Real-Time Digital Signal Processing
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Lecture - 28
Overlap-Add

Welcome back to real time digital signal processing course. So, today we will discuss about overlap add method, how to use our FFT in for the continuous time signal. So, coming to the recap of it, so, we have completed FFT, its butterfly structure. And then we have seen some examples using both DFT and then FFT.

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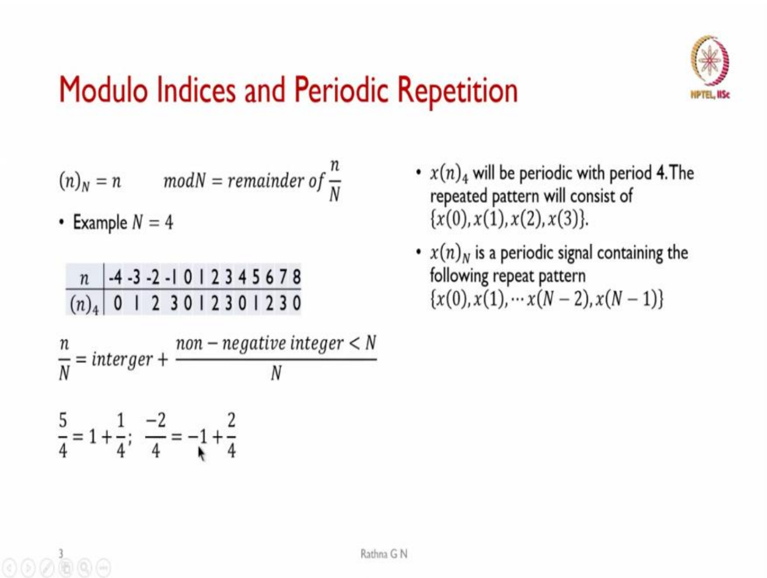

So, how to use FFT for overlap add method so, we will see some of the definitions, so, we will see modular indices and how it is going to be used for the periodic repetition. So, as we can see here, anything n with N is our length of FFT. So, it is equivalent to n itself what does it mean that is $(\text{mod } N)$ is nothing but remainder of $(n)_N$. So, in this case $x(n)$ if we have 4 samples, so, we say that it is periodic with period 4 the repeated pattern will consist of what is it $x(0)$, $x(1)$, $x(2)$ and then $x(3)$.

And the length is N, then we say that the repetition is going to happen after $x(N-1)$ samples, N is usually we take it power of 2. So, as we have seen in the example, N = 4 what we have taken so, we have seen the repetition what is going to happen, so, we will see visually here, n is going to vary from - 4 to 8 is the long sample what we have taken. And then small n which is going to repeat after 4 that is 0 1 2 3 and then later on again 0 1 2 3 and 0 1 2 3 and then the last one is 0.

So, we say that n by N is nothing but integer plus non-negative integer which is less than N divided by N as an example, so, because we have taken N = 4. So, the 5th sample what we have to take it then what is the thing is going to happen this is $\frac{5}{4}$ which is nothing but 1 + 1 by 4, so, this is what the integer plus non-negative integer what we are taking it here if it is $\frac{-2}{4}$, then the integer is going to be $-1 + \frac{2}{4}$ they should be non-negative integer what we have to assume. So, this is how we will be taking the modular indices and then after that it is going to repeat.

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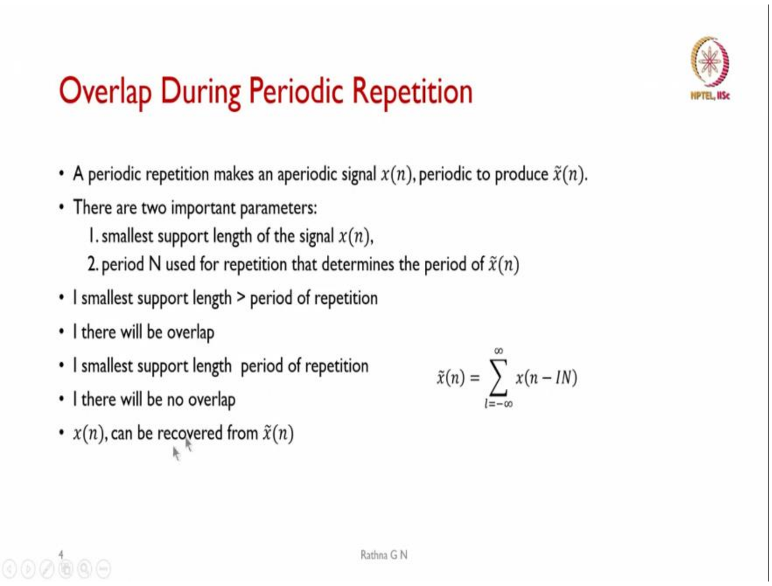

So, what we are going to do with overlap during the periodic repetition. So, we say that periodic repetition makes an aperiodic signal that is $x(n)$, so, periodic to produce $\tilde{x}(n)$. So, there are 2 important parameters in this case, that is the smallest support length of the signal $x(n)$ what we need it and period N used for repetition that determines the period of our $\tilde{x}(n)$. So, we call I smallest support length, which is greater than period of repetition and repeating the thing. I there will be overlap, I will be the smallest support length period of repetition and I there will be no overlap, these are the conditions there will be overlap and no overlap condition and x of n can be recovered from our periodic signal $\tilde{x}(n)$ so, how do we represent our $\tilde{x}(n) = \sum_{l=-\infty}^{\infty} x(n - IN)$. So, I will be defined with these parameters.

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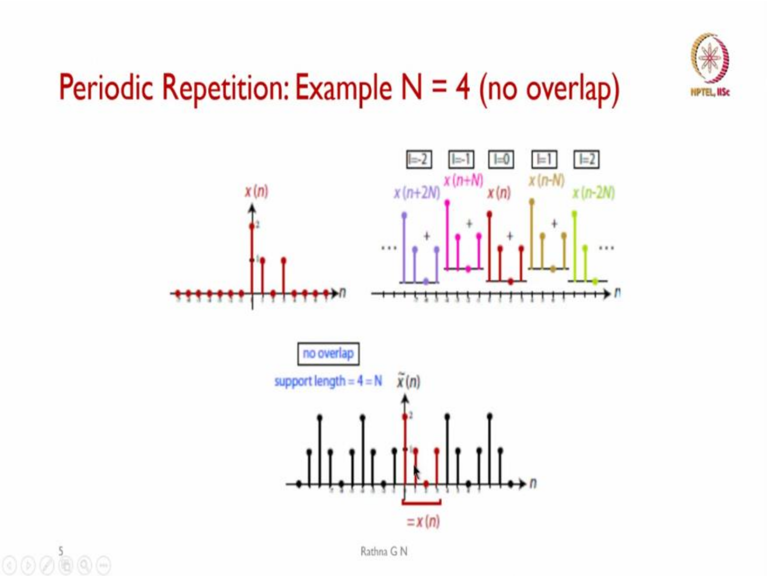

So, to show that how the periodic repetition is going to be that is if we consider no overlap, and then N is taken as 4 this is our $x(n)$,. Then what happens to our the repeated signal at l = 0, so we know that we have 4 samples here, then this is our $x(n)$,, then at l = 1, it is going to be $x(n-N)$, that is we are repeating the 4 samples, same thing at l = 2, we will be repeating $x(n-2N)$, so, that is what dot dot what we have it.

And then what happens on the negative side of it, it is going to be $l = -1$, it is going to be $x(n+N)$, actually, so the 4 samples are repeated here, same way, and then all of them get added. And then we will be seeing that there is no overlap the support length what we have taken is a 4 is which is equal to N. So, our $\tilde{x}(n)$ you will be seeing that this is the $x(n)$ and then this is l = 1 and this side l = -1. So, you will be seeing the repetition of it, this is the periodic repetition what is shown pictorially.

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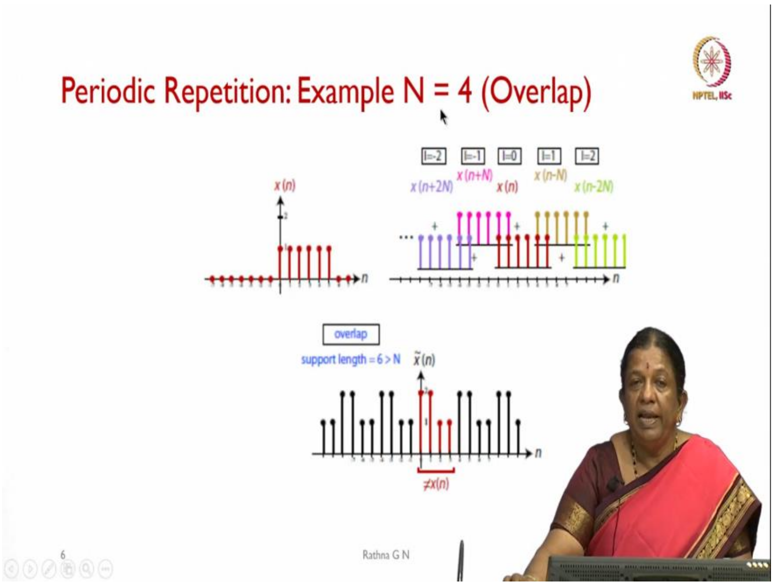

So, coming to if there is a overlap, that is periodic repetition, how it is going to look what we will see it here. So, here also taken as N = 4. So, what we have is our $x(n)$ sample has 6 samples in this case, and then we are taking N = 4. So, initially, what we have is 6 samples at the 0 basically, then $x(n)$ is put in since it is repetition what we have is N = 4 you will be seeing that there is an overlap on the positive side as well as on the negative side.

So, you are seeing that there are 2 of them have been repeated on this. So, you will be seeing that they are overlapping, even here it is going to overlap, then what happens to our $\tilde{x}(n)$, so we say that support length is 6 which is greater than n, then we will be seeing that because we are adding them up. So, you will be seeing in the magnitude which has got increased with these 2 samples, the other 2 samples remain same.

So, you will be seeing that this repetition is going to happen on the positive side as well as on the negative side. So, one has to keep it in mind that how the periodic repetition is going to have an overlap when our length is greater than our N.

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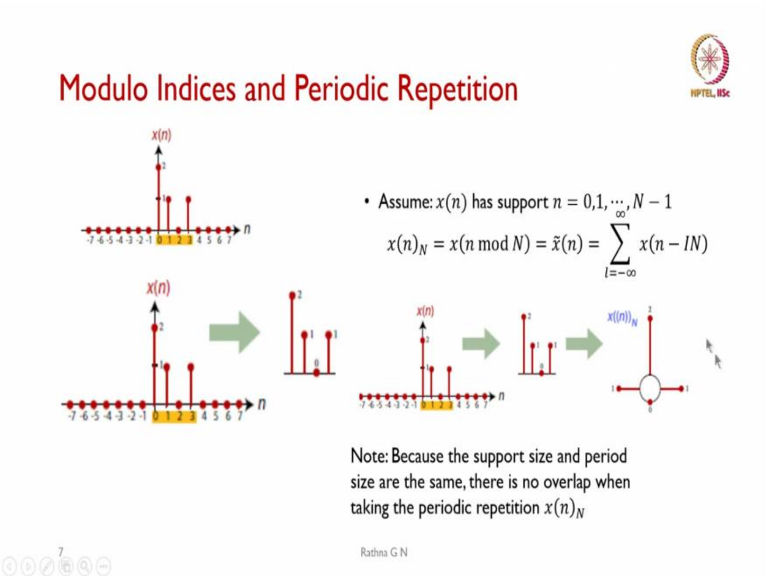

So, coming to the thing, we will see that how we are going to have the periodic repetition once again what you will be seeing the thing, so you have $x(n)$ here. So, then what is the thing is going to happen only we are taking the samples here, which are length 4. So, you will be seeing that the magnitude of them is 2 1 0 1. And then $x(n)$ has support for $n = 0,1,\cdots,\ N-1$, then our $x(n)_N$ will be it is $x(n \bmod N)$ what we will be taking as $\tilde{x}(n)$.

Which is given by the equation $\sum_{i=-\infty}^{\infty} x(n-IN)$. So, how this is going to be represented, so, this is our $x(n)$, this is what what we have taken the thing. So, when we do the repetition, you will be seeing that (mod N) is going to be 2 1 0 1. So, because the support size and period size are the same, there is no overlap when taking the periodic repetition of our $x(n)$. So, you will be seeing that it is same.

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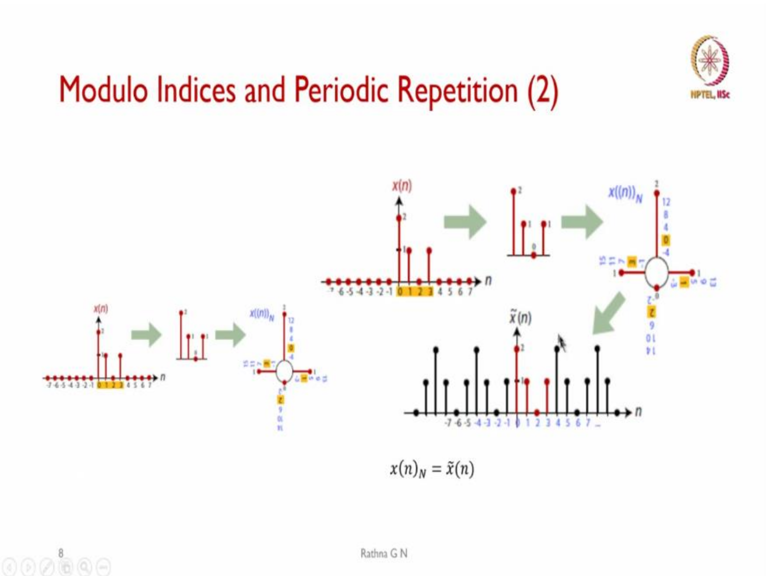

So, now coming to this modulo indices, same thing periodic repetition how it is going to look like what we will be seeing with the thing. So, you will be seeing that the same values have been taken 2 1 0 and 1 and then when we take the modulo of it, so you will be seeing that it is going to go in the 0th here, then -1 will be this is the way and then 1 next, what we will go into have the thing. So, it will be repeating the value at different intervals what it is shown here.

So, which is shown again in this manner so from here to here, what will be going? From here to here, and from there, we can show it pictorially that this is the way what we are going to represent in the periodic of form. So, that is $x(n \bmod N)$ is nothing but our repetition $\tilde{x}(n)$, which is shown there so from here, what you can draw your diagram as x and y coordinates repetition of $x(n)$ in this manner.

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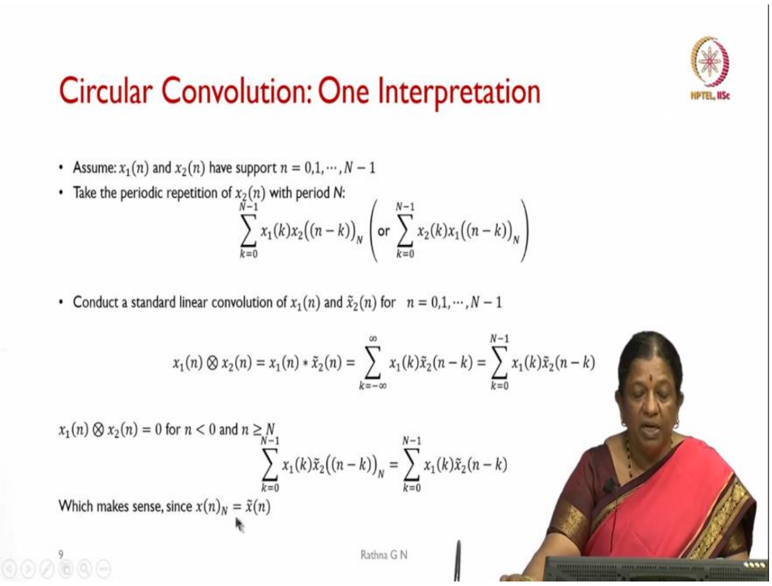

So, now we have to look in for the circular convolution. So, you must be remembering, we have taken some examples, and then we repeated the computation of the circular convolution. So, there are 2 interpretation, the first interpretation what we will see it, so, we are assuming x_1 and x_2 have support n = 0 to N - 1, then take the periodic repetition now our $x_2(n)$ with period N we are assuming both are of the size N in this case.

Then what happens, it is going to be $\sum_{k=0}^{N-1} x_1(k)x_2\big((n-k)\big)_N$ what we are going to take it or we can write it $x_2(k)$ into my this thing modulus of our x_1 basically here. So, both are the same thing, so we can use any one of them. So, conduct a standard linear convolution of x_1 and

$\tilde{x}_2(n)$ for $n = 0,1,\cdots, N-1$. So, we remember that circular convolution has the symbol like this.

$x_1(n) \otimes x_2(n)$, or the other way around what we will be seeing it $x_1(n)$ convolve with our this thing linear convolution $\tilde{x}_2(n)$ what we can give it which is same. So, this is the equation. So, what we have it, so, we will be seeing that this is $-\infty$ to ∞, so, we cannot compute in our hardware, so, it will be reduced from $\sum_{k=0}^{N-1} x_1(k)\tilde{x}_2(n-k)$. So, what happens to the circular circular convolution of it, it is going to be 0 for $n < 0$ that is what, what we have assumed.

And $n \geq N$ this is what we have given the equation. So, that is it makes sense that is what, what we take it since $x(n \bmod N)$ is nothing but our repetition $\tilde{x}(n)$.

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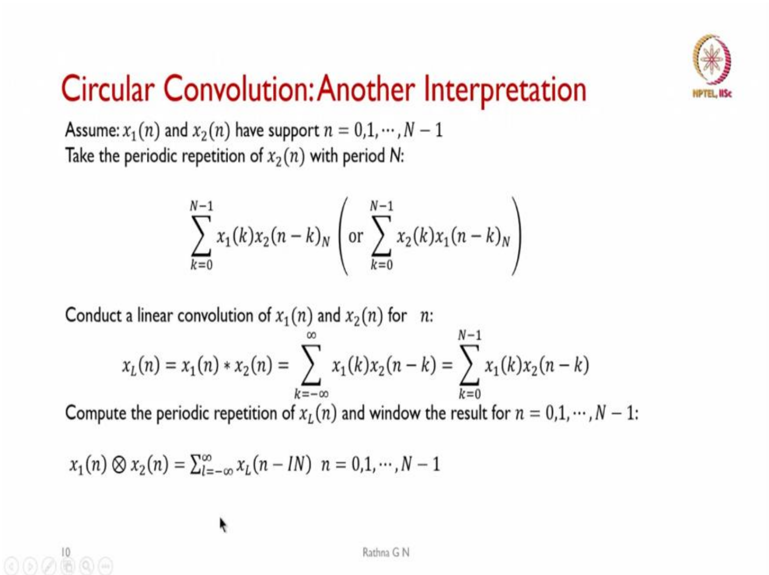

So, another interpretation is what is it? So, they have the support of 0 to N - 1. So, we are going to take the periodic repetition of $x_2(n)$ with period N in this case, so, you will be seeing that $x_2(n-k)_N$ and then we conduct the linear convolution instead of circular convolution, we will be doing the linear convolution of $x_1(n)$ and $x_2(n)$ then what happens x_L is represented as the linear convolution of n is given by $x_1(n)$ linearly convolve with $x_2(n)$.

So, equation what you are seeing $-\infty$ to ∞, so, which is going to be reduced to $\sum_{k=0}^{N-1} x_1(k)x_2(n-k)$. So, compute the periodic repetition of your $x_L(n)$ and window the results for $n = 0,1,\cdots,N-1$. So, you will be doing x_1 circular convolution is equal into $x_2(n)$ which is nothing but our $\sum_{l=-\infty}^{\infty} x_L(n-IN)$ $n = 0,1,\cdots,N-1$.

So, what are the 2 methods first we can do the circular convolution by taking the modulo of one of the signal or we can do the linear convolution of the signal and do the circular convolution at the output that is what, what it is show.

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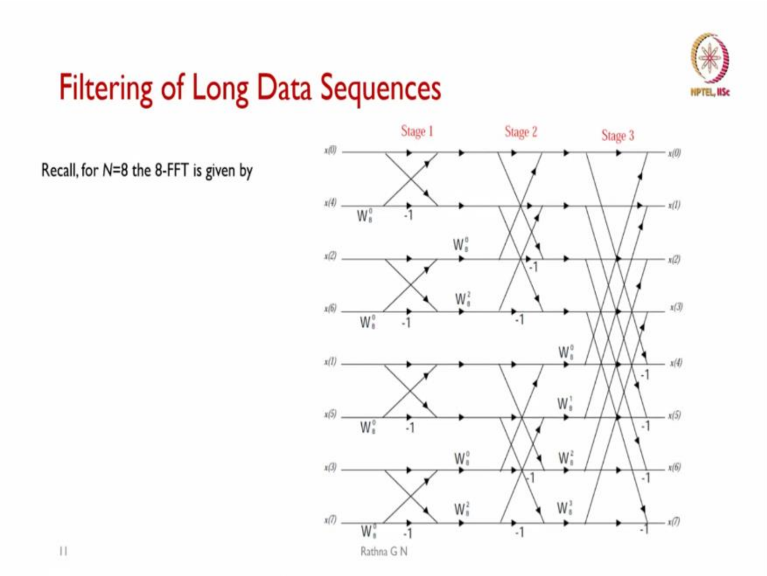

So, we know that filtering of long data sequence how we are going to use our FFT. So, as we have already discussed, we have N = 8 and 8 point FFT is given by this butterfly structure. So, you have seen the thing so, if it is 16 you can extend it and 32 and so on. So, it is easy to write N = 8 so, which we can use it.

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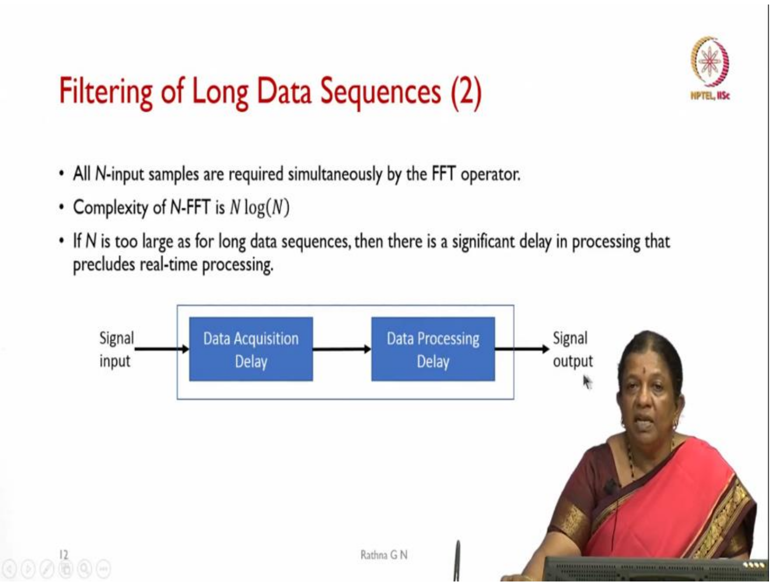

So, how we are going to take care of the long data sequence that we want to filter it, we say all N input samples are required simultaneously by the FFT operation. So, complexity of N FFT is what we have taken $N \log_2 N$ basically and if we use the symmetry property it will be

$N/2 \log_2 N$. So, if N is too large as for long data sequences then there is a significant delay in processing that is a precludes our real time processing.

That is, we are going to have a time constraint on the thing because we do not know the length of the signal which is going to come continuously in real time. So, how we are going to take care of this we cannot have as long as we want. So, then what we do is we give the input signal here we get it from the data acquisition delay what we are going to consider, and then we will be adding our data processing delay also, this is acquisition as well as processing, then we will be giving the signal out, this is our real time processing what it is going to happen.

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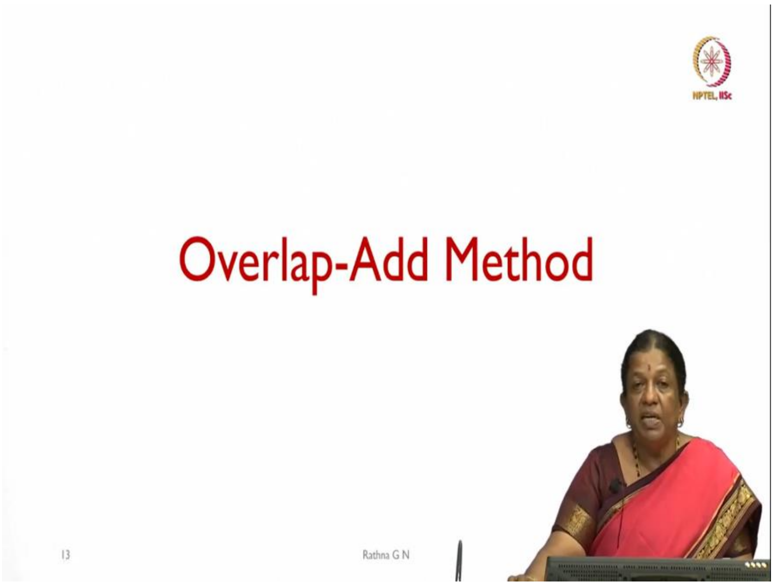

So, we will see the thing. So, for the first case, today, what we will consider is the overlap Add method. So, in the next class will come consider the overlap save method.

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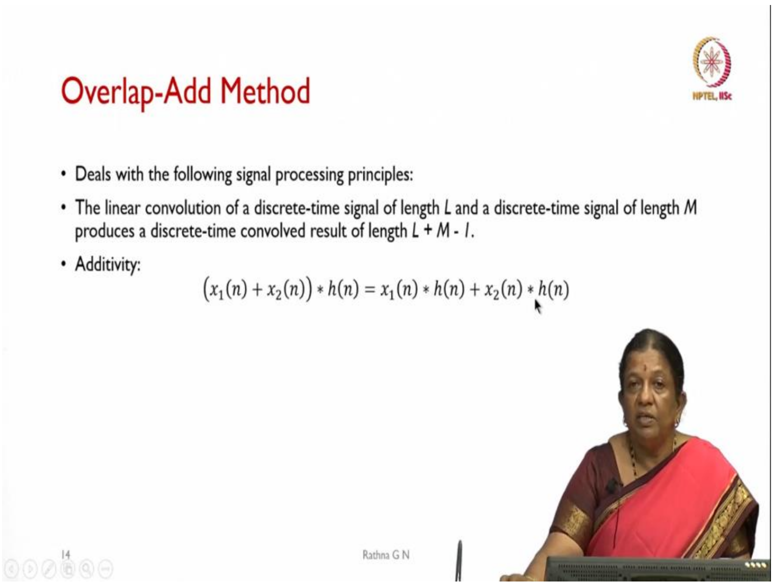

So, what is it overlap add method? This is going to deal with some of the signal processing principle what it is written here. So, what is it, we are going to have the linear convolution of a discrete time signal of length L, what we are going to restrict and a discrete time signal of length M, what we are going to get the output is $L + M - 1$. So, that is we are using the additivity property here, $(x_1(n) + x_2(n)) * h(n)$ is going to give us our $x_1(n) * h(n) + x_2(n) * h(n)$ so, that is what, what we are looking at it.

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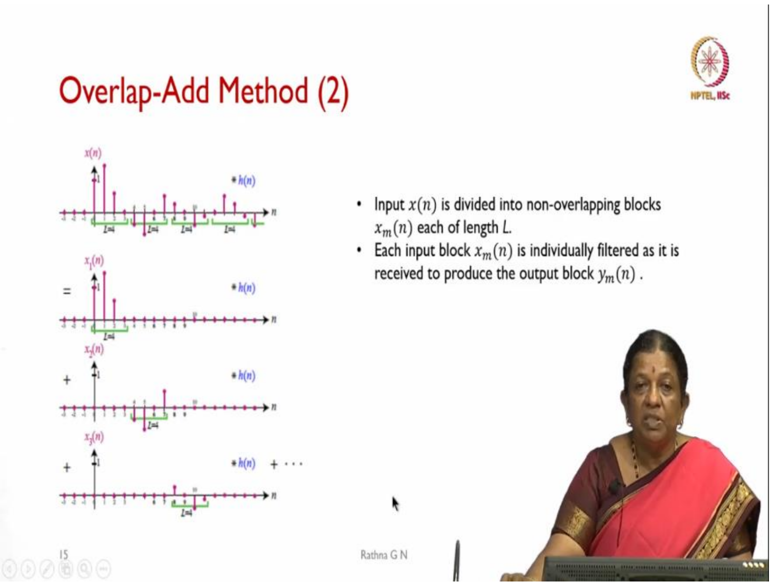

So, now we will see the thing. So, what we have is input $x(n)$ is divided into non-overlapping blocks basically, and $x_m(n)$ each of length L what we will be considering in this and then each input blocks $x_m(n)$ is individually filtered as it is received to produce the output block $y_m(n)$. So, you will be seeing as a picture really here. So, you are considering L = 1 first, that is the length in this case is N = 4 what it has been taken.

So, this is the first length, which is going to be convolved with $h(n)$. So, the other length, what we will be considering is L = 2, 3 and then 4. So, in this whatever $x(n)$ signal, so, we have 4 L basically, now, what we have done the thing, so, we have separated according to the length, so I have 4 values in this, we will do the convolution of it and the next one is convolved with $h(n)$ is convolved with $x_2(n)$. Then later on as you can see it we are adding with the previous one. So, the next one is getting added with the $x_3(n)$.

So, you will be seeing that these are the 4 samples which is in $x_3(n)$ which is getting convolved and then you will be going on till you finish your input or if it is continuously coming, this is the process which is going to repeat it for ever.

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So, what is the filtering stage. So, we make use of that is N-DFT and N-IDFT where N is the length of the this think or DFT or FFT length, which is going to be L + M - 1 and then we are going to do the 0 padding of $x(n)$ and $h(n)$. So, that are of length L what we call it M is less than N is required in this case. So, the actual implementation of the DFT, IDFT will use the as we know that for fast Fourier transform what we will be using it and then inverse fast Fourier transform for computational simplicity.

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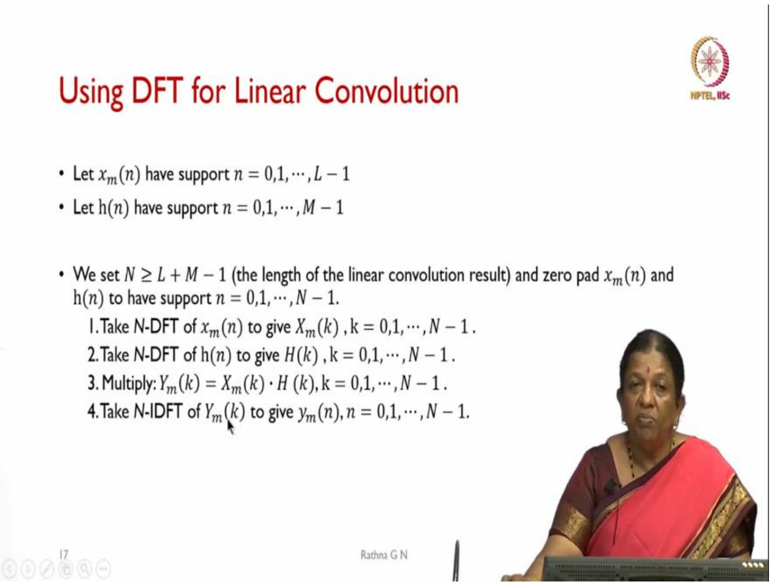

- So, how we are going to use the DFT for linear convolution so, first we are going to take $x_m(n)$ which has the length 0 to L - 1 and $h(n)$ is the other length which has the support $n = 0,1,\cdots,M-1$. So, we set that is in this case one of them is the signal $h(n)$

if it is the filter case, we will be seeing that if it is a FIR filter, we will be having the B coefficients basically or h of n can represent the coefficients of the filter.

So, this is one like the other one is L length input length, then we set $N \geq L + M - 1$ that is the length of linear convolution result and 0 pad $x_m(n)$ and then h(n) to have support for $n = 0,1,\cdots,N-1$. So, in the lab class we have seen that how we can do the zero padding even 2 problems what we have taken out by zero padding, we can make it same length and then do the FFT on the 2 signals.

So, then what we are going to do it so, we are going to take first N-DFT of x of m to give $X_m(k)$, k is varying between 0 to N - 1 in this case, and then the next one is take N-DFT of h(n) to give us $H(k)$ which is k also will be between 0 N - 1 then we know that in the frequency domain when we have 2 DFT's it is only the multiplication between the 2 signals that is $Y_m(k)$ is nothing but $X_m(k) \cdot H\ (k)$ for k $= 0,1,\cdots,N-1$.

So, once we have done the multiplication, we know that the result is available then we can take the IDFT, N-IDFT of $Y_m(k)$ to give our $y_m(n)$. So, you must be wondering why we have to do all these things why not sigma. So, in the last class, we have seen that with the 512 points DFT computation cost and then FFT computation cost and what the frequency will be achieving, although we are doing this 4 times basically, or 3 times we call it because N-DFT to of it and N-IDFT cost is same basically.

And in between, we have to do the complex multiplication in spite of the thing our computation time or the frequency of operation is higher than the normal DFT and even which is equivalent to our linear convolution.

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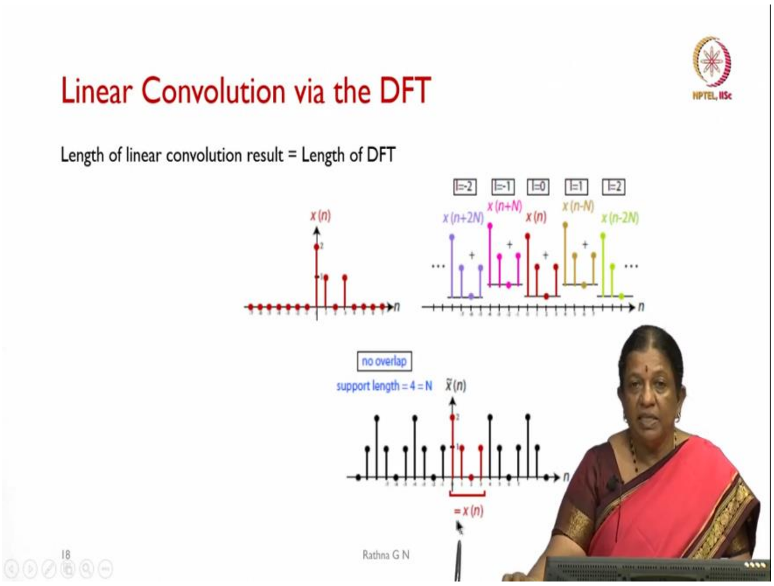

So, now we will see how we are going to do this linear convolution via the DFT. So, length of linear convolution result is equal to length of DFT what we are going to get it so, this is our 4 samples. So, we are repeating the things same 4 samples on both the sides left and then right hand side and we have assumed there is no overlap. And then we have assumed support length is 4, which is N = 4 and we can see that here $x(n)$ what we have is the 4 length and then the repetition on the $\tilde{x}(n)$.

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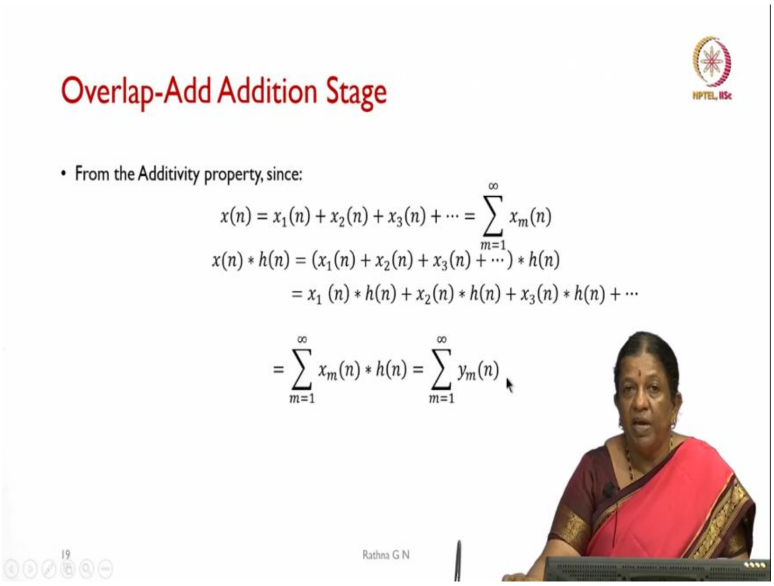

So, now we will see how we are going to overlap add addition stage. So, from the additivity property since our $x(n) = x_1(n) + x_2(n) + x_3(n) + \cdots = \sum_{m=1}^{\infty} x_m(n)$. So, this is what what we have the convolution, so we will be using $x(n) * h(n)$ which is nothing but $(x_1(n) + x_2(n) + x_3(n) + \cdots) * h(n)$ or because of the our additivity property so we can do them individually.

So that is $x_m(n)$ convolved with $h(n)$, m is varying between 1 to ∞ and then what we call it as summation of all of them individually done $x_m(n)$. So, we will be adding all voice basically that is $\sum_{m=1}^{\infty} y_m(n)$ will be giving us the complete overlap add output.

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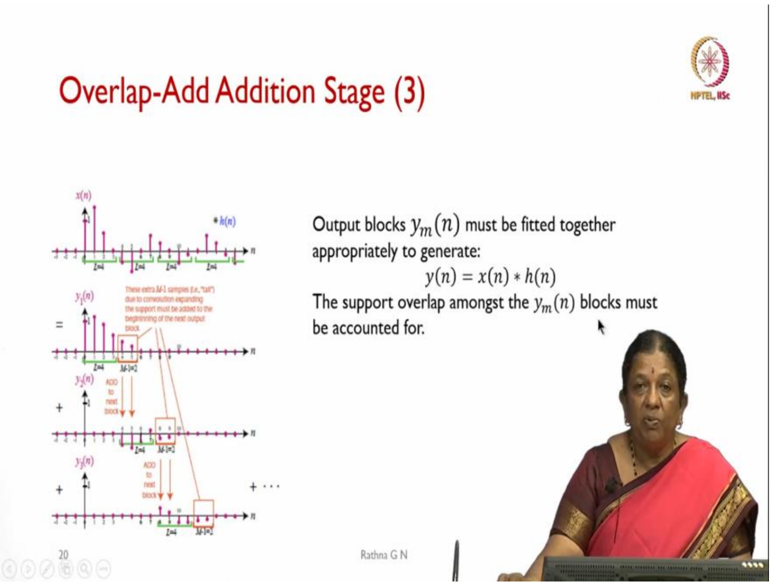

So, as we can see the thing here, so this is the length, what we have it we are going to convolve with $h(n)$ and we will be generating the linear convolution. So, we know that it is going to give us L + M - 1 length here. So, that whatever this extra M - 1 samples, that is we call it as, which is going to be in connection, we will be discarding them, or we can take it to the next one and then use that is add to the next block basically.

So, this is first $y_1(n)$, and then these 2 are going to be added to the next block. And then we will be making it as L = 4 length here, as you can see the thing, so, 2 samples will be coming from there these 2 are from M - 1, which is 2 will be from this block what will be coming, and then we will be using that to compute our here also $y_2(n)$ and then later on it is $y_3(n)$ so, you will be adding these 2 samples as you can see that they are getting repeated in successive.

That is the M - 1 samples are repeated in the next blocks for our computation. So, we call y of n is nothing but x(n) convolved with our $h(n)$ so, thus support overlap amongst $y_m(n)$ blocks must be accounted for that is what the one has to keep it in mind.

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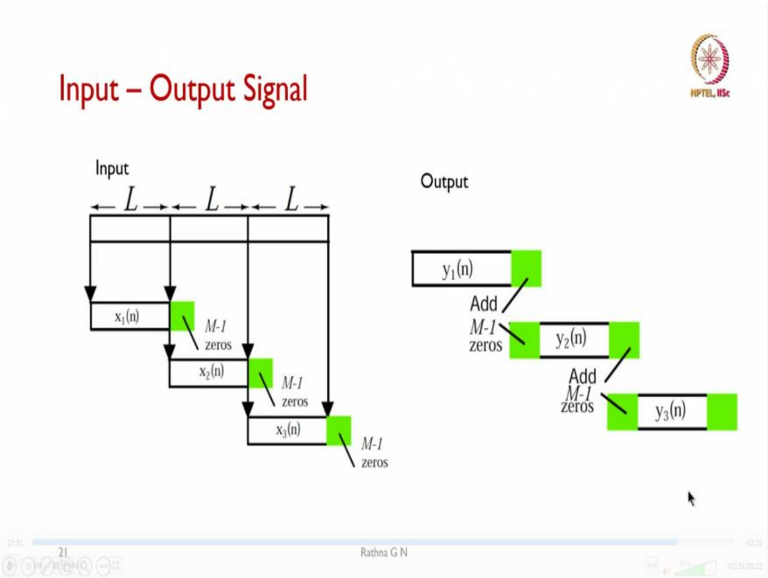

So, how the thing is going to happen, we have this figure shows in I think that was a little small so, you will be understanding the thing, this is my complete input. So, we have bifurcated into L chains of them as you can see it we call them as $x_1(n)$ and $x_2(n)$. So, you are seeing from this M - 1, what will be adding as zeros in this case. And here also we will be adding M - 1 zeros so, you will be seeing that you are adding m - 1 zeros with $x_1(n)$ $x_2(n)$ and then $x_3(n)$.

Then what will be our output this is $y_1(n)$ and add this M - 1 zeros to this one that is you will be appending on the $y_2(n)$ and then we have on the other side also M - 1 zero. So, this one is going to be added M - 1 zeros with $y_3(n)$ and so on what will be having the output.

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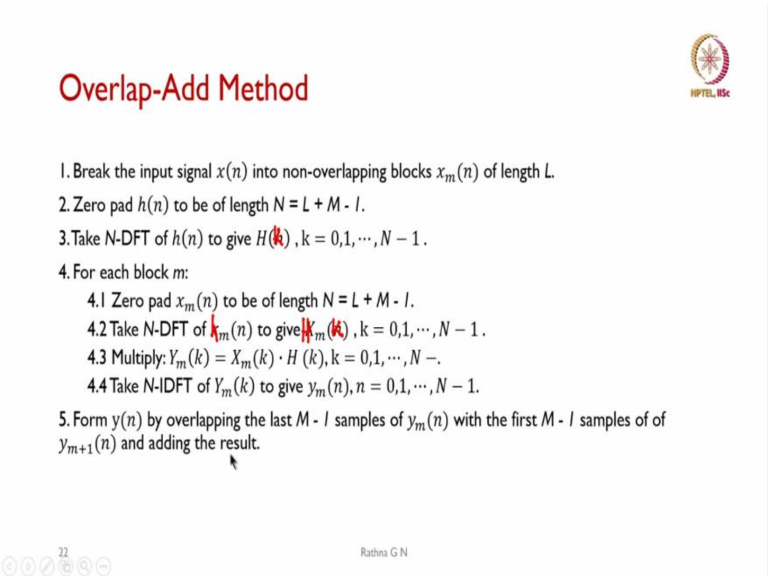

So, we have overlap add method how it is going to work, what is shown in here algorithm what has to be written. So, what is it first break the input signal $x(n)$ into non-overlapping blocks of

$x_m(n)$ of length L and then we are going to do the 0 pad, $h(n)$ to be of length N = L + M - 1. And then we take N-DFT of $h(n)$ to give our $H(k)$ basically $k = 0,1,\cdots,N-1$, $h(k)$ here for $k = 0,1,\cdots,N-1$.

So, for each block m, we are going to 0 pad $x_m(n)$ to be of length N is equal to same thing L + M - 1. And then we are going to take N-DFT of $x_m(n)$ to give $X_m(k)$ here for k = 0 to N - 1. So, multiply $Y_m(k)$ with $X_m(k)$ what we have got it here with that of $H(k)$ this should be h here, h m of n to give our $H_m(k)$, then these are the 2 signals what we have taken the DFT and then we will be multiplying it and then take N-IDFT of $Y_m(k)$ what we have got it to give $y_m(n)$.

For n will be because inverse DFT n will be varying between 0 to N - 1. So, then the 5th step is in form $y(n)$ by overlapping the last M - 1 samples of $y_m(n)$ with the first M - 1 samples of $y_{m+1}(n)$ and adding the result, so this is how we will be doing it. We will take up an example and then show that how we have done the overlap add method. Thank you. And in the next class we will overlap save method so both of them should give us the same results. Thank you.