Mathematical Aspects of Biomedical Electronic System Design

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Week – 07

System of Linear Equations and Scaling Laws

Lecture – 21

Thermal Properties of a tissue

So, we will continue on from our previous lecture where we did a deep dive further into electrical properties. Now it is time for us to move on to the next property of tissues, which is thermal properties.

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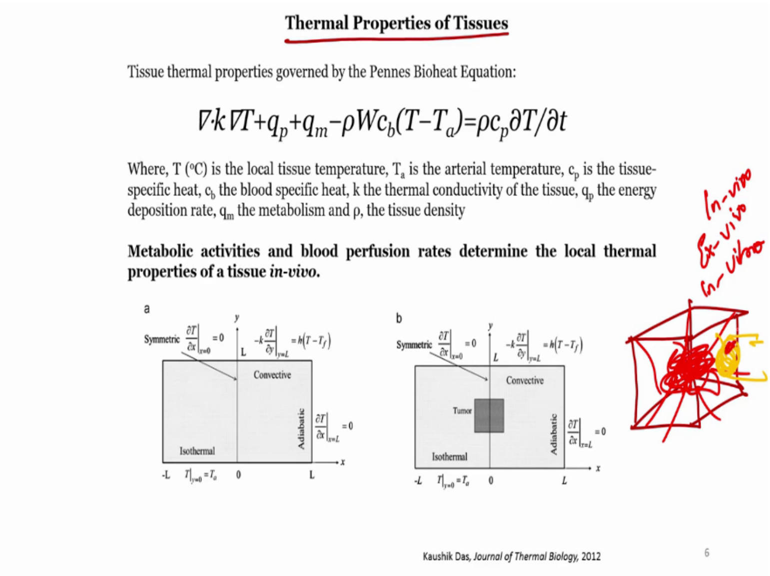

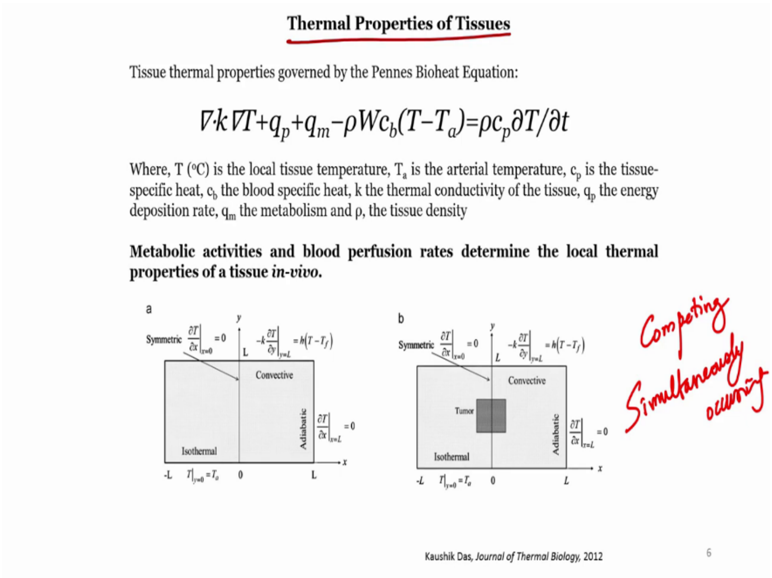

So let us look at the slides. So, thermal properties before we start, there is something very interesting, electrical property, if you look at it, it is a much more recent discovery, even then, we cannot call it recent, it is still many, many centuries old. But thermal properties have been the most oldest properties that we have studied of tissues, because heat is something that we were scientists will automatically get fascinated about right. Because it is right there in front, but electricity is something that came electrical conduction, those models came comparatively more recently.

So, thermal properties are very well it is not experimentally that well done on biological samples, but materially thermal properties are very well understood. So, people have tried to formulate equivalent thermal property equations for tissues. So, that is what we are going to study now. That is basically the thermal properties of tissues and how does a basically what is thermal properties of tissue, what are we trying to understand?

Let us say I take tissue, let us say I take a tissue piece, let us draw a bigger tissue piece. Let us say I take a tissue, tissue piece. Now here, there are two things that can happen. Tissue is not a non-living entity. It is not a immaterial object, there are a lot of activities happening inside the tissue. There is cells are undergoing metabolic activities, cells are digesting food a lot of things are happening.

So, there is actually a lot of heat generation from inside as you will know that we are warm blooded animals, humans are warm blooded animals and we actually generate a lot of heat from inside our body and that is what keeps us warm. Even in a cold environment, this heat internal heat generation is what keeps us warm. So, that is one part of the heat, the internally generated heat also translates or transmits inside the tissue outward.

Now, there is another heat like if I try to give heat from outside using a coil or something and I try to heat the tissue, that heat will also get transmitted through the tissue. And these two activities can happen simultaneously especially, when the tissue in question is not really ex-vivo, if it is in-vivo. What is ex-vivo and in-vivo? We have already seen. In-vivo is inside the body, ex-vivo is tissue outside the body and in-vitro is tissue on a tissue culture dish or cell plate.

Now, this internal heat generation will be of very much importance when we are trying to measure the thermal properties of a in-vivo tissue. Ex-vivo that much not important but in-vivo, it will be very important. Now, this is the challenges involved in trying to understand the thermal transport or the transport of thermal energy in tissues. So, there are a lot of components to it.

There are a number of competing simultaneously occurring components to it. There are two things, one is competing aspects and one is simultaneously occurring. How can we summarize all these simultaneous activities in a single equation, so that we can estimate its thermal, estimate the thermal parameters of the tissue? That is a question that we want to ask. So, how do we estimate the thermal parameters of issue?

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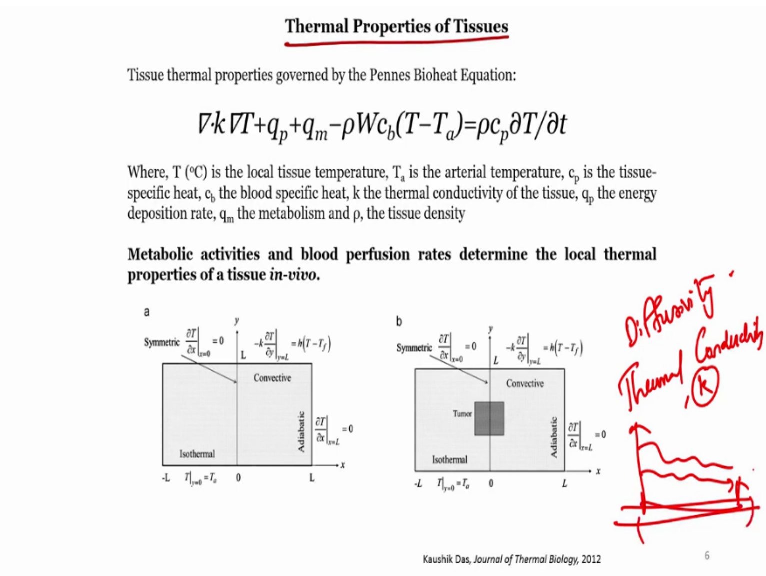

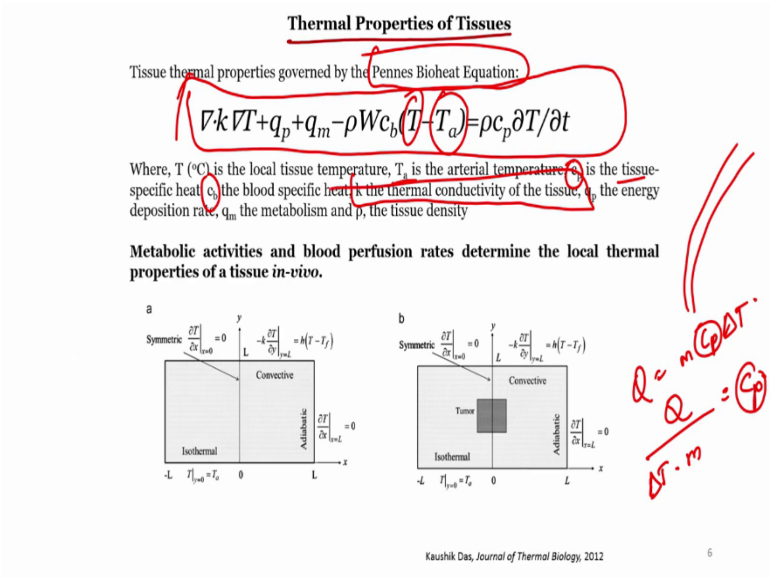

The main thermal parameter, there are different thermal parameters called one is called diffusivity which we will come to later in the course. Then there is called thermal conductivity or K. Today, we will cover about thermal conductivity. Thermal conductivity is a measure of how well the tissue conducts the heat energy from in a heat gradient.

Let us say, one end has very high temperature and one end has low temperature, the heat will flow obviously from high temperature to lower temperature. How quickly does this happen in time? How efficiently does this happen? Thermal conductivity is a measure of that. Diffusivity is a slightly different quantity; we will come to that in a later lecture.

Now, as I told there are different competing phenomena that happen during thermal conduction. How do we summarize all that in a mathematical equation so which can help us estimate the K value that… (()) (5:24) – (()) (8:19)

Thus to understand these competing phenomena that happened in thermal transport in biological tissues, there is a very famous equation that is called the Pennes Bioheat Equation, which is what is written here. This looks like a very overwhelming equation for you. But what we will do today is we will break down the individual components of this equation. Calmly we will sit down and we will peacefully understand what are the different components of this equation and understand how we can intuitively write up this equation for a tissue.

There is it is a very interesting concept. So, we will see what this Pennes Bioheat Equation. So, we see a lot of things

$$\nabla . k\nabla T + q_p + q_m - \rho W c_b \left(T - T_a\right) = \rho c_p \frac{\partial T}{\partial t}.$$

Let us see what is this. T, T here is basically the tissue temperature, overall bulk tissue temperature is T. T_a is the arterial temperature. By arterial temperature, I mean the temperature of the arteries that carry the blood that is T_a clear. C_p is a specific heat capacity of the tissue.

What is specific heat capacity? Yeah, specific heat capacity is a specific measure of per kg or per weight, $mC_p dT$, $Q = mC_p \Delta t$. So, what is C_p for a given amount of what is the amount what is the increase in heat energy per unit rise in temperature per unit mass of the quantity that is called C_p.

So, that is specific heat capacity of the tissue that is C_p. Cb is this specific heat capacity of the blood vessel. Why blood vessels coming in here, we will see. Specific heat capacity in the blood vessel. K is what we are trying to solve. K is the thermal conductivity of the tissue. Here then there are two quantities q_p and q_m.

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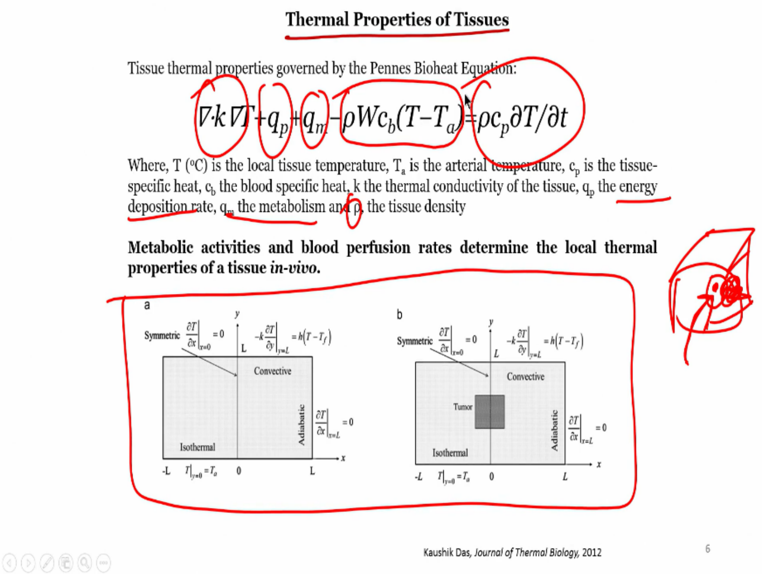

q_p is the deposition rate and q_m is the metabolism rate. We will see how what are these things soon. q_p is the energy deposition rate and q_m is the metabolism rate and ρ is the tissue density. That is understood. Now, this so local, local tissue temperature would be inside if you take any point inside the tissue what would define the local tissue temperature is a combination of the energy deposition rate which is the energy supplied from outside that is energy deposition rate, internal metabolic activities and the blood perfusion rate.

Now in this equation, some parts contribute to heating the tissue and some parts contribute to cooling the tissue. So, we will see which parts contribute to heating the tissue and which parts contribute to cooling the tissue. Now, for the time being let us not look at this, I will come to this later in a future lecture.

Let us completely try to understand Pennes Bioheat Equation. Now what I am going to do next is I am going to break down this big equation into its individual components. We have added few-few terms right to get this equation, let us break down that.

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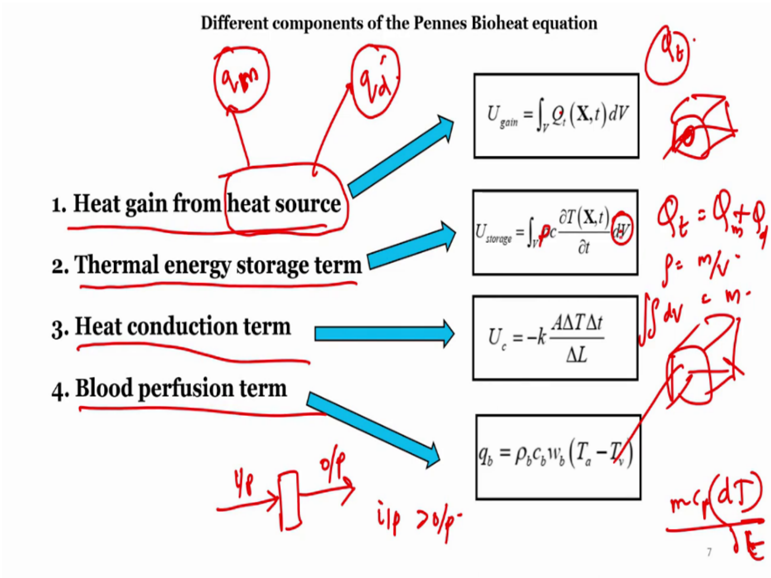

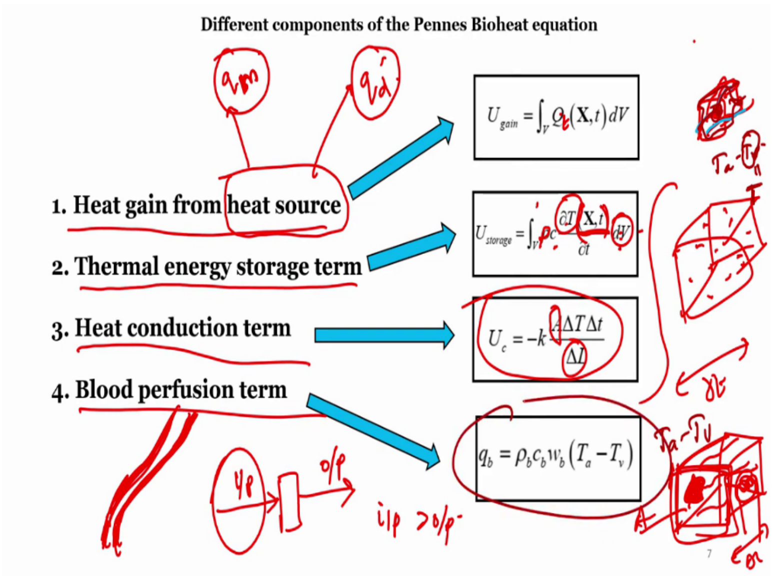

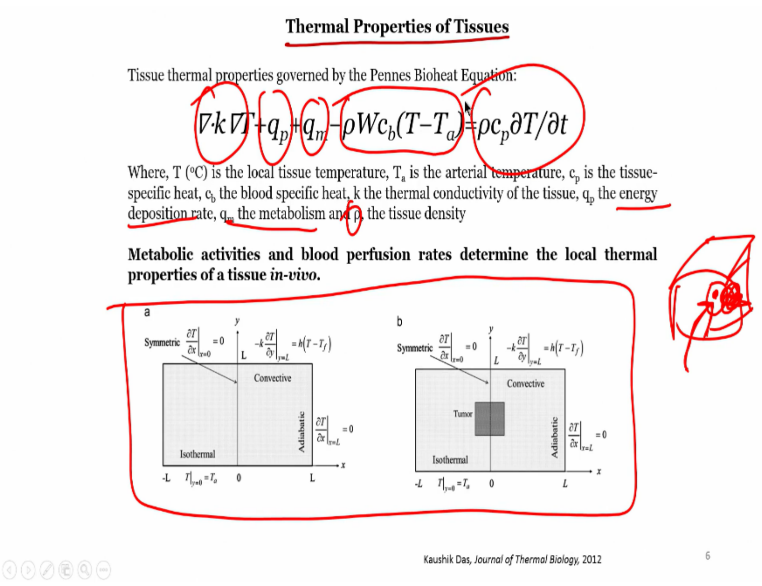

So, what are the different components of that equation? So, we have a heat gain source from heat gained from a heat source. Then we have the thermal energy storage term. Then we have heat conduction term. Then we have blood perfusion term. So, now what is heat gained from heat source? Heat source, simple just tried to see it in a very simple way. What are the heat, let us we have seen so many things about the tissue.

What could be possibly, what could possibly be the heat sources for the tissue? Q_t is the total of the heat sources. What all could be the heat sources? Just now we saw. So, what are could be heat sources are basically q_b or q_m and q_d. What is q_d? q_d is a deposition heat rate, q_m is a metabolic rate. So, the metabolism that is what is metabolism? The breaking down of food to release energy, so that the body can use the energy and perform activities. So, that is basically a catabolic process which releases energy that means there will be heat generated.

Why is heat generated? Because the conversion of food to energy is not may not be a very efficient process and the lack of efficiency will lead to dissipation of heat, loss of energy in the form of heat. That is one form of source of energy, heat source. Another heat source is the deposition rate. Let us say I am taking tissue and I am heating the tissue from outside using a coil or let us say I am sitting in a room heated environment with this the room is heated to 40 degree, so that all that heat energy will try to get into my body.

So that is a deposition rate, that is the two heat gains sources. Basically, the metabolism rate and deposition rate. So, the Q_t=Q_m+Q_d, deposition rate and plus metabolism rate. Now, what is thermal energy storage term? What does that mean? I told you that there are some aspects that are trying to heat the tissue and there are some aspects that are trying to take away the heat from the tissue.

So, if you take tissue like this 2D, I am supplying heat and I am also taking out heat. This is input, this is output. Input is the ways by which we heat the tissue, output is the ways by which we take out the heat from the tissue. When input is greater than output, that means, we are heating the tissue much more than we are cooling the tissue that means there will be an excess amount of heat energy that might be there.

So, that energy will get stored in the tissue and lead to an overall increase in temperature of the tissue when input is greater than output. That is the thermal energy storage term. That is again what is that that will again fundamentally lead mean $mC_p dT$ only. That measure of that energy would anyway been $mC_p dT$ only and if you want to have a power of it, we can divide it by time, ∂t.

Power means we will get a rate of change of that stored energy as a function of time. So, what we do here is so, the stored energy is ρ. ρ is basically for the mass, ρ, ρdV is there here. We are integrating it across the volume to get the mass. What is density? Density is equal to $\frac{m}{V}$. So, $\int \rho dV$ is total mass that is what we have done here. $\int \rho dV$ is done, $\int \rho dV$ is carried out to get the total mass to integrate the total residual heat energy in the tissue integrated over the entire volume to get the overall stored energy.

That is what it is, ρdV will give you mass $mc\partial T$ as a function of x and t. Basically what it means is the temperature of the tissue may not be uniform throughout. So, this T(X,t) is the metrics of the tissue temperatures at different points and at different times of the tissue. So, integrating it over all points over all time for a specific period of time ∂T, we get the total amount of energy stored, that is this integral. Same thing here, this energy gain is basically we are integrating only the input part and integrating it over the entire space of the tissue over volume.

So, we understood that overall, two things which is heat source and thermal energy storage term. Why would thermal energy storage happen? It happens when you are heating the issue much more than you are cooling the tissue. Now, next is heat conduction term. This is simple. What is heat conduction term?

Let us say I am, I am having a very big tissue this small tissue I am showing, it can be a whole human body but let us say metabolism is happening only once one part of the tissue and we are giving also heat from using a coil at some part of the tissue. Now, these two energy sources are supplying energy to the tissue but this applied heat energy will actually spread through the tissue and over a period of time it will become a uniform tissue temperature, if you are sufficiently allowing it to stabilize.

That process is the heat conduction term which depends on the thermal conductivity of the tissue like how well does the tissue translate or conduct the heat generated at a local point to and make it universally distributed. So that and that depends and the time it takes or the total amount of energy required to particularly elevate a particular amount of quantity of tissue to a particular temperature would depend on the dimensions of the tissue A and ΔL with A would be here, this would be A and this would be this width of the tissue will be ΔL.

So,

$$\frac{A\Delta T\Delta t}{\Delta L}$$

what does it give what is the time available? In a given time in a given rise for a given rise in temperature for a given thermal conductivity of the tissue how much energy will be conducted that is what this this unit is doing. Next is blood perfusion term, it is a very interesting aspect. What that means is that we are having blood to carry blood vessels carrying blood.

So, blood vessels are carrying blood and this blood vessel has is a is also a tissue. So, this blood vessel is also a tissue and it has a particular specific heat capacity, particular density. So, that is what this is. So, what this means is how well is the blood taking out the heat blood because blood is a liquid and it is covered with the blood vessel which has a specific heat capacity.

And the blood also has a specific heat capacity and how well is the blood vessel taking out the heat from the tissue. So, what that means is let us say I have tissue here and there are blood vessels running. Now there is a heat generation here and I have a blood vessel running close to this heat generation point. I will use a different color.

There is a blood vessel running close to this heat generation point. Now, what happens is the blood vessel or the artery will have a particular temperature, this heat generation point we will have another temperature and how well now how well this heat generated at this source gets transmitted in the blood vessel once that heat is transferred to the blood vessel, the blood vessel will then take that heat and distribute it everywhere.

How well this transmission happens depends on the thermal temperature gradient. So, that is basically (T_a - T_v), here we a means artery, v means vein. Vein is the return path of the blood but then most of the time it is already in a saturated state such that we can make a assumption that the vein temperature is equivalent to the tissue temperature. That is why we write usually as (T_a - T_v) if you see the previous equation you can see it is written (T_a - T_v) this is because there is a minus sign here.

It is basically (T_a - T). So, why we have written like this because blood can usually what happens the heat conducts from the tissue to the blood. So, T will usually be higher than T_a. So, this is basically a heat removal process that is why minus is put, so that you can understand it. So, it is a heat removal process where the temperature gradient between the tissue and the artery will cause that temperature heat to be transferred to the blood which then carries it to different parts of the body.

So, these are the different aspects of the Pennes Bioheat Equation. I hope it is very it is very clear to you what are the different aspects of the Pennes Bioheat Equation. Now, let us see now we have understood all the individual parts. Now let us go back to the equation and see if we can easily understand whatever is the equation now.

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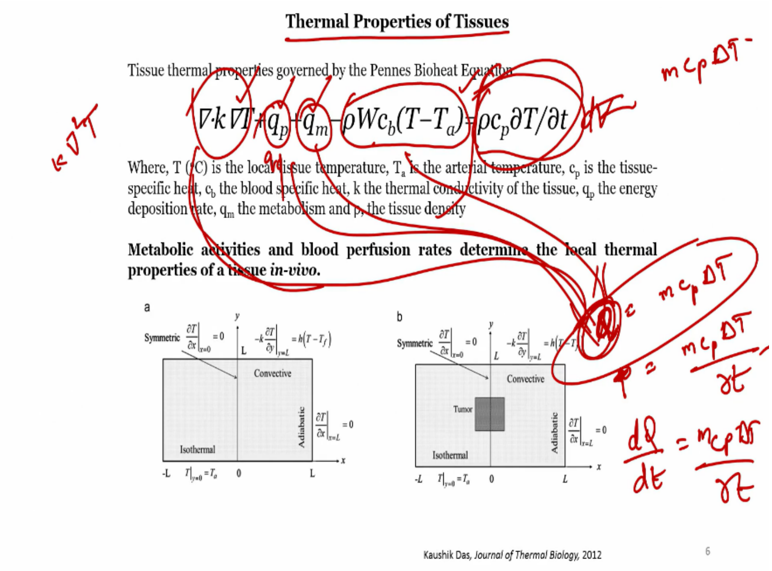

So now let us look at the equation after knowing all these things. Let us look at it in a simple way, what are the plus terms and what are the minus terms here. So, q_p is a plus term, q_m is a plus term. This $\rho W c_b \Delta T$ is a minus term this is also a plus term. What does that mean? Q_p is what? It is a deposition rate or qd that we saw. Q_m is what? The metabolic heat generation rate.

So, these are all adding temperature to the tissue. That is why they are positive. Now, what is $\nabla. k\nabla T$ or $k\nabla^2 T$, it can also be written as $k\nabla^2 T$. What that means? That means the generated heat is conducted throughout the tissue. So, this is again adding positive effect of heat that is getting permitted throughout the tissue. Now, $-\ \rho$ this is the blood vessel taking out tissue from taking out heat from a specific tissue location.

So, this plus and that minus effectively leads to an overall rise in temperature of the tissue which is this $\rho c_p \frac{\partial T}{\partial t}$, T term which is equivalent to the $mC_p dT$ that you are familiar with. So, this is basically what is Pennes Bioheat Equation? Penned Bioheat Equation is simply

$$Q = mC_p dT$$

or let us say

$$\frac{Q}{t} = \frac{mC_p dT}{\partial t}$$

which is basically

$$\frac{dQ}{dt} = \frac{mC_p dT}{\partial t}.$$

So, if we integrate this Penned Bioheat Equation over entire volume dV, we are basically getting $mC_p dT$ only, but then this Q is a very tricky thing. This Q has several contributing factors which is what we have seen here. It has q_p, it has q_m it has $\nabla. k\nabla T$, it has the blood perfusion rate.

So, you understood how a basic equation such as $Q = mC_p dT$ has been modified by the scientific community to take into account the different competing biological processes that are actually happening inside the tissue for us to better estimate, it is finally about estimation problem, for us to better estimate this Q value that is the overall idea. We can simply put $\nabla. k\nabla T = \rho c_p \Delta T$ but that may not be the actual value if we experimentally measure.

So, these things have come to be after people have experimentally measured these values and found out that okay simple two phrase equations like this one phrase on one side and one phrase on another side is not really helping me estimate the actual parameters. There might be some other things that are happening. Then people went into understand the physiological processes that are happening inside the tissue.

They understood that there is internal metabolic heat generation they understood that we are externally heat providing heat, they understood that the blood is actually taking out the heat from the tissue and spreading it to different parts of the body. And then overall because of this overall heat conduction from the thermal conductivity of the tissue, there is an increase in temperature which is finally translating into $mC_p dT$. So, these things people have understood that is how we connect mathematical methods to actual physical biological principles.

That is why mathematical modelling for biomedical systems and bio-engineering is required. I hope you are able to appreciate the importance of mathematical modelling from this very basic of equations but very profound of equations. So, this I think now, without even looking at the equation if somebody asks you what is Pennes Bioheat Equation what will you say.

So, you do not even need to write an equation, you have to say Pennes Bioheat Equation is fundamentally the heat transfer equation only which is $Q = mC_p dT$, but the Q term there is contributed by multiple factors. There is lot of things happening inside the biological tissue it is not a very non-living, inactive entity. A living tissue is a site of intense activity.

So, there are a lot of things happening. There is internal metabolic heat generation, there is internal deposition of external heat to the tissue, there is internal conduction of the heat that is generated throughout the tissue then there is the separate conduction of the that is the physical conduction by conduct then there is separate conduction liquid conduction of conductive or conductive heat transfer by the blood vessels from the site of heat origin to the different parts of tissue.

All these competing phenomena contribute to the heat capacity or the changing heat properties of the tissue. So, this is how you should talk to somebody. So, if somebody asks you okay, tissue is just like any other material, we can just use the normal $C_p dT$ formula. So, you tell them that yes, you can use a normal $mC_p dT$ formula, but the quantities that you estimate may not be experimentally correct, when you actually experimentally measure the values it may not be correct. And then they will ask why do you think so?

Yeah, because there are a lot of competing processes that happen such as internal heat generation, external heat deposition, blood perfusion and internal conduction that can actually lower the amount of that can help you under estimate the current Q value. Otherwise, what happens you will be overestimating it because there are some negative terms which are not considered when you are simply estimating it.

So, this is how biology and physics gives insights to mathematical modelling and mathematical modelling in turn gives insights to better understand unknown biological principles. So, what would have happened? Historically, people are not measured, they found out that the value is not matching experimentally, then they went on to understand the structure of the biological tissue. So, the mathematics would also have helped us to understand the biology also. So, it is a two-way street.

So, mathematics requires biology, biology requests mathematics to understand each other better. That is how science grows. So, in the next lecture, we will actually go into the details of this another diagram that we have seen and then we will take a deep dive into the mechanical properties of the tissue. We will also see as small examples from across the world, how people have tried to measure thermal conductivity and what are the challenges in it. Those are the next two sessions.

Next session, we will see the actually the adiabatic isothermal interfaces of tissue or how do we mathematically model them, then we will see specific examples from literature where people have tried to measure thermal conductivity then we will see mechanical properties of tissues. Thank you.