current I and a series resistance R and voltage V. Now, we will see the analysis of this system. With respect to the earlier analysis like here, we are giving electrical energy as a input right, that is equals to the energy with respect to the magnetic circuit like magnetic field energy plus mechanical energy plus the losses in the system. Here, the losses are one part is I square R loss with respect to the electrical circuit and second part is magnetic core loss with respect to the magnetic circuit, that core loss are nothing but hysteresis losses as well as eddy current losses. And other side with respect to the mechanical side, mechanical losses like friction and windage losses.

So, this is the energy equation. We will stick to the same equation with respect to this magnetic circuit. So, if I will consider the lossless system, where the losses in the system is equals to 0. Now, the electrical energy is equals to the energy present in the magnetic circuit, that is magnetic field energy plus mechanical energy. If I will represent this equation in a differential form, d We the change in electrical energy with respect to the differential form, change in magnetic field energy plus change in mechanical energy.

This equation is in a differential form. Now, consider the system where the mechanical output is equals to 0. There is no mechanical output. For the analysis, first we will consider mechanical output is 0. After that, we will consider the actual mechanical output.

If I will consider Wm equals to 0, the differential form, the change in electrical energy is equals to the field energy. Change in electrical energy is equals to change in magnetic field energy in a differential form. As per the Faraday's law, V equals to what? The rate of change of flux linkages from the Faraday's law d psi by dt. Here, psi equals to flux linkages L into I. L equals to what we have discussed in the last lecture, either N phi by I or N square by reluctance.

I am considering N phi by I into I that is equals to N into phi. So, phi we can replace with flux magnetic fields over an area B into A. Now, what is the power B into I? V equals to the rate of change of flux linkages d psi by dt into I. This is equals to the rate of change in magnetic field energy. If I will bring this equation 1 in this equation 2, then the differential form change in electrical energy is equals to I d psi.

So, if you want the field energy with respect to the magnetic circuit is nothing but integral of I d psi from 0 to maximum value of flux linkages psi, the integration limits. If we will see the flux linkages versus current waveform, it will be in this manner, flux linkages and currents. This is with respect to the displacement x0 and displacement x1. Here, the displacement x is nothing but from the fixed support to the reference point that is x that is a displacement with respect to the I type core. Here, the forces acting on the moving core, this is the moving core.

Here, the forces are with respect to the spring force that is F spring is acting in this direction and this side electromagnet force where exciting with the coil and the induced MMF will results to attract this iron piece and here B fields will be in this direction. So, based on the attractions, this core will try to move in this direction, but this spring will try to pull in this direction. Now, again I am coming back to the flux linkages versus current waveform. For a given flux linkages, we can see the area here. The area with respect to the x1 is this one.

Area with respect to the x0 is yellow color 1 and area with respect to the x1 addition of this yellow color highlighted area plus this green color area. So, for a given flux linkages psi, the field energy is depending on the flux linkages as well as displacement. It is a function of flux linkages as well as displacement. Now, I am considering the same flux linkage versus current waveform. This is the psi maximum psi and this is the current.

The area above the curve that is this one is nothing but field energy. The area below the curve that is this one, yellow color one that is represented as co-energy. Co-energy is represented with W fld. The co-energy does not have any physical significance, but we will utilize it for the mathematical analysis. So, here W' fld and apostrophe is nothing but the co-energy and which does not have any significance.

It is a virtual quantity and we will utilize it for only mathematical analysis. So, area with respect to this rectangular box, area with respect to this one is nothing but psi into i is equals to field energy plus co-energy. Area above the curve as well as below the curve is equals to psi into i rectangular box area. Now, we will see the analysis for the force equations with respect to the field energy. Now, the field energy is equals to from the energy equation that is d change in electrical energy with respect to the differential form and field energy plus mechanical energy in differential forms.

Here, I am considering the mechanical output where Wm is not equals to 0. I am considering back mechanical output is there in the circuit. So, the change in magnetic field energy is nothing but change in electrical energy minus change in mechanical energy in a differential form. Here, the dWe is nothing but integral of i d psi. What we have seen earlier is the differential form of change in electrical as well as differential form of change in magnetic field energy is equals to i d psi.

Here we can say dWe is also i d psi minus mechanical turn. So, here mechanical output the change in mechanical output with respect to time is equals to force into velocity d x by dt. The differential form is nothing but the change in mechanical output is equals to force into displacement dx change in displacement. So, substitute this equation here that will result in i d psi minus F into dx.

So, this is equation 3. Now, from the flux linkage versus current waveform, we have concluded that the field energy is a function of flux linkages and displacement x. If I represent this function equation 4 in a differential form that is dW field energy is equals to partial derivative of field energy with respect to the psi where x is constant into d psi plus partial derivative of field energy with respect to the displacement where psi is constant dx. So, from equation 3 and 5, if we will compare equation 3 and equation 5, the current i equals to one mistake here the change in electrical energy is equals to i d psi. We can see from this equation. So, if we will compare the equation 3 and equation 5, i equals to partial derivative of field energy with respect to the flux linkages where x is constant that is displacement is constant and force is equals to minus the partial derivative of field energy with respect to the displacement where psi is constant.

So, this is with respect to the linear displacement. If we will consider the rotating missions where torque is there with respect to the angular displacement, then torque equals to change in field energy partial derivative of field energy with respect to the theta. This is nothing but torque equation in a rotating missions. Next, we will derive the force equation with respect to the co-energy. So, co-energy is represented with this one and from the rectangular area equation flux linkages into current is equals to field energy plus co-energy.

Now, if we will represent this equation in a differential form, field energy is equals to sorry field energy plus derivative of co-energy is equals to derivative of flux linkages into current. So, this will result in psi di plus i d psi. So, the differential form of co-energy is equals to psi di plus i d psi minus differential form of field energy. So, this is equation 7. So, this complete set I am representing with equation 6, then this is equation 7.

So, from equation 3, we can see here from the equation 3, the differential form of field energy I will substitute in equation 7. Then the change differential form of co-energy is equals to psi di plus i d psi minus i d psi minus F dx. Now, these two terms will cancel each other. The remaining terms are psi di minus F dx. So, this is the differential form of co-energy.

So, co-energy is a function of what? If we will see the flux linkages versus current waveform again. For a given value of a current, the area under the curve and this one is with respect to this one, this is x0 and x1 with respect to the change in displacement, the area under the curve or the co-energy is changing. The co-energy is a function of I comma x. So, if we will represent the co-energy in a differential form, the differential form of co-energy is equals to partial derivative of co-energy function with respect to I as well as with respect to x. So, partial derivative of co-energy with respect to I, where x constant d I plus partial derivative of co-energy with respect to the displacement, where I is constant into d I.

So, this is equation 9. So, from equation 8 and 9, the current equals to sorry flux linkages equals to psi equals to partial derivative of co-energy with respect to the current, where x is constant and current remaining thing force equals to partial derivative of co-energy with respect to the displacement, where I is constant. So, this is equation 10. So, the force can be derived from the co-energy as well as field energy. So, the final force equations, force with respect to the field energy is a partial derivative of field energy with respect to the displacement, where psi is constant and torque equals to partial derivative of field energy with respect to the angular displacement. And with respect to the co-energy, partial derivative of co-energy with respect to the displacement, where I is constant and torque equals to same thing partial derivative of co-energy with respect to the angular displacement, where I is constant.

So, these are the final force and torque equations. Now, we will see an example for a linear system. Whatever the magnetic circuit we have seen, for this magnetic circuit, if I will consider as a linear circuit, as of now we have done the analysis, the circuit is a non-linear with respect to the characteristics and other. Now, I am considering a linear circuit, where the energy stored in the magnetic circuit, that is, field energy is equals to half l I square. Here, I is equals to what? Flux linkages divided by inductance, right.

We can represent in this manner. So, field energy is equals to half psi square divided by l of x. Substitute this in a field energy equation in this one, okay. Final equation if I will say this is equation 11. Substitute equation 12 in equation 11. Force with respect to the co-energy, sorry, force with respect to the field energy is nothing but partial derivative of field energy with respect to the displacement.

So, that is equals to minus half partial derivative with respect to the displacement x half psi square divided by l of x. This will result in I square by 2 dl of x divided by dx. So, this is the final force equation with respect to the field energy. Further, with respect to the co-energy, it is simple like co-energy is equals to field energy is equals to half l I square.

For a linear system, both are equal. So, the force is equals to partial derivative of co-energy with respect to the displacement that is equals to half I square into dl of x divided by dx. Directly, we can derive it. So, from equation 13 and equation 14, we can conclude that force is same with respect to the field energy and with respect to the co-energy, okay, and torque is nothing but half I square dl by d theta. For rotating system, we have to consider the angular displacement that is theta. Now, what is the actual force acting on I bar with respect to the spring constant? So, the force is minus the spring coefficient into displacement is equals to mass into acceleration.

This is the actual force, right. So, the force generated by the electromagnet should be equals to m into acceleration plus spring constant into displacement. If you want to design a magnet, the electromagnet that has to produce a force of Fm is greater than the mass into acceleration plus spring force, okay. So, in this class, we have discussed the force equations in a magnetic circuits with respect to the linear or non-linear. This equation will valid equation 11. For linear systems, directly we can consider field energy and co-energy both are equal and we can solve with respect to this equation for a linear system, okay, and torque equation is this one.

So, with this, I am concluding this lecture. Thank you.