

Real – Time Digital Signal Processing
Prof. Rathna G N
Department of Electrical Engineering
Indian Institute of Science - Bengaluru

Lecture – 08
FIR Filters

Come back to real time digital signal processing course. Today we will be discussing about module 2, unit 7 FIR filters.

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Recap



- In Module 1- W01U1 - W01U6 discussed Introduction, Design concepts and Real-Time Constraints.
- In this class, we will discuss Module 2, the first part have it is Digital Filters

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So, as a recap we saw in the module 1 which goes from unit 1 to 6 introduction, design concepts and real time constraints what we have considered. In this class in the module 2 first we will discuss about digital filters and later with FIR filters.

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Digital Filters



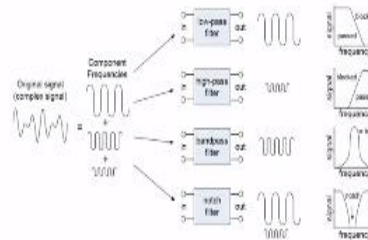
Why filters are used in digital signal processing?

- (1) separation of signals that have been combined, and
- (2) restoration of signals that have been distorted in some way.

• Analog (electronic) filters can be used for these same tasks; however, digital filters can achieve far superior results

Where are digital filters used?

1. Digital Signal Processing
2. Communications Systems
3. Control Systems
4. Electrical Systems
5. Biomedical Systems



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So, why do we need digital filters? So, where we are going to use it in signal processing? For separation of signals is one of the application we say, as you can see in the example here, your original signal has multiple frequencies components present in it, and you can see that how they have been added and each one how we can separate it from the signal using different techniques. So, we will be seeing in a while here it is low-pass filter, this gives the high-pass filter and this is bandpass and this we call it as notch filter or band stop filter if only one frequency has to be removed, then we call it as notch filter. The one application is restoration. So, from these signals, I want to reconstruct my signal then as you have seen, the thing will be adding all of them and I will be getting back the original signal. So, if I want to restore back I will be using that. When coming to analog that is electronic filters can be used for this same task. However, digital filters can achieve far superior results.

So, we will see in a short while how it is going to be done with digital filters and the application of digital filters is given here one is in the signal processing what we are considering it, it can be in the communication systems or in control or in electrical systems or in biomedical systems everywhere we need the filters.

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Advantages of Digital Filter



- Many input signals can be filtered by one digital filter without replacing the hardware.
- Have characteristic like linear phase response. ...
- The performance does not vary with environmental parameter.
- Unlike analog filters; the digital filters are portable.



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What are the advantages of digital filters? So, we know that many input signals can be filtered by one digital filter without replacing the hardware. And then they have the characteristic like linear phase response. We will see it in a while and the performance does not vary with environmental parameters. So, if you are in a cold place or is in a hot place, as we know in the analog filters, they are components how to be highly permanent for those conditions, if it is not the environment is going to play an havoc.

Hence, digital filters will not have this degradation for environments and that is what, what it says and it is unlike analog filters, these can be portable. So, from place to place and from one application to the other application.

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Disadvantages of Digital Filter



- The **bandwidth** of the digital filter is much lower than an analog filter.
- Quantization noise is present.
- The accuracy of the digital filter depends on the word length used to encode them in binary form.
- It requires more design and development time compared to an analog filter.

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The other disadvantage of digital filter is basically the bandwidth of the digital filter is much lower than that of analog filter and we are going to have quantization noise in this filters. The accuracy of digital filter depends on the word length used to encode them in binary form. So, we have seen the number system. So, we have used fixed point and then floating point. So, there are pros and cons. So, this will be, constituting whatever we say that accuracy is going to be affected.

And it requires more design and development time compared to analog filters. So, one has to see what should be the order of the filter whatever you are designing and then you have to pass through input and then if it is available, and then see and it is going to take more time.

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Difference between Analog and Digital Filters



- The main difference between the two methods is that a digital filter circuit has to sample the analog signal and convert it into a set of binary numbers.
- In contrast, analog filters do not have to do this type of conversion and the signal remains in its pure analog form throughout the filtering process.

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The difference between analog and digital filters is this. The main difference we will consider here 2 methods is that digital filter circuit has to sample the analog signal first and convert it into a set of binary numbers. And in contrast, analog filters need not have to do this conversion. They can be used directly wherever it is required.

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The slide is titled "Filters on Social Media" in red text. In the top right corner, there is a small circular logo with a star and the text "IITM, RSC". Below the title, there is a list of three bullet points:

- Augmented Reality (AR) filters are computer generated effects layered over the real life images in your camera displays.
- In Instagram Stories, an AR filter alters the image of your front or back camera displays.
- Think of Instagram's face filters.

At the bottom of the slide, there is a photograph of a woman with dark hair, wearing a green and yellow sari. To the left of the photo, the number "7" is visible. Below the photo, the text "Ratna G N" is written.

So, coming to some of the recent in the social media what kind of filters is being used is shown here. So, most of you may be using this augmented reality filters are computer generated effects all of you know about it. So, they are layered over the real life images in your camera displays and in Instagram Stories, so, you will have most of this augmented reality filters, they alter the image of your front or back camera displays.

You can think of Instagram's phase filters, how they will be working and then you can imagine what is the role of filters in social media.

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Digital Noise Filter



- In Smart TV it reduces the analog noise that is created during signal transmission.
- Helps to eliminate excess noise in a picture and reduces flickers caused.

How to Eliminate Noise on an FM Receiver?

- Keep any cell phones or two way radios at least 20 feet from an FM receiver.
- Cell phones, even when not in use, send out pings that are picked up by FM receiver.
- Choose a station and adjust the dial to the setting if you are using an analog radio.
- Add a larger external antenna to the receiver.



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Coming to noise filter, digital noise filters, why we need it? In basically in Smart TV, it is going to reduce the analog noise that is created during your signal transmission. Helps in eliminate excess noise in a picture and reduce your flickers caused by them and how we can eliminate noise on an FM receiver. So, you will be knowing that frequency modulated receivers most of the time what we will be using it how we can eliminate the noise in that.

Usually keep any cell phones at 2 way radios at least 20 feet from an FM receiver. And then in cell phones even when not in use, all of us know that send out pings that are picked up by your FM receivers you can choose a station and adjust a dial to the setting if you are using an analog radio. Add a large external antenna to the receiver that is how you will be eliminating your FM receiver noise.

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Noise



How do I get rid of static noise on my mic?

- Try unplugging and re-plugging in your mic.
- Try unplugging your headset or standalone microphone from the computer or device and then re-plugging it back in. If possible, try using a different USB port.
- How do I remove hiss from audio?

- Low pass filtering

How do I stop my mic from picking up background noise?

- Click the "Recording" tab in the Sound window, select your microphone device, and click "Properties." Click the "Levels" tab.
- If you're dealing with background noise, try lowering the Microphone Boost option—perhaps to +10.0 dB instead of +20 dB

Which filter is used in ECG?

- The notch filter is used to eliminate the line frequency and is usually printed on the ECG (e.g. ~60 Hz).

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So, in other way what will see noise is in your microphone basically whatever mic microphone you are using for your applications, they will have some, what we call it a static noise. So, how have you can eliminate the static noise? You can unplug and re-plug in your mic. That way it will try to adapt to the noise which is present there and then it can eliminate it. So, other way of doing it is try unplugging your headset or standalone microphone from the computer or device to which it is connected and then re plugging it back.

So, if possible, try using a different USB port if it is USB connected you can try on a different USB so that your static noise is gone. So, if there is a hissing noise in the audio, all of you know the thing then use low-pass filtering to avoid this hissing noise. And then how we are going to stop my mic from picking up background noise. Here it was the static noise here background noise because you will be at different places. How you can do that?

So, what it says is click the recording tab in the sound window and select your microphone device and click properties click the levels tab. So, in that what you will be doing is background noise try lowering the microphone boost option perhaps if you have kept it to 20 dB try to reduce it to + 10 db. So, the other one is we know that most of the ECG signals carry line frequency in them how to avoid the line frequency in that case we use the notch filter to eliminate this line frequency.

So usually now, abroad it is 60 hertz or USA and in India we call it as 50 whatever line frequency is 50 hertz signal.

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FIR Filter – Advantages and Disadvantages



Advantages	Disadvantages
FIR filter is always stable.	Large storage requirements
It is simple.	Cannot simulate prototype analog filter
Design complexity generally linear.	For implementation, complex computational techniques are required
FIR filter are having linear phase response.	Expensive due to a large order
It is easy to optimize.	it is hard to implement than IIR
Non causal.	Expensive due to a large order
Transient have a finite duration.	Require more memory
Quantization noise is not much as a problem.	Time-consuming process

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So, we will see today FIR filter. So, first we will see why we have to go for FIR filters. Their advantages and disadvantages. So, the advantages as it are listed here. So, we say it is stable, simple and design complexity gets really linear in this case, and then it has a linear phase response we will see in a while and it is easy to optimize on the order of the filter what we want to have it and this is a non causal system. Hopefully you would have done your causal and non causal systems earlier in your signals and system.

So, how to make non causal system into causal all of you must be knowing it. And it has a transient, which is going to be finite duration. So, what we call it is for finite input, the output is going to be finite. So that is why there would not be any transient which is going to be finite in this case. And we know that quantization noise is not much as a problem. So, this is one of the advantages of our FIR filter. What are the disadvantages? So, we know that the order of the filter that we usually call FIR filter is equivalent to an open loop systems.

It requires larger storage basically. So, it needs storage requirements what we say and then cannot simulate prototype analog filter. So, IIR filter infinite impulse response filter, we will be taking it in further classes. So, we know that we most of the design is done in the analog domain and then

we convert it into digital domain, because the earlier was more than 100 years analog filter have survived and then they are very well working with the system.

But whereas this FIR filters we would not be able to simulate using our analog filters. So, for implementation complex computational law, we say is techniques are required. So, and then it is expensive due to a large order, we have to pay in for the memory as well as the speed of it. And then it is hard to implement than IIR filter, we will see how we will design it in the lab class. And it is expensive, as it is large order it is cost is going to be more, require more memory and time consuming process because we are talking about real time.

So, if the order of the filter is very high, and then we are unable to complete within the next sample comes if it is interrupt driven, then we will be losing the sample. That is why they are very time consuming in the case of FIR filters.

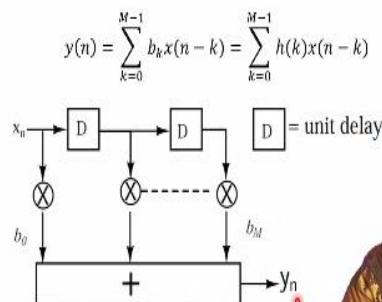
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FIR Filters



Let us consider an FIR filter of length M (order $N = M - 1$, watch out!

order – number of delays)



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So, we will see that how they can be used they are represented in this figure. So, we will consider FIR filters of length M order what we call it. So, then what we say is order of $N = M - 1$. So, you have to watch out it is that is order minus number of delays, what we will be considering as the length of the filter basically. So, we are given the equation $y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$. x is our input, b_k is our coefficients.

So same thing, we said that it is equivalent to convolution if it flashes to you, $h(k)$ is the impulse response, what we take it and $x(n - k)$ is your input. So, we can represent it in this way. But since we use the filter coefficients usually we represented as b_k and how in the flow diagram, we are going to represent $x(n)$ is the input. So, these are the square brackets or the unit delay basically. And then $x(n)$ is multiplied by the b_0 coefficient.

And this is the result what is available for our adder, same way here the after the delay, which is going to comes output is going to be $x(n - 1)$ into if I am considering it as b_1 , then this is going to get added with the previous one. So, on because we have going to have $n - 1$ delays basically. So, the last one is going to be b_M into $b_m \cdot x(n - M)$ will be the thing whatever the order of filter M , which is going to be multiplied with your b_M coefficients, and added together, you will be getting $y(n)$ as the output.

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FIR Filters (2)



- Can immediately obtain the impulse response, with $x(n) = \delta(n)$

$$h(n) = y(n) = \sum_{k=0}^{M-1} b_k \delta(n - k) = b_n$$

- • The impulse response is of finite length M , as required
- Note that FIR filters have only zeros (no poles). Hence known also as all-zero filters
- FIR filters also known as feed forward or non-recursive, or transversal

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So continuing with the thing, so if we represent our input as a impulse response, if I want to take it, then $x(n)$ is going to be $\delta(n)$ basically, then we will be getting the, what will be the impulse response of the filter we will be getting it so as $h(n)$ is equal to what we will call it? $h(n)$ is the impulse response which is equal to $y(n)$, which is given by $\sum_{k=0}^{M-1} b_k \delta(n - k)$. So, you are replacing $x(n - k)$ with the impulse response or $\delta(n - k) \cdot b_k$. So, the output is going to be b_n what you will be getting n will be varying to 0 to $M - 1$.

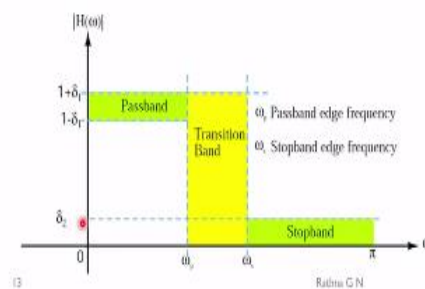
So, the impulse response is of finite length M as required. And then note that FIR filters have only zeros. As you can see from the thing they do not have any poles that are the reason why we call this filter as a stable filter. So, or this name is all zero filters also, FIR filters also known as a feed forward or non recursive or transversal filters what we call it, that is, as you can see in the previous case, the output will be only going in the forward direction that is why it is a feed forward filter also.

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Digital Filter Design



- Desired filter characteristics are specified in the frequency domain in terms of desired magnitude and phase response of the filter; i.e., $H(\omega)$ is specified.
- Filter design involves determining the coefficients of a causal FIR or IIR filter that closely approximates the desired frequency response specifications.



Coming with the design aspect of it some of the parameters what we have to look in, the characteristics as specified in the frequency domain in terms of desired magnitude and phase response of the filter; we call it as $H(\omega)$ in the frequency domain. So, the filter design involves determining the coefficients of our causal FIR or IIR filter that closely approximate the desired frequency response specifications. So, what are the design specifications is given with this diagram?

So, what y axis represent is the magnitude of $H(\omega)$ and then x axis will be represented with omega, which varies between, in this case 0 to π , what has been considered, most of the time it will be $-\pi$ to π or 0 to 2π , but we will be interested in only 0 to π frequency components present in the thing as our sampling frequency is twice that of the highest frequency. So, highest frequency is represented with 2π . Hence, we can consider the frequencies that are present only up to π .

And then we say ω_p is the passband frequency that is from 0 to if here it is the example taken as a low-pass filter. So, I want to allow all the frequencies which are in the passband that is ω_p where I want to stop the collecting my frequency components, then what I have is I am going to say stopband, ω_s is going to say where I have to stop my collection of components. So that is ω_s between the difference between our ω_s and ω_p , we call it as transition band.

For most of the cases, we want direct dropping down, which is not going to happen, we will see in a while it is going to be a smooth transition what it should happen. So, which will be constituting for our transition band. And then we say that is ω_p is my passband edge frequency and ω_s will be stopband H frequency and what are this variation, I am going to have it most of the cases I want the magnitude to be 1 basically, but we know the constraints as and when more constraints are put our design is going to become very critical.

Then we will be having order of the filter very high. So, as we a little bit relaxed on some of the components. So, we can have the order low. So, we say that δ_r is the my ripple what I can allow actually in the passband region between $1 - \delta_r$ to $1 + \delta_r$. So, then passband will have little bit of ripples in the passband. And then I can a little bit deviate from that and how I am going to represent the stopband all the most of the cases I want the flat response, which may not be able to achieve it.

So, have to specify what is the dB that is stopband attenuation, what we call it as δ_s how much I have to come down so that I will not have the frequencies beyond this region ω_s .

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FIR versus IIR Filters



- FIR filters: normally used when there is a requirement of linear phase
 - FIR filter with the following symmetry is linear phase:
$$h(n) = \pm h(M-1-n), n = 0, 1, \dots, M-1.$$
- IIR filters: normally used when linear phase is not required and cost effectiveness is needed
 - IIR filter has lower sidelobes in the stopband than a FIR having the same number of parameters
 - If some phase distortion is tolerable, an IIR filter has an implementation with fewer parameters requiring less memory and lower complexity



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So, coming to comparison between FIR versus IIR so we will take up IIR filter, a more detailed one next few classes. What is the comparison? There we say that FIR filters are normally used when there is a requirement of a linear phase. So, we will take it up a little more in detail in a while and FIR filter with the following symmetry, what we call it as linear phase, what is that symmetry my impulse response has to be equal $h(n) = \pm h(M-1-n)$, $n = 0, 1, \dots, M-1$ order of the filter.

Whereas, in IIR filters they are normally used when linear phase is not required, and cost effectiveness is going to be needed. We say that they have compared to FIR filters, they have lower side lobes and then stopband than that of FIR filter. And for the same type of design, we will see it later also, how it is going to we will be meeting with IIR filter with a less number of coefficients. In some phase distortion is tolerable, then better to use IIR filter implementation it fewer parameters requiring less memory and that lower complexity.

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Design of FIR Filters



- Digital FIR filters cannot be derived from analog filters – rational analog filters cannot have a finite impulse response.

Why bother?

- They are inherently stable
- They can be designed to have a linear phase
- There is a great flexibility in shaping their magnitude response
- They are easy and convenient to implement



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Coming to how we are going to design FIR filters. So, we say that, we cannot derive it from analog filters. So, we have to design in a different way. So, why do we have to bother because we know the advantages of them, which we have already discussed, and then how they are going to achieve linear phase, stability and then how they will have the magnitude of response changes what we can accommodate and then it is easy and then convenient to implement.

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Linear Phase Filters



The ability to have an exactly linear phase response is the one of the most important of FIR filters

$$H(\omega) = |H(\omega)|e^{j\phi(\omega)} \text{ where } \phi(\omega) = -\omega n_0$$

→ A general FIR filter does not have a linear phase response but this property is satisfied when $h(n) = \pm h(M-1-n)$, $n = 0, 1, \dots, M-1$, for linear phase filter types

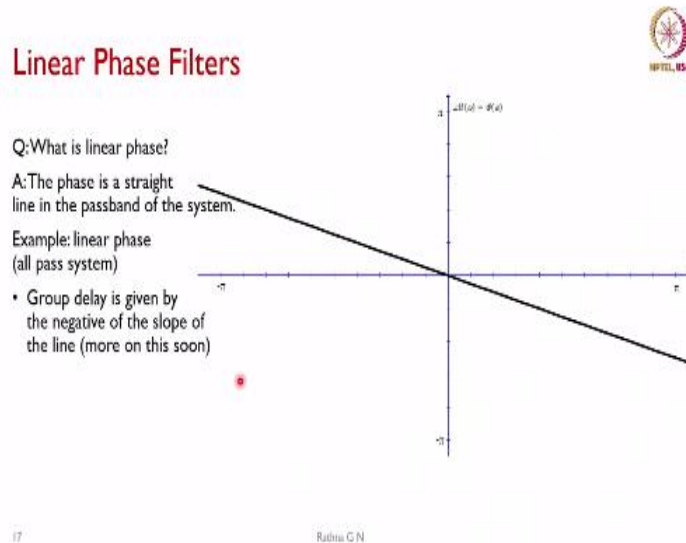
Impulse response	#coefs	$H(\omega)$	Type
$h(n) = h(M-1-n)$	Odd	$e^{-j\omega(M-1)/2} \left(h\left(\frac{M-1}{2}\right) + 2 \sum_{k=1}^{(M-1)/2} h\left(\frac{M-1}{2}-k\right) \cos(\omega k) \right)$	1
$h(n) = h(M-1-n)$	Even	$e^{-j\omega(M-1)/2} 2 \sum_{k=1}^{(M-1)/2} h\left(\frac{M}{2}-k\right) \cos\left(\omega\left(k-\frac{1}{2}\right)\right)$	2
$h(n) = -h(M-1-n)$	Odd	$e^{-j\omega(M-1)/2 - \pi/2} \left(2 \sum_{k=1}^{(M-1)/2} h\left(\frac{M-1}{2}-k\right) \sin(\omega k) \right)$	3
$h(n) = -h(M-1-n)$	Even	$e^{-j\omega(M-1)/2 - \pi/2} 2 \sum_{k=1}^{(M-1)/2} h\left(\frac{M}{2}-k\right) \sin\left(\omega\left(k-\frac{1}{2}\right)\right)$	4

So, coming to the linear phase, so, what is it, the ability to have an exactly linear phase response is the one of the most important of FIR filters, what is that hey $H(\omega)$, it should be equal to magnitude of H of omega into $e^{j\phi(\omega)}$, where $\phi(\omega)$ is nothing but $(-\omega n_0)$. So, when General FIR

filter does not have a linear phase response, but this property is satisfied when you are $h(n) = \pm$ if I have taken $h(M - 1 - n)$, $n = 0, 1, \dots, M - 1$ definitely will have a linear phase filter.

So, different types of impulse response whether it is odd or even. And then, as you are seeing it, there are 4 types, how it can be designed. So, you can go through them and then whichever is convenient, you can select one of them to design your linear phase FIR filter.

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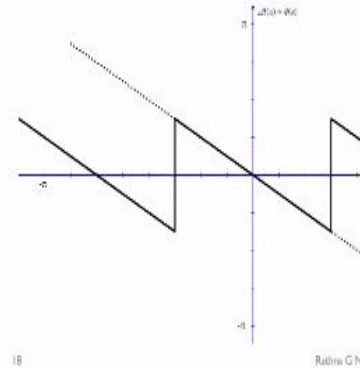
So, why do we say it is linear phase, the phase is a straight line in the passband of the system as you are seeing it this $-\pi$ to π I say that that is my passband it should be linear or we call it as all pass system basically, in this case group delay is given by the negative of the slope of the line. So, we will see it in a while hold on for a little while.

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Linear Phase

Example: linear phase (all pass system)

- Phase wrapping may occur, but the phase is still considered to be linear.



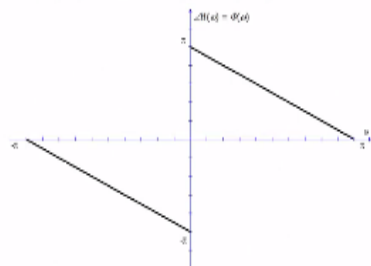
So, coming to the other thing, we said all pass system because we may have discontinuity in the filters or bandpass filter at different frequencies what I want to design it. Then what is the thing is going to happen, we say phase wrapping may occur, that is what you are seeing it. So, but the phase is still considered to be clean here, why, this is the passband region of one this thing parameter the other region of the thing is from here to here, still we have what we call it as a line is linear in this case and then here this side also.

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Linear Phase

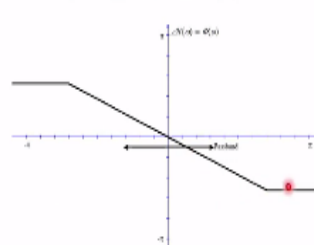
Example: linear phase (high pass system)

- Discontinuities at the origin still correspond to a linear phase system.



Example: linear phase (high pass system)

- Linear characteristics only need to pertain to the passband frequencies only.



Coming to the other part of it, that is if we are considering the highpass system, we call it as highpass filter that is the low frequencies are going to be eliminated only higher frequencies are going to be considered in that case, the phase response looks like which is having a discontinuity

that is from here to here I will be having that is my π to magnitude one in this case to π . And then from here it is -1 to $-\pi$ magnitude if I am considering the magnitude as -1 to 1.

So, then here it is going to be linear and then this side also it is going to be linear both on the negative side as well as the positive side of it. So, the other one, as you can see the thing when we say it is going to be linear phase? So, this is my passband region, if it is going to be linear, I say it is linear phase, and I am not bothered about beyond my interest of region, they are not linear, they are become nonlinear in phase.

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DTFT Theorems and Properties



Property	Time Domain	Frequency Domain
Notation:	$x(n)$ $x_1(n)$ $x_2(n)$	$X(\omega)$ $X_1(\omega)$ $X_2(\omega)$
Linearity:	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(\omega) + a_2X_2(\omega)$
Time shifting:	$x(n-k)$	$e^{-j\omega k}X(\omega)$
Time reversal:	$x(-n)$	$X(-\omega)$
Convolution:	$x_1(n) * x_2(n)$	$X_1(\omega)X_2(\omega)$
Correlation:	$r_{x_1x_2}(l) = x_1(l) * x_2(-l)$	$S_{x_1x_2}(\omega) = X_1(\omega)X_2(-\omega) = X_1(\omega)X_2^*(\omega)$ [if $x_2(n)$ real]
Wiener-Khinchine:	$r_{xx}(l) = x(l) * x(-l)$	$S_{xx}(\omega) = X(\omega) ^2$

among others ...

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So, some of the terms we will see it which is going to help us for our linear derivation. So, discrete time Fourier transform some of the theorems and properties which more we will be taking up when I consider DFT and then FFT. So, the property we consider is the notation what we are going to have it, in time domain, we say $x(n)$, $x_1(n)$, $x_2(n)$ and frequency domain, all of us know it is $X(\omega)$, $X_1(\omega)$, $X_2(\omega)$. So, when we say it is linearity, so, when it meets the superposition theorem.

So, which is both commutative as well as can you guess the other one, so, $a_1x_1(n) + a_2x_2(n)$ when I do the thing, so, I should be able to get even in the frequency domain, $a_1X_1(\omega) + a_2X_2(\omega)$. So, when we do the time shifting, that is if I provide a shift of $x(n-k)$, then here the resultant is going to be $e^{-j\omega k}X(\omega)$. In the case of time reversal, $x(-n)$ will be giving me in the frequency domain is $X(-\omega)$.

So, when we do the convolution of 2 signals x_1 and then x_2 , I know it is convolution in the time domain, whereas in the frequency domain, it is multiplication. So, when we want to do the correlation, basically it can be autocorrelation, as we will be seeing it later are x_1 to x_2 . So, if we consider the thing this is in the time domain, it is convolution, whereas in the frequency domain, it becomes a multiplication with respect to complex conjugate of the other sequence.

So, if we consider x_2 of n is real, it becomes a multiplication of $X_1(\omega)X_2(\omega)$. So, whereas in the sum of the wiener filter, we will be considering in the adaptive filter LMS algorithm so, that time you are autocorrelation with respect to the same signal what you are seeing it that convolution of this that is time reverse signal what you are having it which becomes the square of the $X(\omega)$ magnitude of it, what you are seeing in the filter domain.

So, there are other more properties and then both in time domain and frequency domain. So, you would have studied in your signal processing or signals and system classes, you can look into them.

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Group Delay



Therefore,

$$y(n) = x(n - n_0) \xrightarrow{F} Y(\omega) = X(\omega)e^{-j\omega n_0}$$

group delay

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = e^{-j\omega n_0}$$

$$\angle H(\omega) = \phi(\omega) = -\omega n_0 = -\omega \cdot \text{group delay}$$

In general (even for nonlinear phase systems),

$$\text{Group delay} \equiv -\frac{d\phi(\omega)}{d\omega}$$



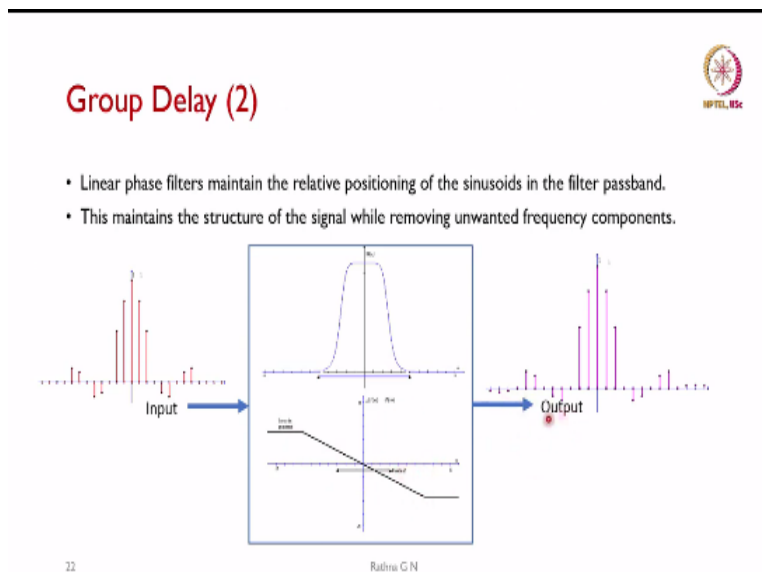
So, now, we are telling that linear phase. So, what we have said is something group delay, which we have to consider, so, $y(n)$ is my output, and then $x(n)$ is going to be delayed by n_0 . So, we call that as group delay, then what happens to the frequency domain signal which is $Y(\omega)$ is the output

which is going to be $X(\omega)$ you are seeing $e^{-j\omega n_0}$ it is going to be so then what happens to the our magnitude and then phase responses basically.

So, $H(\omega) = \frac{Y(\omega)}{X(\omega)} = e^{-j\omega n_0}$ and the phase of it we call it as $\phi(\omega)$, which is going to be $-\omega n_0$.

And because we know that n_0 is the group delay, which is $-\omega$ into group delay. So, in general even for nonlinear phase systems, we consider group delay as the differentiation with respect to our phase. So, that is $-\frac{d\phi(\omega)}{d\omega}$ negative part of it, what we will be taking up.

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So, continuing with the group delay, so, we know that linear phase filters maintain the relative positioning of the sinusoids in the filter passband, as you will be seeing it that is structure of the signal while removing unwanted frequency components. So, we see that some of this is an input and as you can see, this is a low pass filter, which is represented both in the negative domain as well as in the positive domain. So, the phase is going to be linear in this. So, output after passing it through also, you will be having a little bit delay at the output, but they will maintain the same phase of the thing.

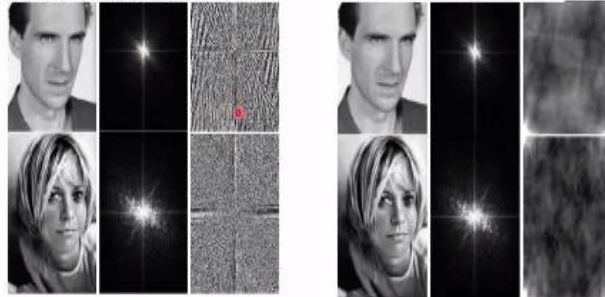
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Importance of Linear Phase

Q: What can happen when there is loss of phase information?

A: To maintain the original "structure" of a signal in the passband frequency range linear phase (or close to linear phase) is required.



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Ruthra C N

So, why do we need a linear phase? So, as you know that in what happens to loss of phase information that is one of the things whatever literature gives it, most of us how do we identify the phases of people basically? So, we say that the phase for 1 person to the other person is different. So, if they are linear phase, that is a passband frequency range linear phase or close to linear phase is required. Otherwise, I may be reconstructing a different one as you will be seeing it some of the frequency domain component how it looks like.

And then what is the phase of it what you are looking at it, and if they are distorted, then you may not see the same face you may be looking at different face.

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Linear Phase FIR Filters

- As mentioned previously, FIR filters with the following symmetry are linear phase:

$$h(n) = \pm h(M-1-n), n = 0, 1, \dots, M-1$$

- Note that this means

$$h(n) = +h(M-1-n) \text{ for } n = 0, 1, \dots, M-1, \text{ or}$$

$$h(n) = -h(M-1-n) \text{ for } n = 0, 1, \dots, M-1,$$

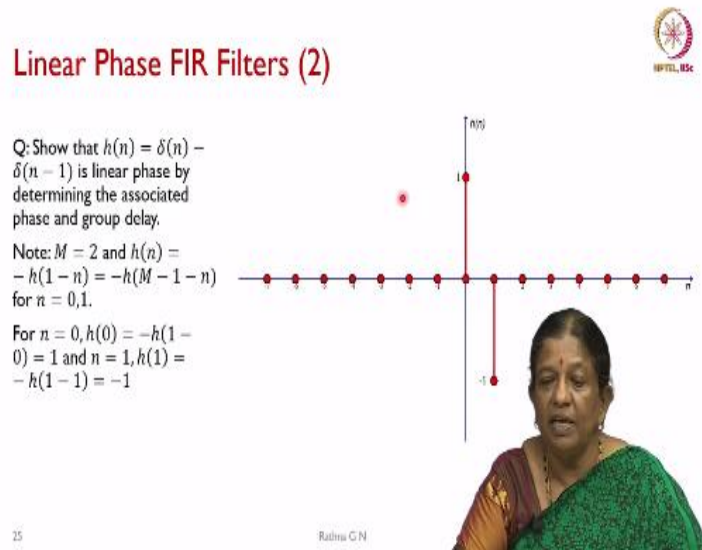
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Ruthra C N



So, linear phase FIR filters, as we said, how we can develop them. That is we follow the symmetry property. So in this case, we will be following only the simple one, we will assume that $h(n) = \pm h(M - 1 - n)$, $n = 0, 1, \dots, M - 1$ what we will consider. That means, what is the thing, so, I am considering the positive side of this for these values, as well as $h(n)$ for the negative of it, what I am considering it.

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So, we will see that what happens if I give $\delta(n) - \delta(n - 1)$ as my impulse response to my system. So, whether it is going to represent a linear phase or not, we will see the thing. So, for that, we have to see the phase response as well as the group delay. So, note in this case, because m is equal to second order, we have chosen the thing. So, it will be going $h(n) = -h(1 - n) = -h(M - 1 - n)$ for $n = 0, 1$.

So, these are the 2 things what will be substituting first $n = 0, 1$ because $M = 2$, when $n = 0$, $h(0) = -h(1 - 0) = 1$ and $n = 1$, $h(1) = -h(1 - 1) = -1$ basically, which is $h(0)$, $h(0) = 1$, so, it becomes -1 . So, this is how you will be representing with respect to $n = 0$, its impulse response is 1 and then $n = 1$, its ripples responses -1 .

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Linear Phase FIR Filters: Example



Q: Show that $h(n) = \delta(n) - \delta(n-1)$ is linear phase by determining the associated phase and group delay.

Note: This system corresponds to:

$$\begin{aligned} y(n) &= x(n) * h(n) = x(n)[\delta(n) + \delta(n-1)] = x(n)\delta(n) - x(n)\delta(n-1) \\ &= x(n) - x(n-1) \text{ (first difference system)} \end{aligned}$$

First difference \Leftrightarrow DST-time derivative \Rightarrow highpass filter



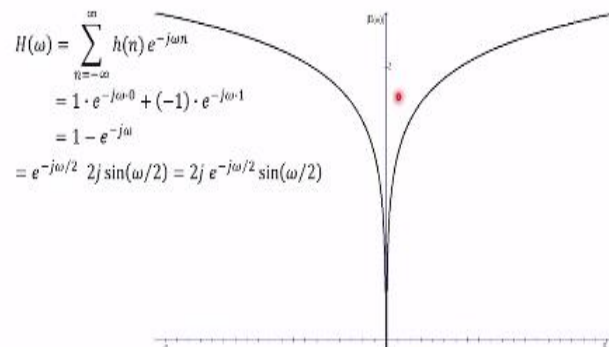
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So, we will see an example, how we are going to show that this has a linear phase. So, what is that equivalent into convolution of your $x(n) * h(n)$ by expanding our function which is nothing but $x(n)[\delta(n) + \delta(n-1)]$ in this case, we have assumed $-\delta(n-1)$. So, this will be negative $x(n)\delta(n) - x(n)\delta(n-1)$. So, which is nothing but $x(n) - x(n-1)$, this we call it as first difference system. So, first difference system what we can go into the discrete sine time our derivative so, we define this as a highpass filter. We will see it in a while how it can be highpass filter.

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Linear Phase FIR Filters: Example (2)



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So, we will be expanding this we have $H(\omega)$ is nothing but $\sum_{n=-\infty}^{\infty} h(n) e^{-j\omega n}$, this is our DTFT equation. So, I am substituting $h(0)$ as 1 and we have $e^{-j\omega \cdot 0} + (-1) \cdot e^{-j\omega \cdot 1}$ which is nothing but $1 - e^{-j\omega}$. So, when I expand in terms of my exponential, so, you will be seeing that simplifying it, it becomes $e^{-j\omega/2} 2j \sin(\omega/2)$.

So, this is what if I move $2j$ into the first place and then put $e^{-j\omega/2} \cdot \sin(\omega/2)$. So, for various values of ω between minus and π if you plot the thing, so, you will be seeing the curl like this. So, that means to say low frequencies whatever there that is going to be eliminated only higher frequencies almost when it becomes 1 you will be allowing those frequencies to be present in the system.

(Refer Slide Time: 36:09)

Linear Phase FIR Filters: Example (3)

$$\begin{aligned}\phi(\omega) &= \angle 2je^{-j\omega/2} \sin(\omega/2) \\ &= \angle 2 + \angle j + \angle e^{-j\omega/2} + \angle \sin(\omega/2) \\ &= 0 = \pi/2 = -\omega/2 = \begin{cases} 0 & 0 < \omega < \pi \\ \pi & -\pi < \omega < 0 \end{cases} \\ &= \begin{cases} \frac{\pi - \omega}{2} & 0 < \omega < \pi \\ \frac{3\pi - \omega}{2} - 2\pi & -\pi < \omega < 0 \end{cases} \\ &= \begin{cases} \frac{\pi - \omega}{2} & 0 < \omega < \pi \\ -\frac{\pi - \omega}{2} & -\pi < \omega < 0 \end{cases}\end{aligned}$$

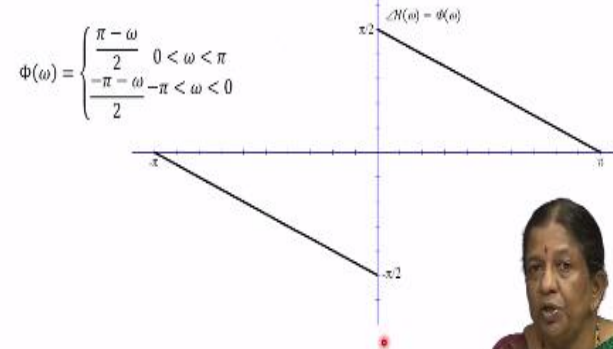


So, how we are going to consider the group delay we have to see $\phi(\omega)$ what we have got it as $\angle je^{-j\omega/2} \sin(\omega/2)$. So, by putting them that is $\angle 2$ we know that it is 0 $\angle j = \pi/2$ and $\angle e^{-j\omega/2}$ is $-\omega/2$ and for $\angle \sin(\omega/2)$ it will be in this range it is going to be 0 , $0 < \omega < \pi$ and it will be $\pi - \pi < \omega < 0$. So, when we put it in terms of ω which will be $\frac{\pi - \omega}{2}$ and $\frac{3\pi - \omega}{2} - 2\pi$ in this range.

So, when you put this simplified one you will be getting it as $-\omega/2$ into the other one it is in the range 0 to π , $-\pi$ to 0 will be $-\pi - \omega/2$.

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Linear Phase FIR Filters: Example (4)



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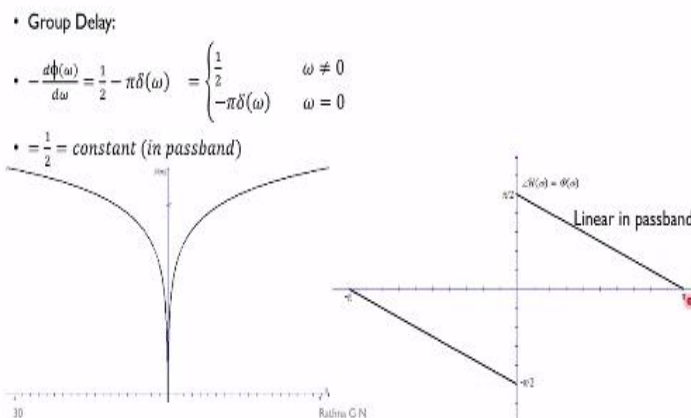
Ruthra G N



So, now, we will see that how is it going to be represented, so, we have $\phi(\omega)$, so, we will be put ω 0 to π then you will be seeing this is shown as $\pi/2$ to π here the magnitude is going to be half here, which is going to go between that to that is π here. So, whereas, in the case of negative region, so, you will be seeing that $-\pi$ to $-\pi/2$ because we are representing even my y axis is in phase of the thing. So, which is going to be $-\pi/2$ to $\pi/2$.

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Linear Phase FIR Filters: Example (5)



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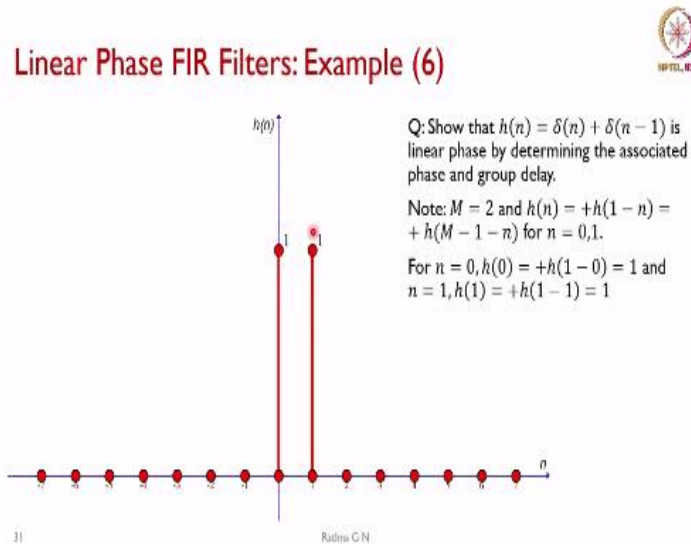
Ruthra G N

So, how we are going to say that group delay is going to be constant in this case, so, I am going to take the derivative at negative derivative of my phase. So, which is nothing but $\frac{1}{2} - \pi\delta(\omega)$. So, which is going to give me $\frac{1}{2}$ and then $-\pi\delta(\omega)$ which $\omega = 0$ and wherever $\omega \neq 0$ it is going to be

$\frac{1}{2}$. So, we consider in the passband region because I am not bothered about when $\omega = 0$. So, we know that in the passband region it is becoming constant.

So, we say that, the delay is constant or we call it as a linear phase basically, that is the reason why you will be seeing that it is $\pi/2$ to $-\pi/2$ what you will be seeing the linear slope of the line here.

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So, coming further with the example. So, what we have is, this is an assignment for you to work it out, we showed that the FIR filter for a highpass filter is linear phase. Now, it is your time to show that the low pass filter that is you have been given $\delta(n) + \delta(n-1)$ is my impulse response that this also is a linear phase. So, you can consider here also you have $M = 2$ and these are the values what you have it when $n = 0$ it is going to be 1 and then when $n = 1$, it is $+h(0)$ which is also 1. So, you will be seeing that both of them are 1 you are supposed to show whether it is linear phase filter.

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Linear Phase FIR Filters: Example (7)



Q: Show that $h(n) = \delta(n) + \delta(n-1)$ is linear phase by determining the associated phase and group delay.

Note: This system corresponds to:

$$\begin{aligned} y(n) &= x(n) * h(n) = x(n) [\delta(n) + \delta(n-1)] \\ &= x(n)\delta(n) + x(n)\delta(n-1) \\ &= x(n) + x(n-1) \text{ (scaled averaging system)} \end{aligned}$$

Averager \Rightarrow DST-time smoother \Rightarrow lowpass filter



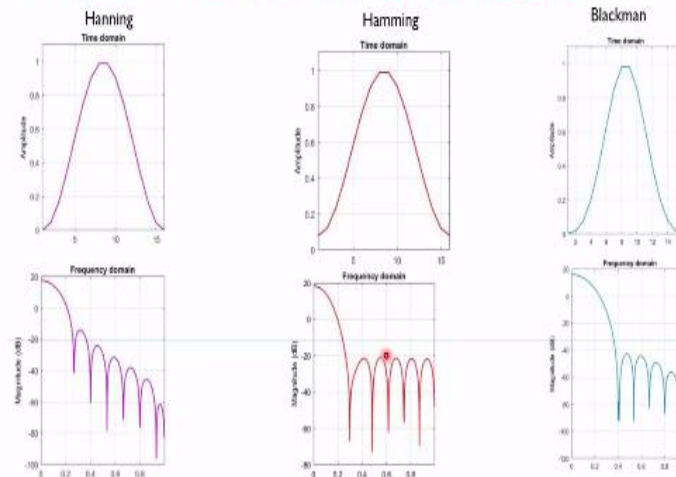
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Ratna C N

So, as you can see that when you are giving your $\delta(n)$ representation input $y(n)$ is nothing but because we have an impulse response $\delta(n)$ and $\delta(n-1) = 1$. So, the output $y(n)$ will be equal to $x(n) + x(n-1)$, it is the scaled averaging system what we call it. So, the Averager is also represented as discrete time smoother, and we call it as low pass filter.

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Windows – Magnitude of Frequency Response



So, some of the techniques to design our FIR filter which we will be taking it up more in our lab class, just to give you a flavour of it, so, I can design a hanning using the window technique, there are different ways of doing it. So, most of the time we will be using MATLAB FDA toolbox with older versions and latest version use the filter design toolbox. So, here it is the Hanning window

what you are seeing $M = 16$ what we have considered in this case plotting this figures from the MATLAB.

So, you will be seeing that this is a smooth as you will be seeing between 0 and the other thing. So, whereas the hamming window so, you will be seeing there is a little drift in the thing and then that is what it will be represented use a Blackman window as we will be seeing that it will be having the passband is an narrow band whereas, it is going to drop down the dB whatever a frequency response what you can see that is if we have put this as a reference line, so, which will be below this line that is -20 dB point.

Whereas, in the case of these are the sidelobes, what we call it weather I want a narrow this think passband lobe or more ripples in the sidelobe what we have to take it into considerations. So, for the Hamming window you will be seeing that it goes a little more than -20 dB. Whereas, in the hanning window also but it is going to come down as you will be seeing when you are using the hanning window. Most of the filter equations will be using the hanning window to design FIR filters.

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FIR Filter Design using Windows



Many alternatives have been proposed

- Hanning
- Hamming
- Blackman

Window's Name	Mainlobe	Mainlobe/Sidelobe	Peak $20 \log_{10} \delta$
Rectangular	$4\pi/M$	-13dB	-21dB
Hanning	$8\pi/M$	-32dB	-44dB
Hamming	$8\pi/M$	-43dB	-53dB
Blackman	$12\pi/M$	-58dB	-74dB

Window type	Mainlobe width	Peak sidelobe (dB)
Rectangular	$4\pi/M$	-13
Barlett	$8\pi/M$	-25
Hanning	$8\pi/M$	-31
Hamming	$8\pi/M$	-41
Blackman	$12\pi/M$	-57

We see clearly that a wider transition region (wider main-lobe) is compensated by much lower side-lobes and thus less ripples.



Rathna G N

So, that is, this table shows that how they are represented the mainlobe is going to be $4\pi/M$ which is equivalent to -13 dB what they will have it and if I consider the peak $20 \log_{10} \delta$ sidelobe. So, it will be coming down to you will be seeing that -21 dB, whereas, in the hanning window it

comes to -44 dB when I want to go with the increasing the order of the filter basically if I want to achieve it M has to be increased and then I can go up to -53 dB maximum in the hamming window and if you consider the blackman window, I can go up to 74 dB from 58 dB.

So, some of the design if you want to have a peak sidelobe -13 and other things, you can select any one of these windows and if you want a variable you would have heard of cases of window which you can selected which one of the parameter what beta parameter what you can vary alpha and beta parameter there and select your own window.

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M2U8



- FIR Filters - Pipelining and Parallelism for Low Power

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Rutima GN



So, this gives you a flavor of FIR filter. So, in the next class we will be taking up pipelining and parallelism for low power how we can design this FIR filters, thank you for your listening and then happy learning in this course.