

**Mathematical Aspects of Biomedical Electronic System Design**  
**Indian Institute of Science Bangalore**  
**Lecture 20**  
**Scaling Laws**

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**Mathematical Aspects of Biomedical Electronic**  
**System Design**  
**Scaling laws**

Hello everyone. Welcome to the course mathematical aspects of biomedical electronic system design. Today, we are going to cover the topic scaling laws. Before we start with this topic scaling laws, we will first do a recap on the topic that we covered in the last TA session which was on percolation theory.

The reason we are setting scaling laws as you will understand in the coming slides is that these scaling laws actually help us quantify to get some numbers to get some patterns in the biological systems be it tissues be it cellular structure or be it organization at an organ level. So, if you talk about a cellular structure or a tissue or at an organ level at all these three levels, you will find that these scaling laws help us quantify that how a particular structure or a particular tissue or a particular organ is evolving.

It also helps us in understanding physical properties like the way we saw yesterday with respect to percolation theory, that how the system disorder in biological tissues that we had covered in the first TA session in the series and how that system disorder helped us to understand and appreciate the heterogeneity which is present or inherent in biological tissue and also in materials which can be used for making biomedical devices. So, let us begin with our description of scaling laws.

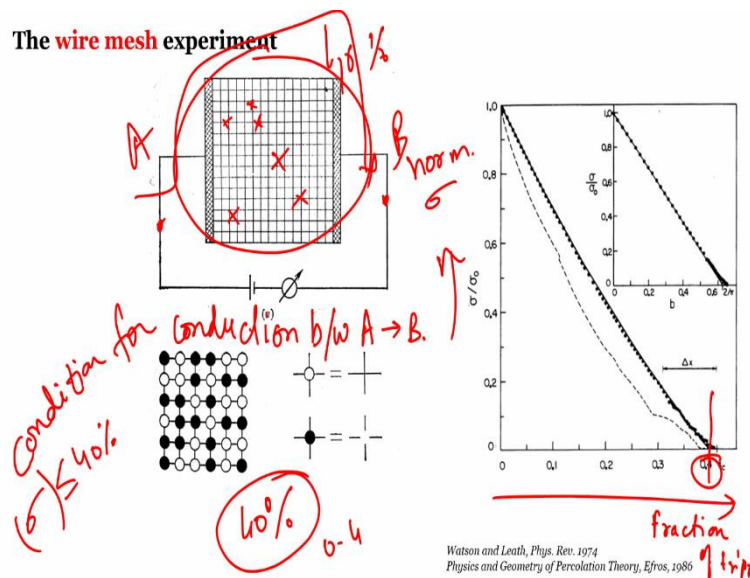
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# Percolation Theory



Before that, we will have a quick recap on the percolation theory that we had discussed in the previous session. So, in the previous session, I hope you would have understood most of the things. If you would have missed something, please, please feel free to put your questions in the forum.

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So, last time, we discussed about the wire mesh experiment, a very simple experiment, but very profound thoughts that could be deduced from this particular experiment. We saw that how this experiment can be used to understand the percolation theory. What we did was we tripped some of these junctions and then when we trip as we tripped, we try to plot as a function of conductivity.

So, in the x axis, it is the fraction, fraction of tripped junctions and this is the normalized conductivity and what we found, we found that there is exists, there exists a linear relationship like as we keep on reducing the fraction of the nodes or the conducting junctions in this particular wire mesh, the conductivity also reduces linearly. What does it tell us? After a particular point, it tells us that after a particular point is reached, which in this case was 0.4 or in terms of percentage is 40 percent.

So, when 40 percent of these junctions are tripped, the connection between point A let us say somewhere here and point B let us say somewhere here is lost, the circuit stops conducting from point A to point B, when 40 percent of the junctions or the nodes that we see here, by junction I mean this, are tripped.

What does it mean that 40 percent is the percolation threshold, the circuit needs to have at least 40 percent less than equal to 40 percent in order to allow conductivity. So, this is the condition for conductivity between A to B. Now this particular condition is only specific to this particular example. So please do not mistaken it and make it universal in nature.

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**What does it mean by the term "percolation"?**

Let each site be associated with a probability  $p$  of being occupied with charge

Let be  $N$  such sites in a **square lattice**

Then  $pN$  sites are occupied and  $(1-p)N$  are empty

(a)  $p < p_c$  (b)  $p > p_c$

$\sigma \propto (p - p_c)^t$  critical fraction

For:  $p > p_c$  and  $(p - p_c)$  is very small

Each site of a very large lattice is occupied randomly with probability  $p$ , independent of its neighbours. Percolation theory deals with the clusters thus formed, in other words with the groups of neighbouring occupied sites.

*Journal of Applied Physics, 92, 7, 2002*  
*Reviews of Modern Physics, 72, 3, 2000*

*Introduction to percolation theory, Dietrich Stauffer and Amnon Abraham*

Then what we saw? Then we tried to understand in terms of probability, because all these experiments, be it wire mesh experiment or any other disordered system, which has a characteristic percolative transport, the associated junctions which we saw getting tripped are just our means to understand the effect of percolation theory.

What does it mean? Let us say we have multiple junctions here, this wire mesh experiment was just to make you understand that the entire system if you have to select which particular

junction will be tripped off cannot be systematically determined a priori that is why the process is randomness, the process is random and hence, there is a probability associated with the entire process of percolative transport.

And then we try to understand the same thing by looking at an example where there is a maze and there are two cases. In one case the condition or the percolative path has connections less than the percolation threshold,  $p_c$  is percolation threshold or critical fraction, there are different names to it as we had discussed previously, critical fraction and at this particular point, the critical fraction is bridged.

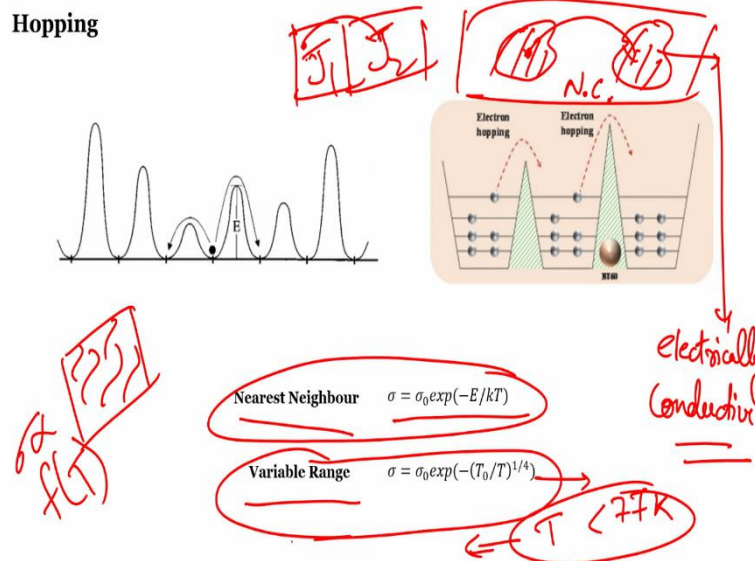
And we can see that when it comes to connection between let us say point A here to point B here, there was no single continuous path, there did not exist any single continuous path. The reason being this condition was not satisfied where the percentage fraction should be greater than or equal to the critical fraction.

Whereas in the second case, the percentage fraction or the volume fraction or the critical fraction condition has been satisfied and that is why we are able to see that there will be a continuous path from point A to point B and then we try to understand the same aspect by associating a probability  $p$  for occupation of each site by a charge.

So, let us assume that this is a square lattice which was the case study that we took for the similar study can be extended to triangular and honeycomb lattice. So, this is the case for square lattice and what we found was if we consider these each nodes as sites were charged or electrons can occupy a position, then the probability of them occupying this particular location spatially is  $p$  and if there are  $N$  such sites in this junction, so they say there are  $N$  such sites, then how many number of sites are occupied, it is a simple probability math, which is  $pN$  and then how many are vacant will be  $(1 - p)N$ .

So, by knowing what is the probability and by knowing the number of sites, one can reduce that what is the percentage fraction or  $p_c$  at a given point in any such system. And then we also tried to see that how this particular relation can be converted to an equation which can help us deduce the conductivity.

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Now, we also looked at how in a percolative network, transport happens. So, let us say I have a node from this junction. Junction 1 to junction 2, how will the transport happen for an electron to go from here to here? Please be noted that this particular network is just shown for the simplest possible case, the junctions could be something like this in the network. So, let us say these are conductive junctions and in between there is a matrix which is non-conductive. So, this is non-conductive region and this is electrically conductive region.

So, what we found was the process through which electrons can jump from one to another conductive junction is the process of hopping. Hopping can be of two types, nearest neighbour and variable range, nearest neighbour is dependent on temperature. And that is why we have also appreciated that why in the polymer composite that we had seen in the first TA session in this series, which was on disordered system, that why a polymer composite filled with carbon nanotubes will have an equation of conductivity which is a function of temperature.

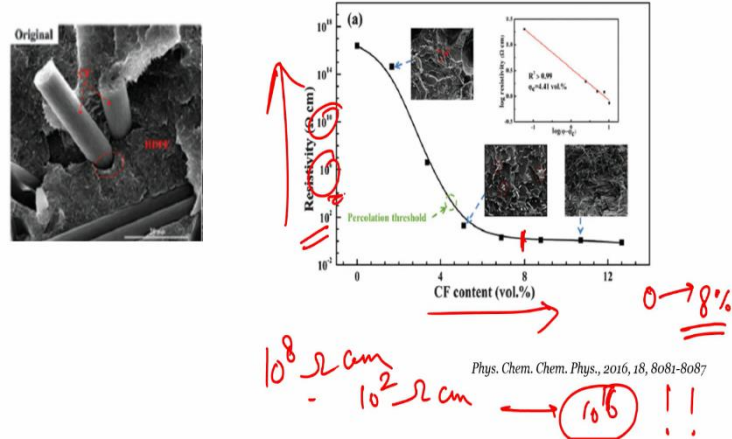
Because temperature provides the necessary excitation energy for the electron to hop from one point to another as there it does not exist any continuous path, that is why hopping is the mechanism. Another way in which electrons can hop is variable range hopping. This particular type of hopping as we saw is found for the case where temperature is at very low value compared to the case where nearest neighbour hopping is generally observed. Such temperature ranges are typically less than equal to 77 Kelvin, which is liquid nitrogen temperature.

This is not a very standard value, I am just giving you a rough idea that what could a lower temperature mean and at this particular temperature, the thermal vibrations are to a large extent quenched and that is why variable range hopping is the preferred mode of transport.

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### Examples from the literature

#### Polymer-Carbon Nanofiber composite



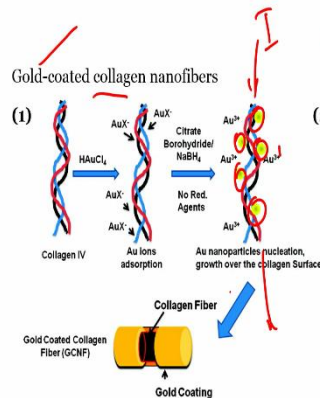
Then we saw that how these theories can be applied to a system, real time real life system. And that is why we took examples from the literature. The first example we took was for polymer-carbon nanofiber composite, we saw that how resistivity versus carbon fiber content plot shows a deviation from  $10^{18}$  ohm centimeter to  $10^2$  ohm centimeter, a staggering  $10^{16}$  times change in resistivity values when the filler fraction is increased by volume percentage from 0 to 12 percent. Rather it is not even 12 percent because it saturates at 8 percent.

Now, why it saturates? This is something that we would mostly see today. It saturates mostly at 8 percent. So, just at 8 percent itself, you could see a staggering change of  $10^{16}$  resistivity change in the overall carbon-polymer composite. So, this gives you an idea that how you can tune the property of a polymer composite just by changing its filler fraction.

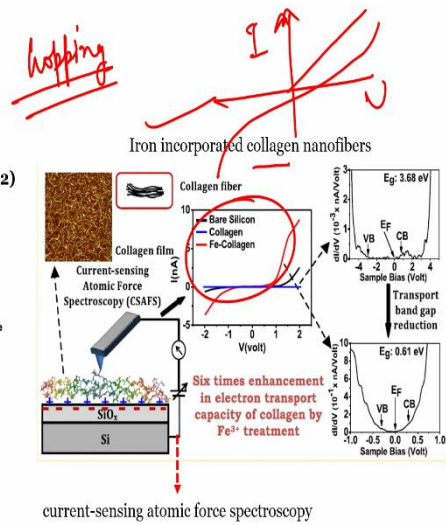
Conductive filler fraction can be used as a means to improve the conductivity or other I would use the word tune the conductivity you may not always require it to be ultra-low conductive, you may also want to match it with some impedance which is somewhere let us say here or here or somewhere here. So, tunability is one of the important part of this particular process.

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### Examples from the literature



ACS Nano 2011, 5, 6, 4490–4503

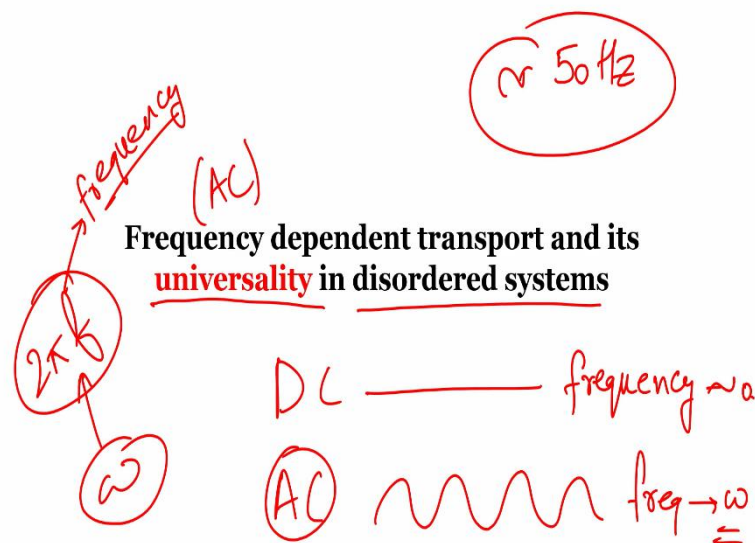


Langmuir 2019, 35, 11950–11957

Then, we also looked at examples, where biomaterials especially, we took the case of collagen in two different literature reports wherein in one case, gold-coated collagen nanofibers and another case iron incorporated collagen nanofibers were used. And they found in both the cases that dominant transport mechanism was hopping. And this hopping how was it deduced that it was hopping mechanism?

By understanding its IV characteristic which was not ohmic, ohmic is generally linear, if you plot for I versus V, in this case, there was a deviation as you can see in this particular figure. Similarly, in the case of gold-coated collagen nanofibers, gold particles, which you can see here, form a kind of percolative network of conducting islands and when current was applied between this point to this point, they form a percolative transport path and by virtue of hopping conduction, the charge was transported from this point to this point.

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So, let us now come to the topic of interest today, which is frequency dependent transport and its universality in disordered systems. This is essentially the part of scaling laws that I was referring to at the beginning of this discussion. So, we will first begin with frequency dependent transport or it can also be called as AC transport, alternating current transport. And then we will look at a more general approach in disordered system that how these disordered systems together are universal in nature.

So, what is that universal aspect, what brings them into the same category of universality as far as a disordered system is concerned? So, let us first begin with frequency dependent transport. So just to give you a brief idea, there are two different types of currents as you would know with your even 10<sup>th</sup> class physics, that there are two different kinds of voltage signals that you will have one is DC which is generally represented by a straight line because the frequency at which the voltage signal travels is almost 0. I am saying almost 0 because there will be some fluctuations in the frequency but it is negligible.

Whereas, in case of AC, there will be some frequency associated. So, frequency will be finite let us say the frequencies denoted by  $\omega$ .  $\omega$  is a standard or standard notation to denote frequency, its expansion is  $2\pi f$ , where  $f$  is standard frequency what we use for. So, for example, in our homes at least in India, we have 50 hertz as the line frequency. 50 hertz as the line frequency and there are many such systems which work on only AC signals. So, having now clarified the difference between DC and AC for the beginners, let us now try to understand that how this AC transport fails when it comes to disordered systems.



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**Brief background for the formulation**

From the classical percolation theory:  
 "ξ" is considered to be the correlation length between 2 connecting nodes/electrical junction

For an AC voltage signal: ξ length is travelled by a charge at ωξ  
 2 implications from here:

1. For ω < ωξ the average distance travelled by charge will be > ξ
2. For ω > ωξ the average distance travelled by charge will be < ξ

*J. Appl. Phys., 92, 7, 2002*

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*J. Appl. Phys., 92, 7, 2002*

We had already spoken about how DC transport happens in polymer composites in a way that we spoke about a polymer composite system, I will again use the same drawing. Let us say these are some conductive fillers, which is carbon nanotubes and this is polymer. And then, if you remember the equation

$$\sigma_E \propto \left( \frac{W_Y}{K_B T} \right)^\gamma$$

So, this was mostly associated with DC transport and we understood that what is the significance of gamma and how it is temperature dependent. So, having understood the DC transport, let us now understand why first AC transport is important particularly in disordered systems and how we can arrive at a particular equation which can help us understand and

appreciate the fact that most of the disordered systems when it comes to frequency transport, they are universal in nature, they show some universal behaviour. So, this is something we will look in next few minutes.

So, we discussed about percolation theory, when we were discussing about percolation theory, at multiple instances where I had used the term nodes, this is something I had drawn again and again which are nodes. This is a very standard example and this may not be the case in real life systems.

So, let us say I take a real life system and there are conducting islands. So, the shaded regions are conducting islands and the blank regions are non-conducting islands or let us say it is polymer or some other material. If you do not like polymer, you can add some other material. These are conducting islands.

So, for any such system, so, this is a disordered system and it has percolative transport which we have already established. So, let us go a step ahead and see how transport happens at microscopic level. In this particular figure that I am that I have just sketched, we can see that there are conducting islands separated over the spatial domain, this is let us say xy plane. Similar thing can be extrapolated to the z plane. So, this is xy plane. This is y, this is x. So, we can see that these are separated.

One characteristic feature of a percolation or a percolative system or disordered percolative system is that the separation between conducting islands is defined by a term called correlation length and standard notation, which is generally used for correlation length is  $\psi$ . So, correlation length is considered to be a kind of fundamental unit for any kind of percolative network.

What does it tell us? It tells us how closely are conductive conducting islands spaced. I will repeat the statement because it is an important statement, how closely what is the separation between these two conducting islands. So, let us say this is conducting island 1 and conducting island 2.

So, what is the separation  $x_{c_1}$  minus I should rather write  $(x, y)_{c_1} - (x, y)_{c_2}$  and this is something we can write in terms of correlation length but if we talk in terms of single direction, then we can safely write  $x_{c_1} - x_{c_2}$  considering we are considering that we are only talking about straight lines we are not talking about any staggered path, so this is the shortest distance. So, this shortest distance is called as  $\psi$  or correlation length.

Now, I would like you to recall our previous discussion that we had in the session of percolation theory where we had spoken that hopping happens when a particular at for two conditions, one, there should be enough energy and that is why temperature came into the picture.

And second, the length should be within the hopping distance or hopping length. If this length is too large, then the charge cannot transport from this point to this point, if this L is very, very large.

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### Brief background for the formulation

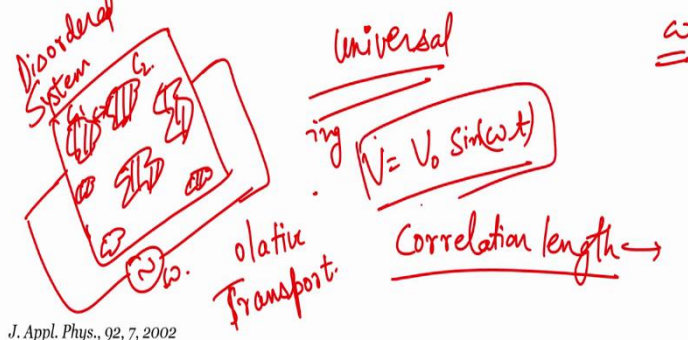
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For an AC voltage signal:  $\xi$  length is travelled by a charge at  $\omega\xi$

2 implications from here:

1. For  $\omega < \omega_\xi$  the average distance travelled by charge will be  $> \xi$
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J. Appl. Phys., 92, 7, 2002

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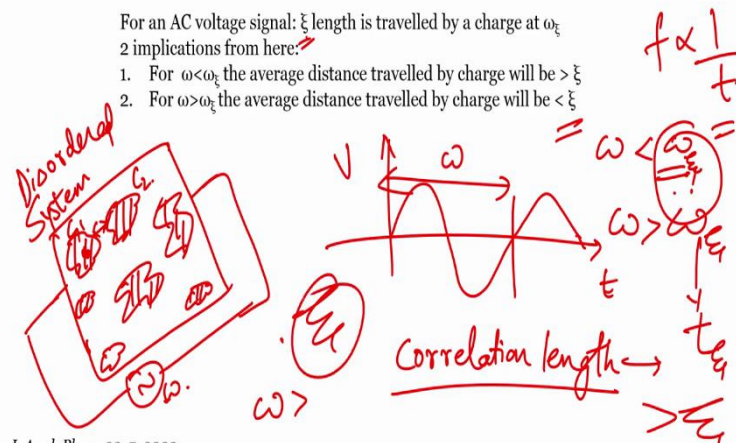
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Now, very, very large is a very generic term and it varies from system to system. But, let us talk about in a general approach, let us take that there is a correlation length for this particular system we will stick to this system to understand this aspect. Now, please pay attention I will

this following few minutes this is something which is new and you will have to pay a little more attention to what we are going to discuss.

So, let us say this correlation length exists. Now, we have discussed that in AC signal, there is an associated frequency  $\omega$ . So, let us say I apply a frequency to the system. By frequency, I mean the voltage signal I am applying a frequency. So, I am applying an alternating voltage to this particular system with some frequency.

Generally, it is if I write in terms of voltage, it will be of the form  $V = V_0 \sin(\omega t)$ . This is a standard notation, this is just for your information. So, let us say we have applied a frequency of  $\omega$  with the voltage signal and the charges are getting sufficient potential to get excited.

This is something that we are going to assume with that frequency let us suppose for the frequency corresponding to  $\omega_\xi$  the distance travelled will be the correlation length. To more understand this particular aspect, I am drawing a sin wave or other sin wave, this is voltage this is time and this is and this defines the frequency.

So, what is this frequency, this is  $\omega$ . So, for the duration of this particular time period, the distance which is scanned by the charged particle from here will be correspondingly equal to correlation length because I am defining this frequency as frequency corresponding to correlation length. Till this point it is clear I suppose.

Then, the next deduction that we can logically make is that for the frequency which is less than this which means lower frequency more time period because frequency is inversely proportional to time. So, in this extra time period compared to the time period corresponding to  $t_\xi$  the charges will travel a distance greater than  $\xi$  and vice versa.

When this frequency is greater I will write here only than  $\omega_\xi$ , which means that time period is reduced, correct because it is inverse relation. When time period is reduced, the distance travelled by the charge will be less than correlation length.

So, there are three aspects there is a correlation length, there is a frequency corresponding to the correlation length for frequencies less than  $\omega_\xi$ , the distance travelled by the charge will be greater than the correlation length and the distance travelled by charges for frequencies greater than  $\omega_\xi$  will be less than the correlation length. Till this point it is clear, correct.

So, let us now understand how this translates and how this can be correlated to the concepts that we had developed with respect to percolation theory such as volume fraction, critical volume fraction and conductivity.

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 2 implications from here:

1. For ω < ωξ the average distance travelled by charge will be > ξ
2. For ω > ωξ the average distance travelled by charge will be < ξ

For a random walk without the electric field : scanned distance along the network  $L \propto \tau^{1/2}$  or  $\omega^{-1/2}$ ; τ is walk time

Under the influence of strong electric fields :  $L \propto \tau$  or  $\omega^{-1}$

Therefore, in general:  $L \propto \omega^{-a}$ ; where  $0.5 < a < 1$ ; or  $\omega \propto L^{-1/a}$

At high frequency then:  $\omega \propto \xi^{-1/a}$

For a network system, ξ decreases as the fraction of filler increases given by:  
 $\xi \propto (p-p_c)^v$   
 where v has the value 4/3 in two dimensions and 0.9 in three dimensions (from percolation theory)  
 Substituting ω we get:  $\omega \propto (p-p_c)^{v/a}$

*J. Appl. Phys., 92, 7, 2002*

So, I will talk about few things most, please if you wish to read in detail, you may want to but most of the things are just for your understanding. You may or may not want to read it in detail, this is just to give you a glimpse into what goes in in deriving these laws which can govern the electron transport in disordered system.

So, we have talked about the correlation between the length ξ which is the correlation length that is the distance between two junctions or two nodes in a disordered system and then we talked how this can be correlated to ω and then we saw two cases ω < ωξ and ω > ωξ.

Now, considering the associated randomness, if I have a system the system will have if I have a charge, the charge will not have any direction per se of course, there will be field but within the conducting island the charge is free to move in any direction and that is why the associated motion is also called as a random walk without electric field, with electric field there will be direction.

Now, without electric field, the scanned distance will be, it has been found from mathematical derivations which can be further referred to from this particular article that a particular length which is scanned by a charge is proportional to the square root of that time which is lapsed in scanning that distance or correspondingly inverse of the square root of the frequency.

Now, under the influence of electric field, under the influence of electric field pardon me, there will be unassociated direction because you are forcing the charge to move in a particular direction. You are applying a voltage from here and then the circuit goes to completion from here with something like this.

So, let us say this is AC or even you can consider the same thing for DC, the charge will be forced to travel from this path to this path. In that case, the length which is travelled by the charge is proportional to is linear or it is a function of is a linear function of the time, unlike the case where electric field was not there, the charge was having a square root relationship with that time lapsed in covering that path.

So, talking about a general aspect, these are two extremes that we have either it is a square root dependence or it is a linear relationship. So, when unless and until there is a strong electric field, the general case that could be defined would be  $L \propto \omega^{-a}$ , where  $a$  would vary from 0.5 which is the lower limit and 1 which is the upper limit under the influence of strong electric field. So, this point is clear, I will just repeat it for the better understanding.

In any disordered system, for a particle which does not have any direction even local electric field does not exist. So, there is no electric field. What will happen? The particle will move or scan a distance and that scanning, the distance that is a scanned will be proportional to the square root of the time lapsed or inverse square root of the frequency.

Under the influence of very strong electric field, there will be direction, the particle will be forced to move from point A to point B and that is why the relationship will be linear. In general, if we consider it will be somewhere between the relationship where electric field does not exist and the case where strong electric field exists.

Now, on a similar lines, we it has been found it has been deduced that at high frequency, if we come back to the discussion that we had for the correlation length what we could find the same thing for correlation length at higher frequencies this would be applicable to correlation length as well. Am I correct?

In the previous case we had just saw just seen that as the frequency increases, the distance travelled by the charge will be less compared to the correlation length and vice versa. So, the same relation I am just substituting  $L$  with  $\xi$  and  $\omega$  with  $\omega_\xi$ . That is it, there is no other change in this particular equation, if we compare this and this equation. Now, now brings now, this brings me to an important point of discussion.

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### Brief background for the formulation

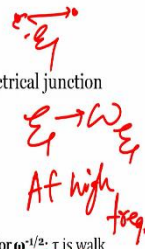
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(L)

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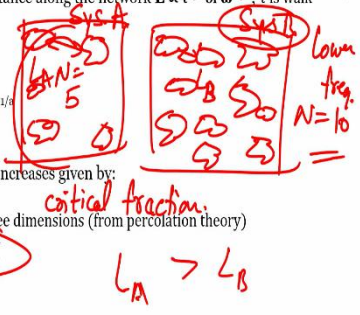
At high frequency then:  $\omega \propto \xi^{-1/a}$

For a network system,  $\xi$  decreases as the fraction of filler increases given by:

where  $v$  has the value  $4/3$  in two dimensions and  $0.9$  in three dimensions (from percolation theory)

Substituting  $\omega$  we get:

$$\omega \propto (p-p_c)^{v/a}$$



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This is very interesting; you will appreciate more about the concept of filler concentration as we go through this discussion. So, let us assume again I will draw the same picture in which there are several charge transporting conducting islands. This is one case and adjacently, I will draw another case where it is a little sparsely distributed. So, in this case, let us say there are over 5 conducting islands,  $N = 5$  and in this case there are  $N = 10$ .

So, it is almost double that of this, it is almost double that of the number of islands present in this particular case. So, what do we see from this particular aspect when trying to correlate with the correlation length and the correlation frequency? Now, let us see as we increase the failure fraction let us say the correlation length for this system, name it as system A and system B.

So, for system A, the correlation length will be  $L_A$  and for correlation and for system B it will be  $L_B$ , where if we see closely just by the look of it we can find that  $L_A$  is greater than  $L_B$  which means the charges have to travel a smaller distance or shorter distance when filler fraction is increased compared to the other case where filler fraction is just half of the case, half of the previous case.

And this is what is explained in this particular equation that as the filler fraction increases, the correlation length decreases and that decrease is given by this particular relation where  $p$  and  $p_c$  we have already discussed,  $p_c$  is the critical fraction which is required to form a continuous percolation network and  $v$  is an exponent. So, when we combine this equation and this equation, the relation that we get gives us a correlation between frequency and filler fraction of a percolative structure.

What does it tell us? It tells us that as we keep on increasing the filler concentration, the associated frequency at which this transport would happen would keep on increasing. It is a proportional relation. Whereas, as far as correlation length is concerned, the relationship is inversed because of this negative sign if you are able to pay attention. I will just make it more clear for you. So, this  $\nu$  has a negative sign associated with it.

So, if you see correlation length has an inverse relationship with the percolation fraction which means if you fill the composite with more and more conductive elements, the correlation length will decrease whereas, the frequency correspondingly will increase. So, what does all these things mean? What does it mean?

It means that if you keep on increasing the filler fraction the point at which the charge has to travel the distance greater than correlation length will be at very high frequency. Whereas, if the filler fraction is comparatively lower, then the frequency at which the charge has to travel from the charge has to travel a distance which is always going to be smaller than the correlation length will be at lower frequency. Now, with what reference we are talking about this high frequency and low frequency?

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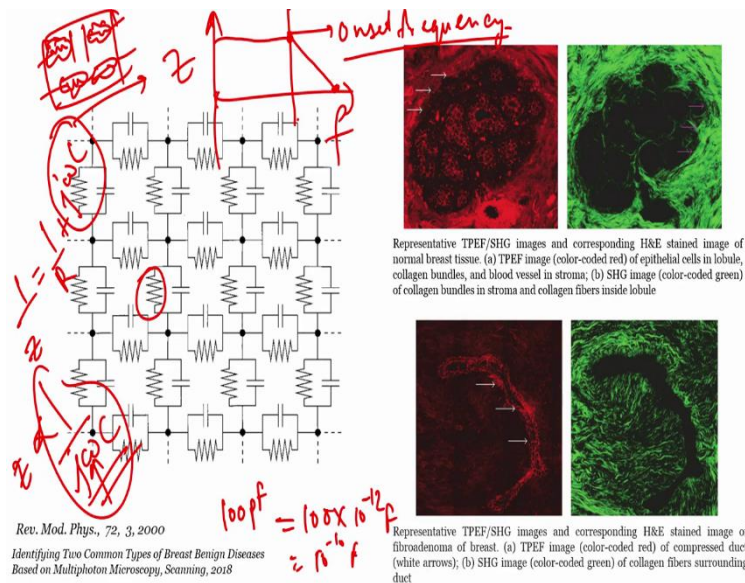
Rev. Mod. Phys., 72, 3, 2000  
 Identifying Two Common Types of Breast Benign Diseases  
 Based on Multiphoton Microscopy, Scanning, 2018

100 pF = 100 x 10<sup>-12</sup> F = 10<sup>-10</sup> F

Representative TPEF/SHG images and corresponding H&E stained image of normal breast tissue. (a) TPEF image (color-coded red) of epithelial cells in lobule, collagen bundles, and blood vessel in stroma; (b) SHG image (color-coded green) of collagen bundles in stroma and collagen fibers inside lobule

Representative TPEF/SHG images and corresponding H&E stained image of fibroadenoma of breast. (a) TPEF image (color-coded red) of compressed duct (white arrows); (b) SHG image (color-coded green) of collagen fibers surrounding duct





This brings me to the frequency dependent conduction in disordered systems. This particular aspect will be clear from this particular slide. So, let us have a look at it. Here what we have done is the entire composite network of conducting islands and non-conducting regions in-between them is converted into a unit resistor connected in parallel with a capacitor.

How do we achieve how do we arrive at that? So, you would agree that this conducting island can be considered as a finite resistor element because it is a normal it can be considered as an analogue of a metal wire. In between there is non-conducting region, this non-conducting region does not conduct electricity, but it can act as a dielectric, that is why this thing can be considered as a capacitor in parallel with the network.

So, if you consider a unit, it can be represented in terms of a resistor connected in parallel with a capacitor. Now, if you see,

$$\frac{1}{Z} = \frac{1}{R} + j\omega C.$$

That is why it will be easy for us to talk in terms of admittance. So,

$$Y = R + \frac{1}{j\omega C}.$$

If we talk in terms of admittance, what we realize is that the entire network if we talk about voltage drop across the entire network, it will be dominated by R at DC. Now, there is something that is interesting that happens when you increase the frequency. Let us see what happens.

So, let us say, there is a particular value of capacitor for so, let us say it is 100 picofarad which means  $100 \times 10^{-12}$  farad or  $10^{-10}$  farad. Now, with this small, with this small value, the entire gamut of this particular system is dominated by resistance alone, if we talk about impedance.

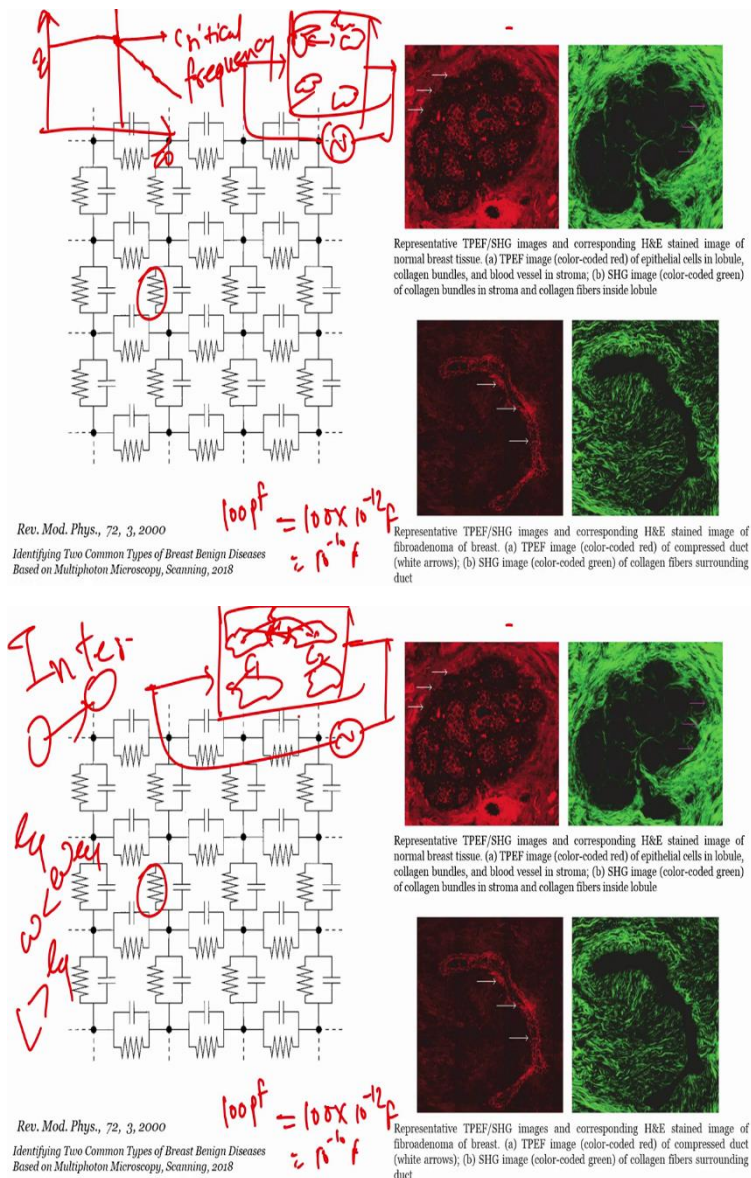
To avoid confusion, let us forget about admittance. We will talk in terms of impedance because most of you may not be comfortable in talking in terms of admittance. So, if we talk in terms of impedance, we can see that the entire system will be dependent mostly on these resistive elements.

But after a particular frequency is crossed, this particular term will also contribute to the conduction. How? Because as soon as so, if we write  $z$ , this term will come in denominator or this will be looking something like this. So, if we keep on increasing this frequency, this term will keep on decreasing as we keep on increasing the frequency.

This particular fraction in this equation will keep on decreasing and that is why the overall value the overall value of the impedance will come down. And that is the reason that when you plot for any standard resistor, impedance versus frequency, the kind of graph that you will see is something like this. At a particular frequency, the impedance will drop from its value to another lower value and then it will saturate at that particular point.

Now, this value at which the impedance drops and it goes to a smaller value in disordered system is referred to as onset frequency. Because, why it is called onset frequency? It is called onset frequency because this is the frequency at which AC conduction or frequency dependent transport dominates the DC transport and that is why since it is the onset of AC conduction, it is called as onset frequency.

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Now, how does it translate to that disordered system that we had studied in the previous slide or previous lectures? So, if we talk about a system where there are grains, by grains I mean this conducting islands. Let us say they are having a correlation length of  $\xi$ . Now, if I am applying a potential an AC potential, and I am increasing the frequency and I am plotting a board plot, a board magnitude plot. For those of you who do not know board magnitude plot, it is the graph between impedance and frequency.

In a similar way, the graph can be plotted for impedance and phase and that is called as board phase plot. So, what it will show? It will have two distinct regions that till a particular point it will be showing almost consistent change. It will almost be consistent without much change as a function of frequency.

After a particular value, it will show that it decreases and then it saturates after a particular frequency is reached. And that is why this particular frequency is denoted as critical frequency or onset frequency because it starts or it marks the onset of AC conduction. So, what happens microscopically is something we will see here.

I will again go back to the same diagram, where we have conducting islands and the charge has to cross this path in order to make that conduction happen. So, what is the best way to understand the concept of onset frequency? So, this is what we are going to understand now.

At lower frequencies, what will happen, remember the correlation length concept, this is the correlation length. At lower frequencies, the particle or the charged particle will get more time to cross a distance which is greater than the correlation length. That is why the transport will happen from conducting island 1 to conducting island 2. If we talk in terms of conducting grains, because in some literature reports, it has also been used the name has been termed as grains.

So, the conducting island transport will be inter-island which will be inter-island or between two islands. Because why will it be between two islands? The correlation length, the correlation length is  $\xi$  but our frequency is less than  $\omega\xi$ . So, the length that it can travel will be greater than  $\xi$ . And that is why it can travel from one conducting island to another conducting island.

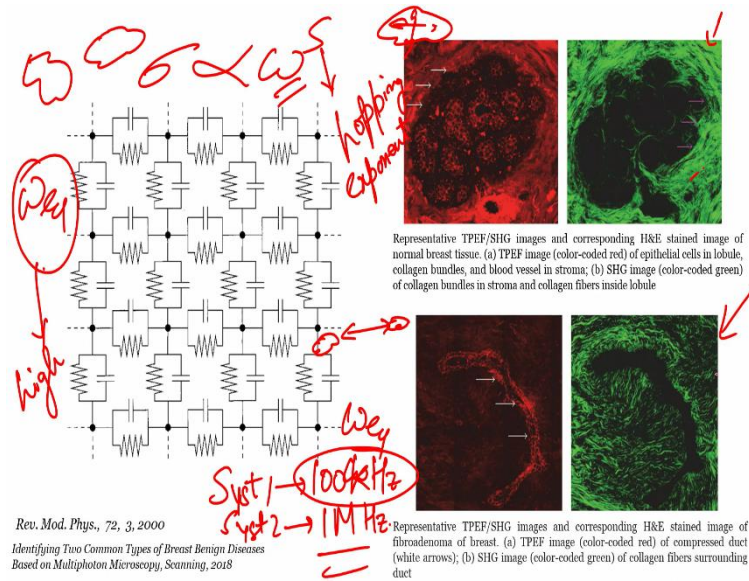
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Handwritten notes:  $100 \text{ pf} = 100 \times 10^{-12} \text{ p}$ ,  $z = 10^{-14} \text{ f}$ ,  $\omega < \omega_c$



Now what will happen if I keep on increasing this frequency? Let us say this frequency has now crossed  $\omega_\xi$ . So now I am talking about a point the same diagram, I am talking about the same diagram, where the frequency has reached frequency has exceeded  $\omega_\xi$ . So, now particles do not have enough time to travel a distance of  $\xi$  or even less or not just even greater than  $\xi$ , but not even equal to  $\xi$ .

So, now we have reached the regime where the length which is travelled will be less than  $\xi$ . What does it mean? Less than  $\xi$  means the particle will travel within the conducting island. Now one would question a particle is traveling within the conducting island, how will the transport happen? So, in this case, that the catch here is if the particle is travelling within the conducting island, if I were to zoom in this conducting island here, and let us say there is a conducting island next to it, the particle would oscillate or jiggle between different directions or different walls.

So, if this is my electron here because of that frequency, the particle would jiggle and it will try to seek the path where it can find a very small potential barrier so that it can get transported from this particular conducting island to the nearest one. I will repeat this discussion again. In case where frequency is much greater than the frequency corresponding to the correlation length, the particle cannot travel any distance greater than correlation length or not even equal to it. It will always be within the island.

But what it will increase, it will increase its probability, it will increase its chance to find a place where it can find a barrier that it is having sufficient energy to jump and contribute to the

conduction. And that is why the energy which was lost in travelling or crossing this potential barrier for frequencies lower than  $\omega_\xi$  will now not happen anymore.

Because the potential barrier offered in this case the chances or the probability that the particle will find such places of low potential barrier will be higher and that is why it will be able to contribute to the conduction and that is why conductivity will increase as a function of frequency. This is a very interesting and important concept and it is the hallmark of disordered system.

And when you have such kind of a system, it follows a universal scaling law and that universal scaling law is given by  $\sigma \propto \omega^s$ . I will redraw it again here.  $\sigma \propto \omega^s$ , where s is also called as hopping exponent.

Now, how one can deduce that just by the look of frequency or rather the onset frequency that how much is the disorder in my system? How much randomness is there in my system? So, there is a quick check to that. If my frequency or onset frequency or I would call correlation length frequency, whatever name you give is very high and I will take some numbers.

So, let us say for system 1, the frequency is let us say 100 kilo hertz, this is  $\omega_\xi$  and for system 2, it is 1 megahertz. So, what does these two frequencies tell about the system? It tells about the system that in system 1, the correlation length is so large that at frequency as low as 100 kilo hertz itself, the particles have started oscillating within the island.

Whereas, in the case of system 2, where frequency is 1 megahertz the correlation frequency, the onset frequency is 1 megahertz, the correlation length or the separation between two conducting islands is very small or it is highly conducting. So, this is what we got the answer. For highly conducting systems, the onset frequency will be higher compared to lower conducting systems if it is a disordered system as a whole.

Now, many of you would be thinking that why we have included these images here on the right-hand side. So, these are fluorescence images of normal breast tissues and breast tissues with which are having which are having an issue of fibroadenoma. Fibroadenoma is a kind of tumour, but it is a benign tumour it is a non-cancerous tumour.

This is just to give you a real time feel that how this disorder can also be translated into biological tissues. So, if you see in this particular system, this is the case where it is a normal breast tissue, the density of collagen fibers as it is found here is higher but the tissue density overall is not that great.

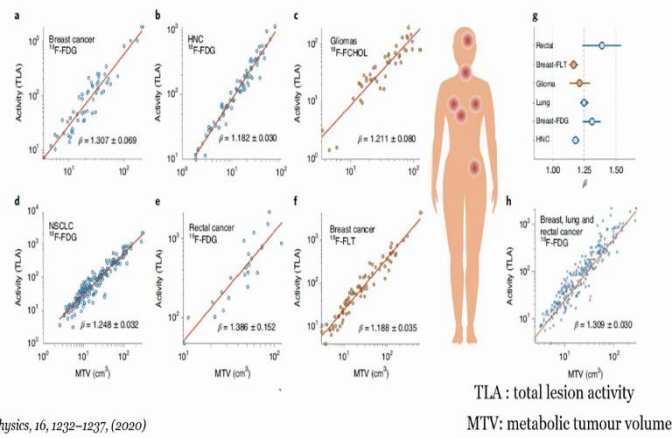
Whereas, in the case of fibroadenoma, the tissue density along with collagen fiber density is very large which means that disorder has increased as a function of, as a function of tumour growth. And this is something can be a signature of a tumour-related, a system which has tumours. This was the only motivation to show that how disorder, the concepts of disorder, the concepts of filler fractions can be translated to biological tissues as well.

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**Examples from literature on scaling laws in biological systems**

$\sigma \propto \omega^2$

Universal scaling laws rule explosive growth in human cancers



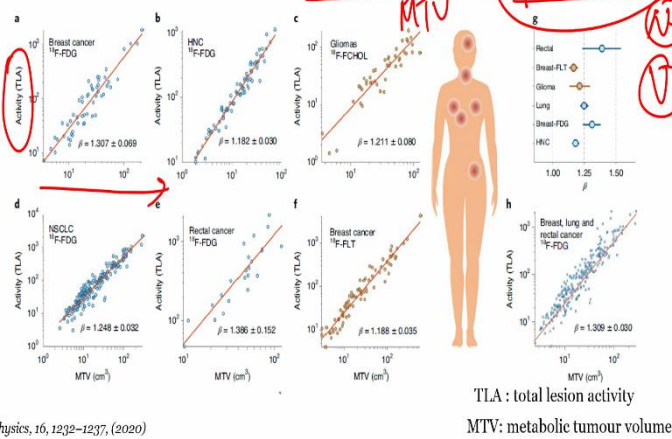
Nature Physics, 16, 1232–1237, (2020)

**Examples from literature on scaling laws in biological systems**

Universal scaling laws rule explosive growth in human cancers

$\sigma \propto \omega^2$   
TLA  $\propto$  MTV

Box A P  
W  
V



Nature Physics, 16, 1232–1237, (2020)

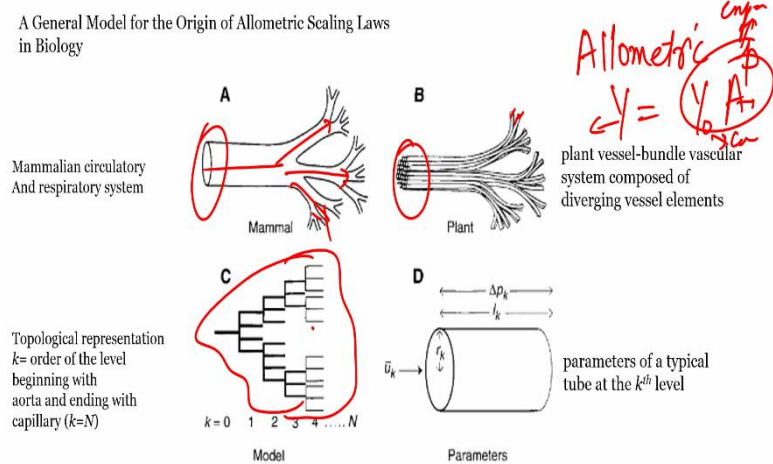
So, this is we have come to the towards the end of the session. And just to quickly help you understand that how these can be, how these scaling laws that we saw in the previous slide which was  $\sigma = \omega^2$  can also be applied in general wherein there is no conductivity or frequency associated but there is growth and there is time and how these two things can be related is something that we are going to see or volume can be seen here.

So, this is a very recent article in which they have discussed how universal scaling laws can govern the growth of cancer cells in human cancers. So, they have taken six different kinds of cell lines and then they saw and then they plotted for the volumetric for the volumetric growth and they plotted a graph for total lesion activity versus metabolic tumour volume. And what does it tell us? It tells us that these plots which are log-log plots, so, if you see it in linear-linear plots, it will look something like this.

There is some so, the same graph, if it is a linear-linear plot, this is MTV it will look something like this, which means there is some scaling relation associated with this. There is some exponent it is not linear; it is some exponent. So, what they are trying to say that almost the six major kinds of cancer which they studied have shown some scaling laws associated with their growth.

So, if there is a volume  $V$ , then and if there are  $n$  number of cells, then the growth of then the growth of this cell in this volume  $V$  can be explained by such a scaling law relationship. This is a very exciting thing because the scaling laws which are seen for frequently dependent transport, though they talk about conductivity but the similar concept of scaling, this concept of scaling I am not talking about conductivity I am just talking about the concept of scaling can be applied to tumour growth as well.

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Science, 1997

In a more general approach, if we talk about or if we try to understand that how different organs of our body or different tissues of our body are have been growing not just in our body, but even in plant tissues, we would be astonished to see that they also follow some kind of scaling



law which are called Allometric Scaling Law. So, Allometric Scaling Law are of the type, let us say we have

$$Y = Y_0 A^p.$$

These so, this  $Y$  is a variable, this is  $A$  constant and this is system dependent constant as and as you can see here and this is the exponent and this is a parameter with which you are referring to that this scaling is happening. So, if we see this case by case, we can see the example of mammalian circulatory system.

So, this is your, so this is your aorta artery, this is your aorta and how this aorta is getting branched into different capillaries and the way it can that branched can be explained graphically using this particular structure. In a similar way, when you have a vasculature from a plant tissue, even the growth from a very plant vessel bundle, a very broad a big plant vessel bundle to very small capillaries can be explained by this scaling law.

So, this is where that our session ends. We discussed about how percolative networks transport charges from point A to point B or one into another end and how there are frequency dependencies in such disordered systems. We also tried to understand that why this frequency dependence happens mathematically as well as qualitatively and what is the origin of such frequency dependence.

We also saw how such disordered systems are present and how they can be used to differentiate tumour tissues from normal tissues in one of the slides. And then, we also saw that how scaling laws which seem which are very which are very universal in nature. So, as we can see here, this allometric scaling law is applicable to all kinds of, to many of the species I would not use the word all kinds of species, because there are always some exceptions.

But they are applicable to many kinds of such species, be it mammalian species or plant species and a similar scaling law can based on frequency dependent conductivity is also applicable to disordered system. So, this session, the motivation to take this session was to make you familiarize with such scaling laws and how they can be used to quantify a disordered system.

I hope you understood some aspects of it. The best case would be you understand most of it. And if you have any kinds of doubt, please feel free to put your questions or doubts on the forum. Thank you very much.