Mathematical Aspects of Biomedical Electronic System Design

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Lecture 02

Basics of Biomedical Signal Processing

Hi, welcome to the course, Mathematical Aspects of Biomedical Electronic System Design. I am Rathin Joshi, I am a PhD student at the electronic system engineering department Indian Institute of Science, I will be mainly discussing and demonstrating some of the concepts taught by Professor Singh and also I will be talking about how we can interpret information from obtained different kind of bio potentials.

Throughout the course as regular intervals, I will be supporting the concepts. So, signal processing concepts like in todays module, we will be touching upon some of the things taught in the first week. So, Professor Singh has already given a brief introduction and some explain some types of signal and systems, anything that conveys an information can be considered as a signal, it can be of, it can be a function of more than one variable.

In this particular course, for most of the biomedical applications, we will be focusing on one dimensional signal that one dimensional signal will have mostly time as an independent variable. So, in this module, I will be talking about some of the types of signal and moving average system demonstration, which was explained by Professor Singh.

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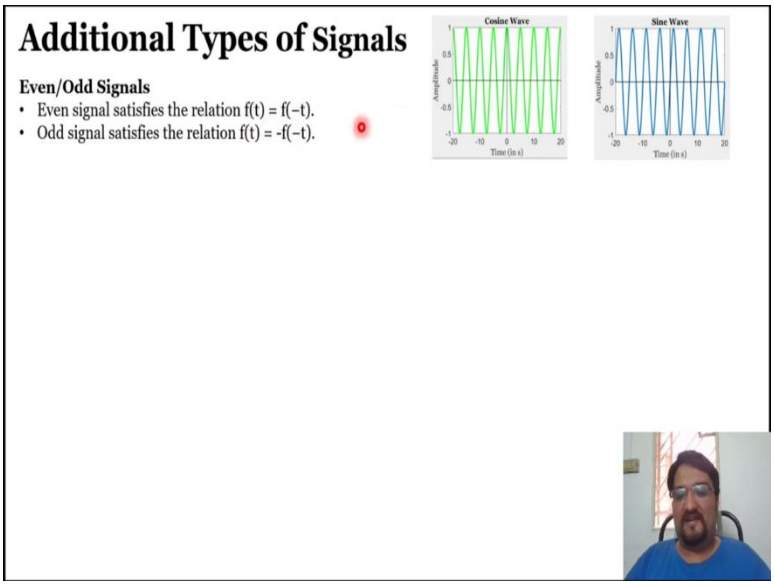

So, let us see some of the additional types for signal. So, Professor Singh has already taught that there are different kinds of signal types like continuous time, discrete times also talked about the properties of system like causality, stability, etc. Here, we will see another set of or another types of signal which is very simple and very frequently used in biomedical signal processing.

So, they are even signals and odd signals. So, from the name it is pretty clear that even signal will have somewhat symmetry or no equal behaviour towards one particular thing whereas, odd signals will not probably have that kind of symmetry. So, another question is, can we classify or can we distinguish or can we say that okay this particular signal is either even or odd, we will come to that in after our demonstration.

So, we will see which kind of waveforms are considered as even and odd. If you can see in your screen, here you see the green trace is cosine wave and which is symmetrical to your y axis, what does it mean if you take any value towards that x axis, you all know that $\cos\left(\frac{\pi}{2}\right) = 0$, cos (90°) = 0 same way $\cos\left(-\frac{\pi}{2}\right) = 0$ or cos(-90°) = 0.

So, if you can consider this thing y axis as your reference, then you will see the mirror reflection in the both sets. So, you can see a symmetry with respect to y axis that is nothing but if your signal will have a symmetry with respect to y axis, it will have even, it can be considered as even signal.

Whereas in case of a sine wave, the same thing would not hold, $\sin\left(\frac{\pi}{2}\right)$ or sin (90°) = 1 as you all know, whereas, if you can talk about sin ($-\frac{\pi}{2}$) or sin (-90°) = -1. So, this signal obeys the property shown here f(t) = -f(t).

Whereas, in cosine wave it follows the property f(t) = f(-t). We will see quickly how we are you know getting it. We will see that quickly using MATLAB. So, you will and we will see some more examples, so you will get some more idea. So, let us see the demonstration.

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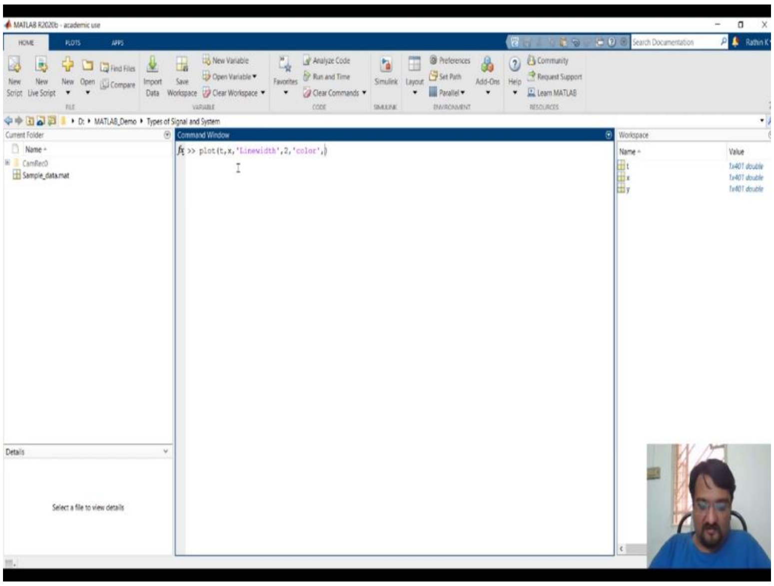

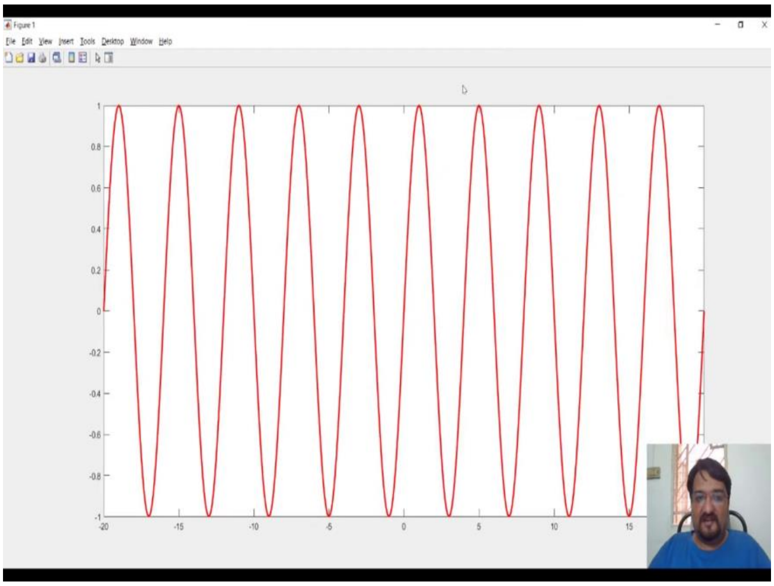

Let us see the demonstration for you, (())(5:00) signals, I have defined some variables, time variable, which is nothing but -20 to +20. Just to demonstrate the behaviour with respect to both sides of x axis, if it is symmetrical to Y axis, it is even signal, if it is symmetrical to origin it is odd signal.

Let us see how it is looking like. So, what I will do, I will just first plot the x, which is my sine wave. With that, I will also try to make it a little thicker. By default, it is 0.5 line width, which is difficult to make out and see. Also, let us put it in a red colour. Let us see how it is looking like. So, this is your x axis. But in order to check it the symmetry with respect to y axis here, it is better if we can show y axis as well.

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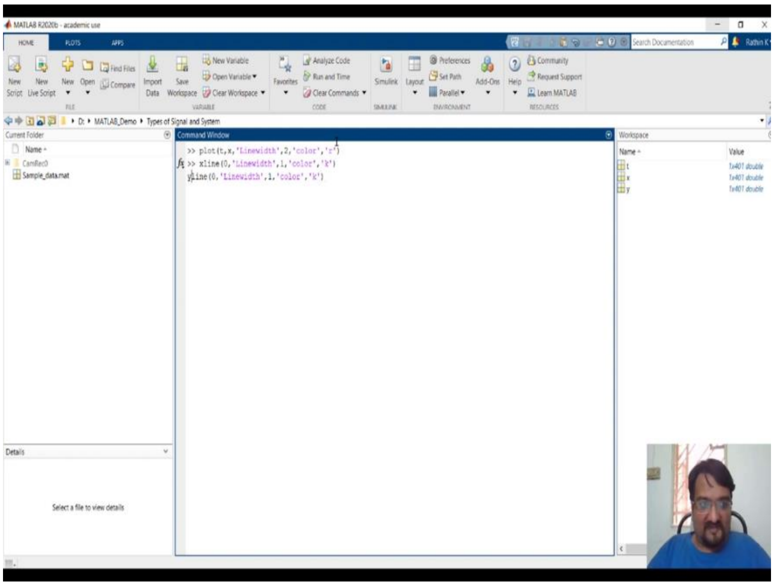

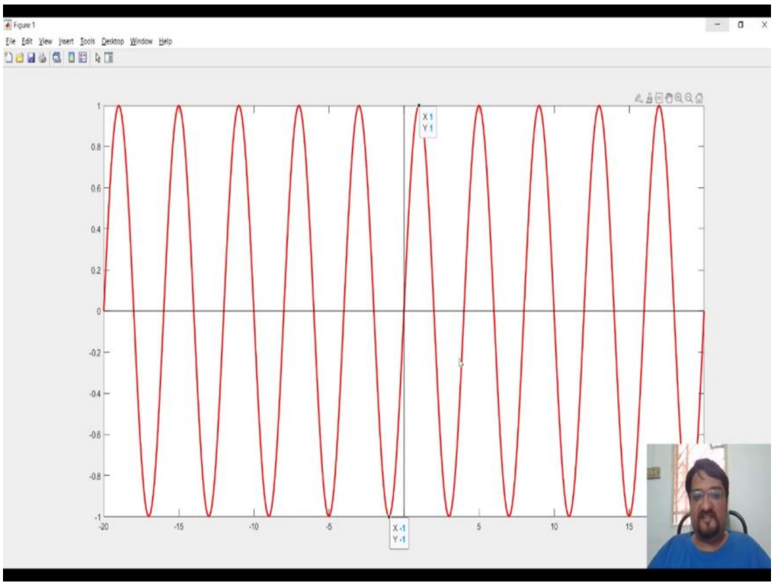

So, I will use the function x line of 0. Also, I will give the same I will give some little less line width and colour being x axis and y axis I will put it in a black colour. Same I will also draw the next axis as well, which is nothing but your x axis and y axis. So, if you can see this is your sine wave. Here if you see at x =1, at x =1, y =1 whereas here x =-1 y =-1.

So, f(t) is equal to a particular value whereas f(-t) is equal to minus of that particular value. So, it is symmetrical with respect to origin, if you can see, whatever shape you are getting, if you consider this as a reference origin as a reference, you will be getting a mirror reflection of that. So, odd signals generally symmetrical are symmetrical towards an origin with respect to the origin. Whereas, the even signals are symmetrical to your y axis.

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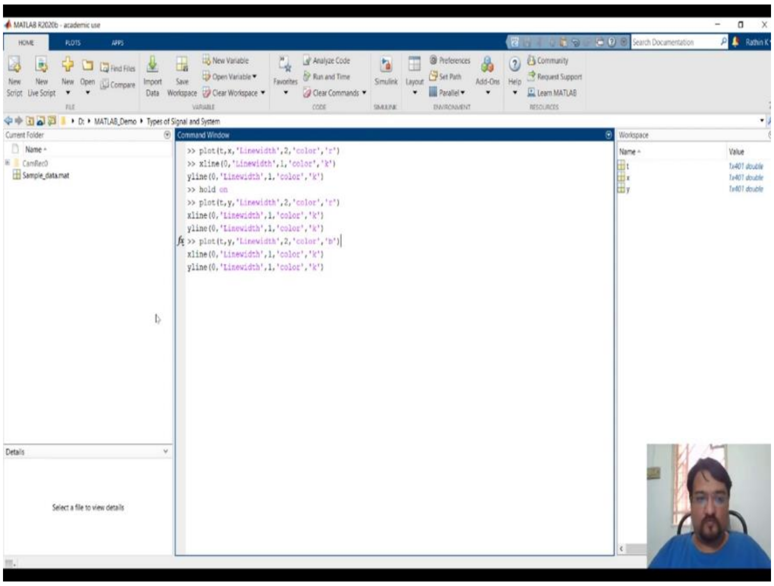

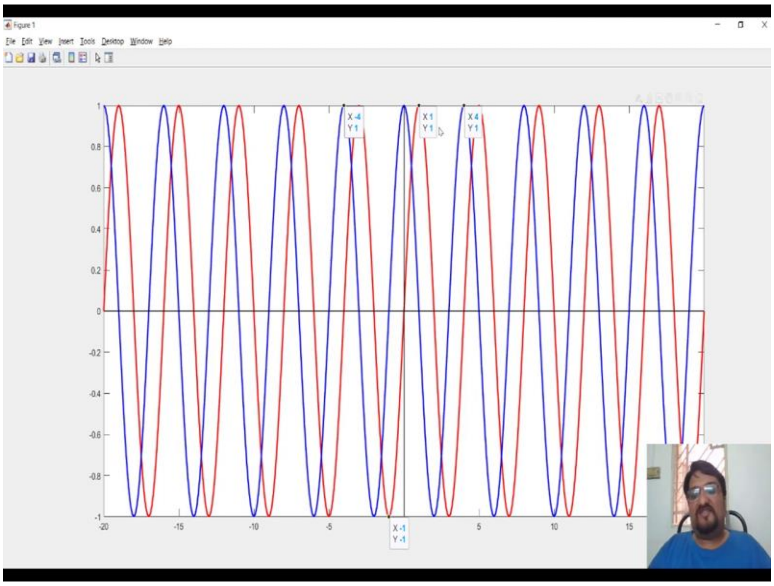

Let us see, I have already made a cosine variable with values are stored in the y. So, before plotting this thing, in order to, I want to show you the same graph, values on the same graph, so I will give the command hold on and then I will plot again it will come as red only. So, let us use a different colour instead of red, put it blue, so it would be easy to differentiate.

So, you can see that the blue colour trace is symmetrical with respect to y axis, which is nothing but your cosine wave, let us see one more value, so it would be better and more clear. If you can see here 3 point, let is keep it as 4. So, for cosine value, the value for x =4 is 1, same thing if we can check for x =-4, for the blue trace we are talking about cosine wave.

So, at x =-4, if I take this data point, we have got zoomed out. Yeah, x =-4 it is 1. So, consider the red trace you absorb the value x(1) = y(1) whereas x(-1) = y(-1) which is obvious the signal property for odd signal whereas x(4) = 1 and x(-4) = 1 this holds throughout your x axis. That is why sinewave is called odd signal and cosine wave is called even signal.

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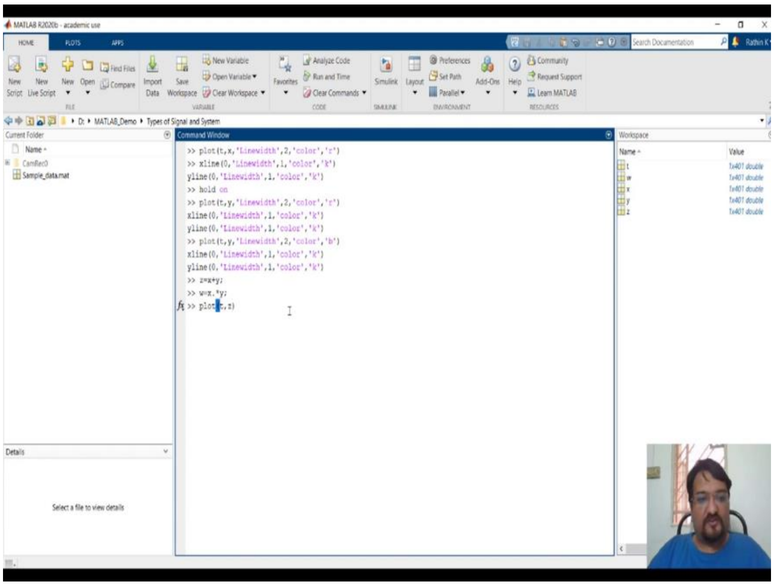

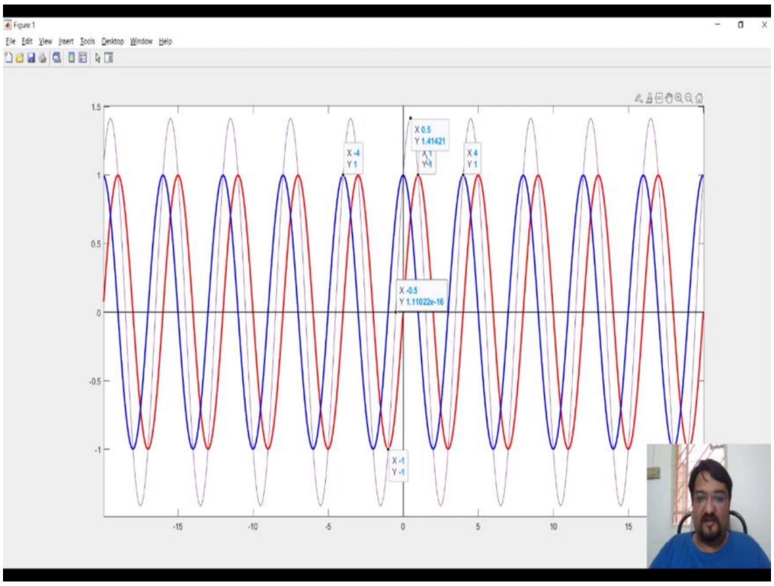

One more question, if I define a variable z which is a summation of x and y, whether z would be even signal or signal, if I define a signal w = x.y. I will do point wise multiplication x into y. So, whether w would be a even signal or odd signal. I will show, I will plot one of them. Other one you can check as a homework. It is a very small homework and you do not need MATLAB for that you can even check it with your Excel as well.

What I will do, I will plot the summation of that because and you have already seen this graph. So, I will quickly show you the summation of that how it is looking like I will first show with the existing graph. You can see this. Focus on this trace, which is there of magenta colour. If you can see at x = 0.5 it is having maximum value.

Whereas and maximum value is nothing but 1.4121. Whereas, if I consider this trace magenta trace and if I try to get the value of -0.5. So, which is almost 0, if you can see this value that is e^{-6}, somehow its cursor is just difficult, a little difficult to show. But yeah it is minus, you can see from this it is almost 0. So, yeah it is 0. So, it is not following property for even signal x(t) = -x(t) and it is also not following x(t) = -x(t).

So, a combination or addition of two even and odd signal may not result in either even or odd signal and also I wanted to inform or you know infer that it is not necessary that all the signals can be either even or odd, there exists a set of signal which can be neither even nor or odd, in most of the biomedical signals, it will be neither even nor odd signals. So, I hope this is clear, you can check about this multiplication thing in your homework and yeah, there might be some of the questions related to what we have discussed in your assignment as well. So, we will go back to the our present.

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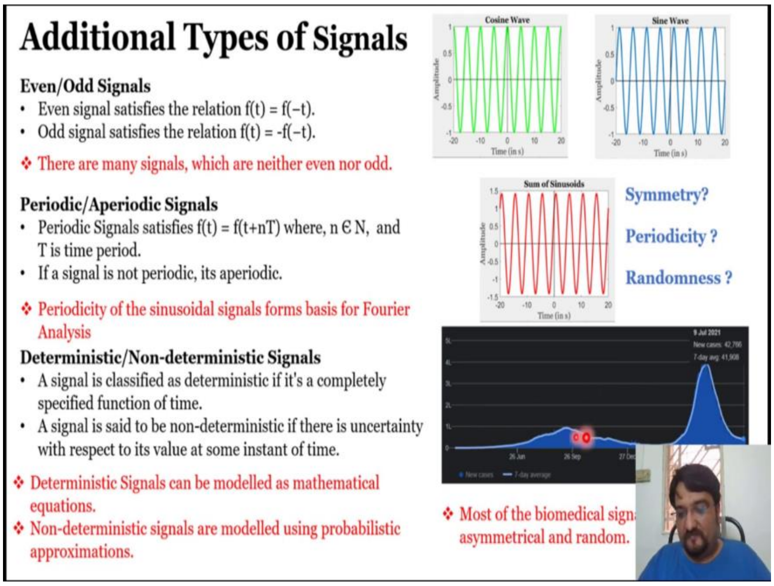

So, I hope it is clear what is even and what is odd signal, how it is look like, just remember even signals symmetrical to Y axis, odd signal symmetrical origin. So, the symmetric point is now clear I suppose you understood it properly, this is the summation of sine wave and cosine wave.

So, I hope this question you should be able to answer the question, also you now know that there are many signals which can be neither even nor odd and if any signal is given you should be able to identify whether it is even signal or odd signal let us see another type of signals which is very important in terms of frequency analysis as well as, Fourier analysis and which would be covered in detail in subsequent lectures as well. But I just want to give you a brief introduction about the these types of signals. So, they are periodic signal and aperiodic signal.

You can see I can directly explain with this illustration or this waveform available here as well, these waveforms are repetitive. The same values is attend after some point of time. So, in mathematical term for any functions f(t), if you can find out this relation for all n belongs to natural number N set that f(t) = f(t + nt) then you can consider this particular signal as a periodic signal, very simple.

All sinusoidal signals and sum of sinusoidal signals are periodic signal, it is very important and you know very, very useful problem in terms of Fourier analysis, communication related protocols etc to know the fundamental frequency, how to find out the fundamental frequency and what is the importance of that, we will see in subsequent lectures.

Also this this particular periodicity is will form the basis of Fourier analysis, Fourier analysis will be covered in detail in the subsequent sections, be it Fourier series or Fourier transform and more importantly, how the analysis with respect to Fourier transform is related to your biomedical data, it is very important for any particular biomedical problem or data to verify it or cross check it your physiological signature with respect to a set of frequencies.

It will also help to identify, what are the signal of interest and what is the noise, we will quickly see one very elementary signal Noise Removal technique in this particular presentation in the upcoming slides. So, I just want to ask you let is consider again the same sum of sinusoid signal it we have seen that it is not symmetrical, it is neither even nor odd signal. But what about periodicity.

So, you can see the same pattern is getting repeated. So we can say that yes, the signal is periodic and as I mentioned that any particular signal or any, like linear combination of sinusoidal signal will be periodic and it is, you should be able to identify the fundamental period there is a condition, we will cover that in the subsequent lecture.

So, I hope this brief just, I wanted you to glance through this periodicity of a signals, very important property and we will come to that we will discuss in detail, we will also see the fast

Fourier transform and everything in the future. So, yeah, we will see one more type of data, signals that is a deterministic and non-deterministic signal.

Like if I talk about sinusoidal signal again, if I talk about any particular signal, let us say, which is like $f(t) = t + 2$. So, when my t range is defined, I can have an idea at any particular point of t what would be the value of my signal. So, that is known as a deterministic signal, sinusoidal signals are deterministic signals, exponential signals are deterministic signals, in short for all signals, for the signals which you can identify the value at any point of time are known as deterministic signal.

Whereas there are some other type of signal if I talk about, as I mentioned signal is anything which conveys the information, if I talk about rainfall in India or rainfall in any particular city over the period of time, so this is basically a non-deterministic quantity you cannot at any point of time say what is the you know, how much rainfall would be there at this particular point of time, you can approximate, you can forecast that is based on your previous data and that is always a function of your probability.

If you see any weather report or something, they will say expected, this expectation is there, it is not a certainty. So, those signals are nothing but your non-deterministic signal where you cannot identify. Instead of you know, giving more example one very, very recent and appropriate example, which all of us are aware and affected in some or the other way is this.

So, this graph if you see is number of COVID cases in India from February March 2020 till today. So, this graph nobody can, you can approximate and using this you can say that might be this many cases, there might be third wave you never know. So, let us hope it would not be there.

So, you can identify you know, number of cases using this approximation, but you cannot formulate an exact equation or you cannot provide the future value of the function using this. So, let us also we will see whether the this signal what we have analysed, whether it is randomised signal or not, whether it is deterministic or non-deterministic signal.

So, we can clearly see that and we have already have the formula also of this particular equation. So, we know that this is not a random signal, this is deterministic signal. Few more things which I have covered, I will just quickly tell you. Deterministic signal can be modelled as a mathematical equation. Non- Deterministic signal can be approximated or modelled using probabilistic approximation.

One more important point is, most of the biomedical signals are aperiodic, let us just simply think about ECG. You all of you, I hope all of you have seen, if not please go and see how ECG looks like, it has, it is, it would be periodic but not exactly periodic, from person to person it differs. If your ECG has uneven rate it is a condition which needs to be treated you have to, you should consult a cardiologist.

So, that ECG signals are aperiodic. You will be having this kind of PQRS waveforms, but that time window is not a certain. It should be in particular range for a healthy human being. Same goes for EEG as well just check the EEGs also. They are also random a periodic signal, random signal and etc. Obviously most of the problems we will consider time as a positive entity unless and until we are taking a reference and observing the properties on both sides. So yeah, most of the biomedical signals are aperiodic, asymmetrical and random.

In this course, we will be mostly focused on one dimensional signals if time permits, we will also talk a bit on image analysis or biomedical image analysis which is a two dimensional signal. So, one more point to notice here, if you can see here, there are two kinds of traces, the one is a line, other one is a plot.

Actually, this all are discrete signal, which is a function of a days, number of days and at every particular day there are values, because of too much, too many values on the x axis, you are able to identify this as a you know, like it is filled or something. So, when you have multiple data and you want to know the trend. Now, we all know that this is a first wave This is the second wave, how can we identify that what is the trend, how a particular quantity or signal is changing, for that here if you can see, very small letters written seven day average.

So, this particular point corresponds to the seven day, previous seven days and it is average or previous seven days, it will give you a trend, it will give you how a particular data is data’s behaviour or its tendency. So, this is something which has been talked by Professor Singh in the first lecture is nothing but your moving average. So, let is quickly see what is moving average, I will also see the demonstration how it will help and everything so yeah, we will see the moving average.

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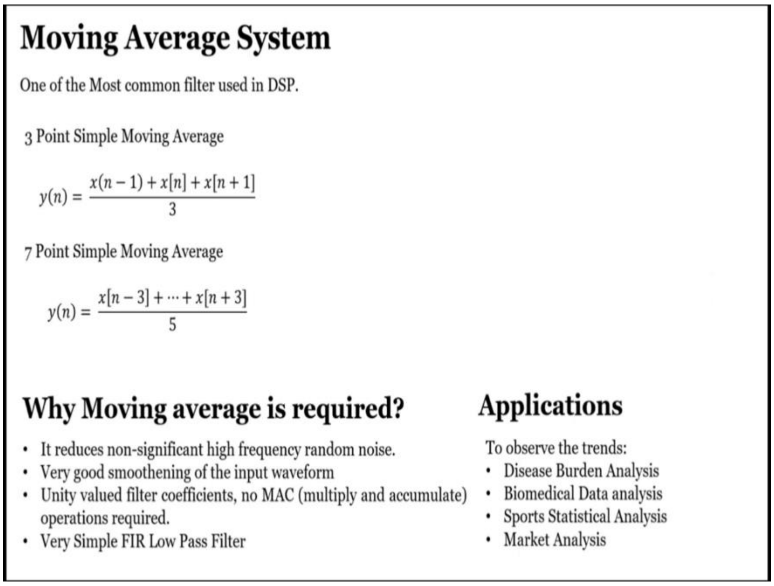

This moving average can be of different type, when you have a data or like when you have a data sequence already available, you can take one sample before and one sample after along with the present time sample and average it out it would be a three point average, you can also consider seven point average, 11 point average, how it affects your data, how it is looking like we will soon see in the demonstration, but before that I will just tell you why moving average is required, what is the applications of that.

So, first of all, most of the biomedical data sets are affected by high frequency noise, if you talk about EEG, ECG and some of the signals which are used they have their own region of interest for frequency. For EEG we do not, mostly we do not care about the frequency is greater than 100 hertz or something. So, in your recording and which is a non-neural if it is something greater than 100 hertz it is very first of all, it is very rare any in yield and if it is there, we need to identify whether that particular signal is because of some artefact or whether it is a neural signal.

Also it depends from where you are taking EEG there are different levels of taking EEG based on the invasively whether you are opening the skull and taking EEG or not, whether you are, even if after opening the skull at which level you are taking and accordingly your range of frequency will differ the point of interest or the why I am saying this is, high frequency noise removal is one of the very essential and basic step is in any biomedical signal be processing algorithm.

So, just one of the main point is it will remove the high frequency noise now as it will remove the high frequency noise. Most of you should who are familiar with little bit of electronics and filter design, it is considered as a low pass filter. Again, it is a finite impulse response low pass filter. So, it has all the advantages which FIR filter has and easy to realise, because it has a finite impulse response, you do not need a lot of hardware to implement it if required, it will smoothen your curve as the name suggests it is a filter, it will smoothen your curve.

Another important thing if you can see here, when I am taking average of this it is just considering sample, it is not considering or it is not getting samples are not getting multiplied by any of the integer 1, 2, 3, 4, 5. So, if it is getting multiplied, there is a model for filter implementation known as mac multiply and accumulate here we are just accumulating. So, it is simple, a complexity is less and that is why it is important in analysing or you know analysing biomedical signals.

So, what are the applications? Main applications are we already, I have already told you about COVID cases, so we can identify the tendency of disease over the period of time how it is moving changing. So, one of the like, all the disease burden analysis, it can be used. Already talked about ECG and EEG analysis it can be used. If you think, if you, most of you would be liking one or other kind of sports.

So, if you your favourite players analysis, if you want, you can use this moving average to see in the last previous games, how many number of game, how he has played, what are his statistics and all and also, this is recently used in many data analysis related to sports, if you are interested in investment in stock markets, etc, a moving average is very, very known terminology. So, it is used that as well.

So, let us quickly visualise what I wanted to say. Also, while it is like 3 point how waveform will be looking like if I use 3 point moving average, how it will be looking like if I use 7 point moving average, how it will be used, if I use 11 point moving average. So, we should know how it looks like. So, let us quickly see the demonstration of the same using MATLAB.

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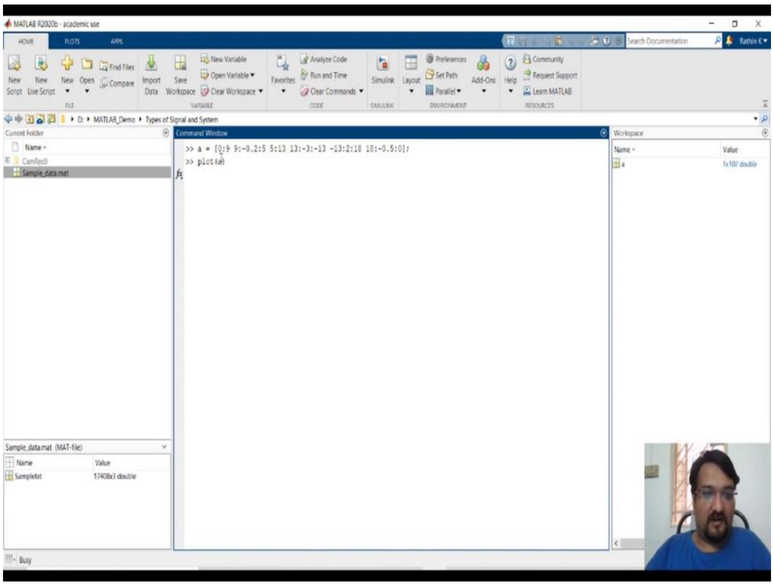

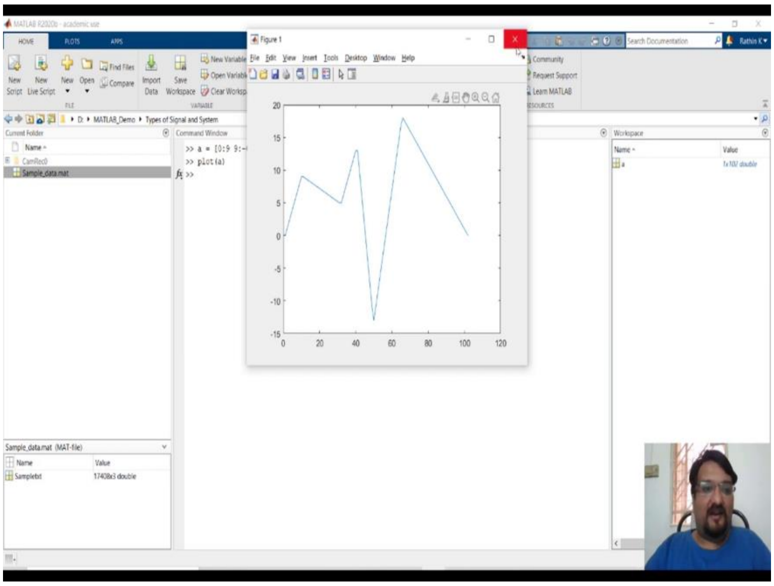

Let us quickly see the demonstration for moving average. For that I need to define one particular variable, it should have enough changes or fluctuations to understand this thing in more detail. So, for this I have defined this particular variable you can (disp) it is a particularly a random original signal. Let us quickly see how it is looking like just to verify. Yeah, it has enough number of variations, this can be used as our sample.

Still, I will add some more. I will add something make it a little bigger sample to include all type of variations. So, I have used a different multiple of the same variable here, as you can see.

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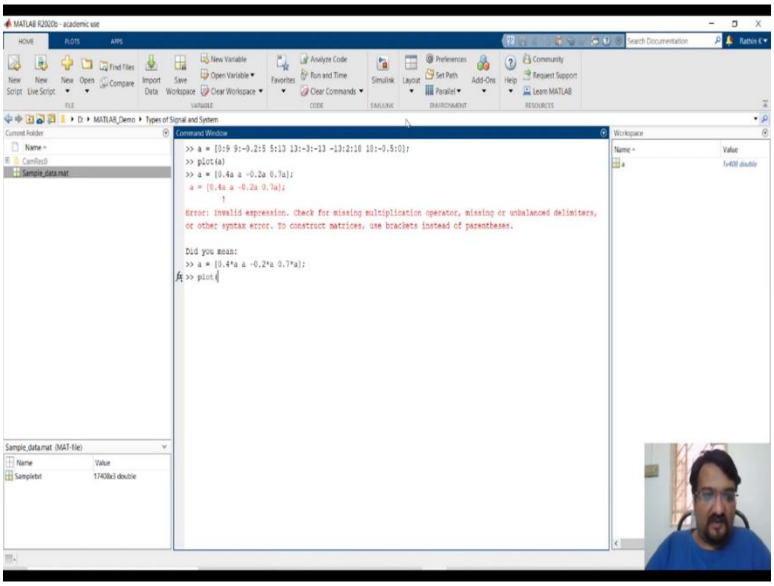

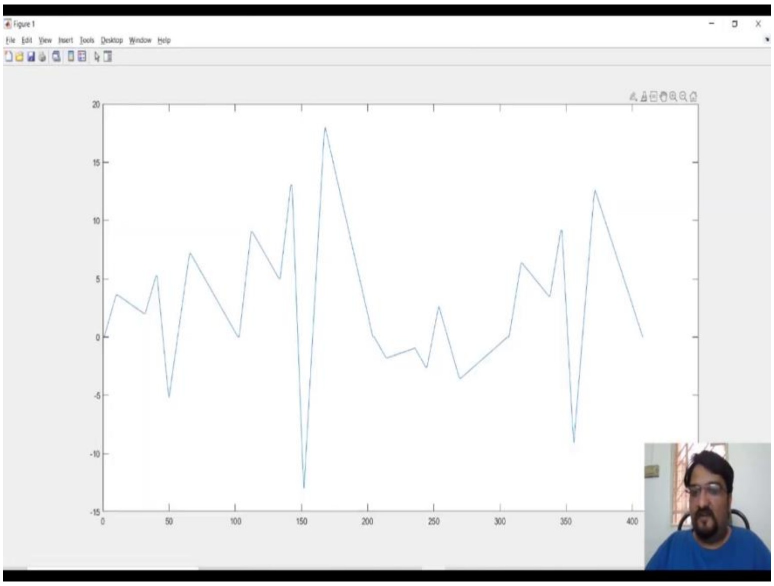

So, it should be 0.4 anyway that is fine. So, let us just quickly see how it looks like, that is around 408 samples and it is looking like this it has enough changes and you know enough slope, so we can consider this it would be a good example to visualise.

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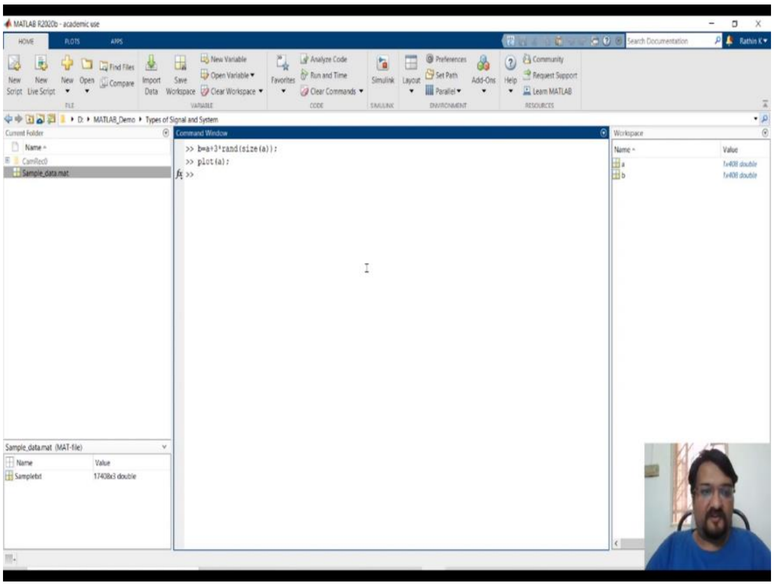

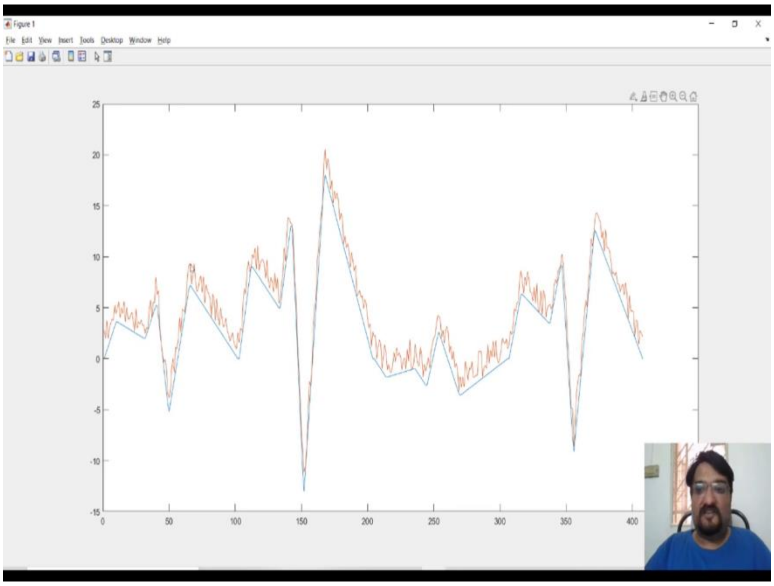

So, also I will show you one more version, after adding a particular noise. This is the sequence, syntax to add noise a data value is very large. So, I will just multiply the noise by 3. So, now I will show you both the plots. First I will, let me draw a, that is our original signal, we have already seen this, or let me also draw also show the version with noise. So, that is nothing but your b.

So, if I plot that I can see. So, this is something which is similar to what you know biomedical signals look like, it will be affected by some noise, it is a random noise also very important when we are considering a frequency based analysis and while talking Fourier FFT, Fast Fourier Transforms and Fourier analysis, I will be also explaining the frequency response of similar kind of waveform it is very essential to understand the frequency response of such waveforms.

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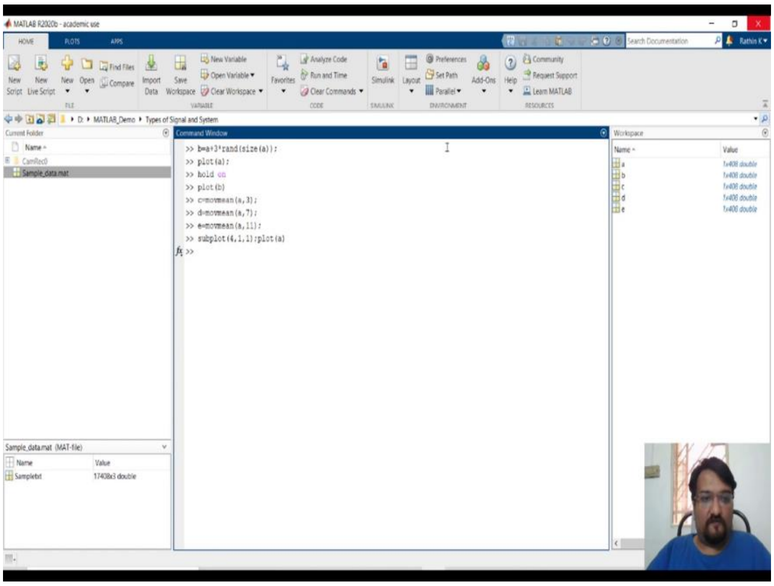

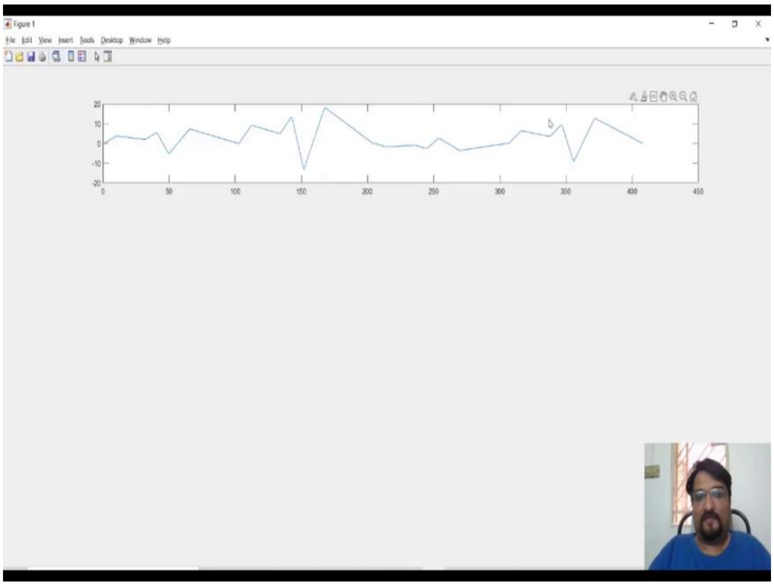

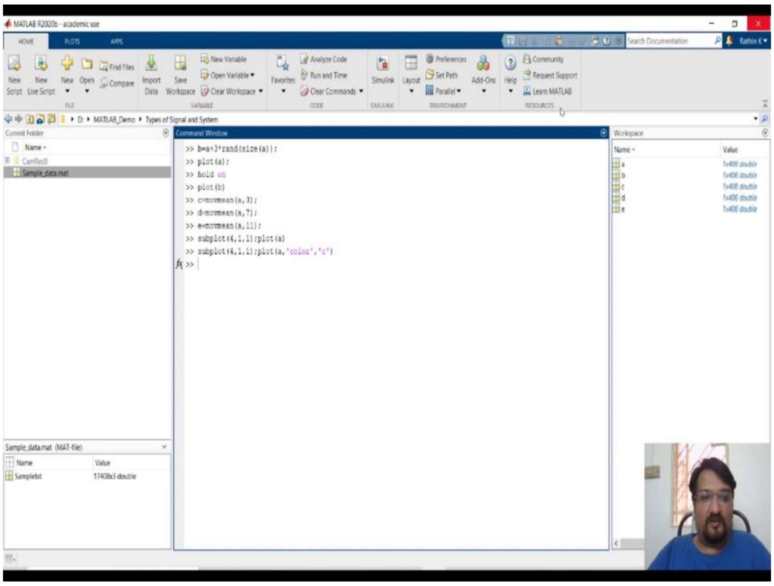

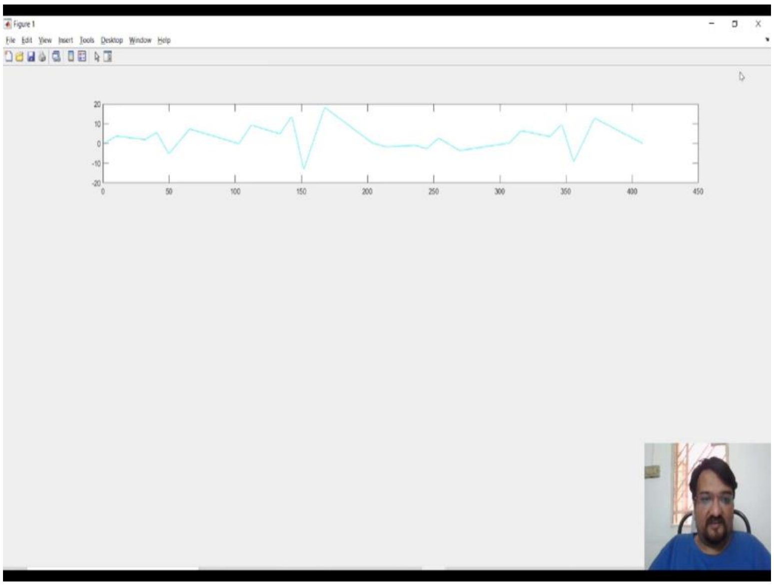

So, let us go back and understand furthermore and we need to verify the moving average system functionality and how different points of moving average will affect your entire original waveform. What we will do is we will have three versions based on moving average, one which is a 3 point moving average other one is 7 point moving average, other one is 11 point moving average. So, you already have your one original signal that is your a.

Then you will have a noise affected signal which is b. Then we will have 3 point moving average 7 point moving average and 11 point moving average. So, let us quickly get those values, so movmean is a function in order to obtain the moving average. Remember c is 3 point moving average, d is 7 point moving average and e is 11 point moving average. So, I have defined the c d and e as different moving average points.

So, we need to compare this b c d e with respect to a noise signal and all three moving average signals with original signal. So, we need four plots So, I am using sub plot, very slowly I am going because of first class. So, you should understand how it is let us just plot a, also we will decide let us just plot a first original sequence. So, out of four sub plot, first plot we can see the a, very easy this is our original signal.

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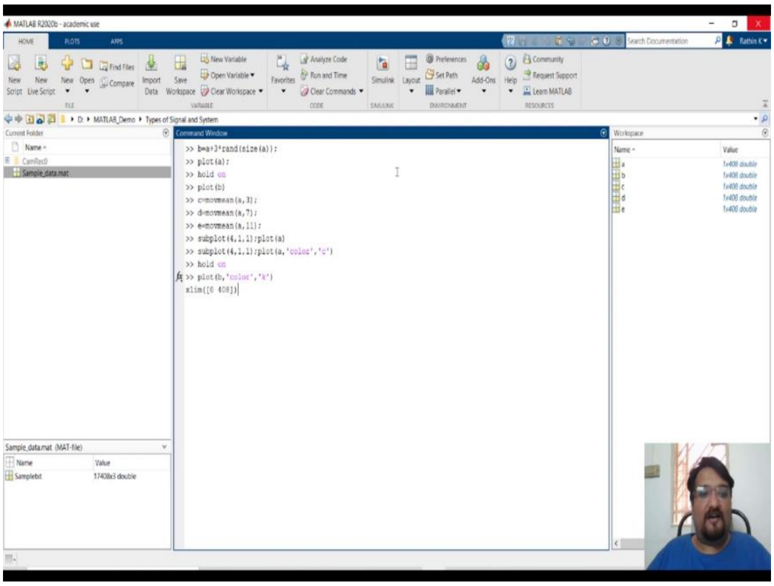

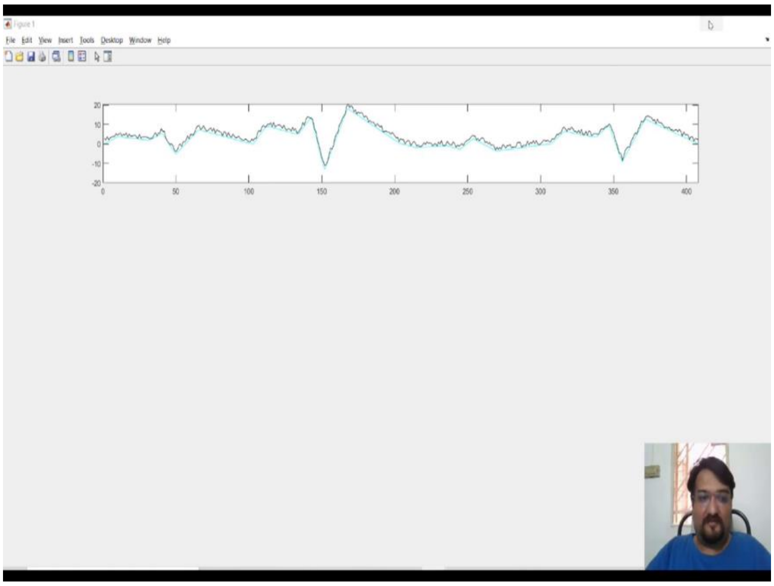

What I will do is I will draw it with cyan colour or light colour. So, it will be easy for you people, when I show the next trace on the same plot you can differentiate between them, this is your input original signal remember cyan colour is input original signal, then we will plot this noise affected version in the same plot, plot b, b is the noise affected version.

Let us draw it with black colour. Let us see how it is looking like also I will change x limit because I do not want samples more than 408, it will look like a discontinuous trace which is not, so I do not want more limit or more values on x axis let us just quickly verify, how it is coming, you can see in the graph. See this black one is nothing but noise affected, where the cyan one is your original signal. Similarly, we will see this comparison for the next all three graphs as well.

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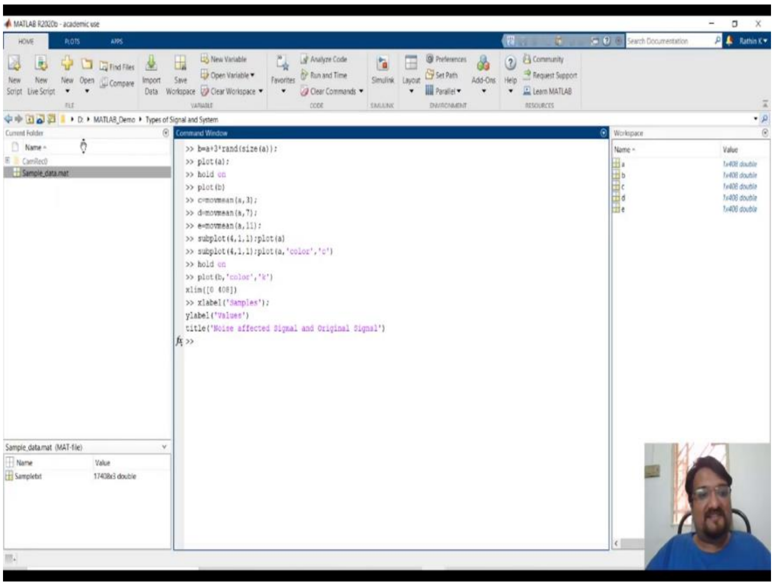

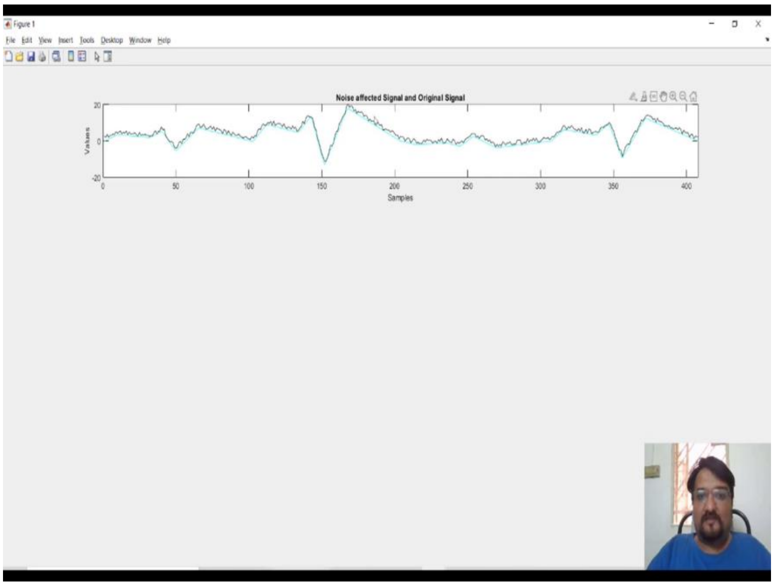

What do we do here, we will plot the next few graphs but if you want you can use this kind of different functions related to plot it is very important to label both the axis, y label as values. Also all the graphs should have a proper title. So, this graph is nothing but your original noise affected signal, noise affected signal and original signal. Similarly, so, let us just run this. So, you will have an idea it has samples noise of affected signal original signal values.

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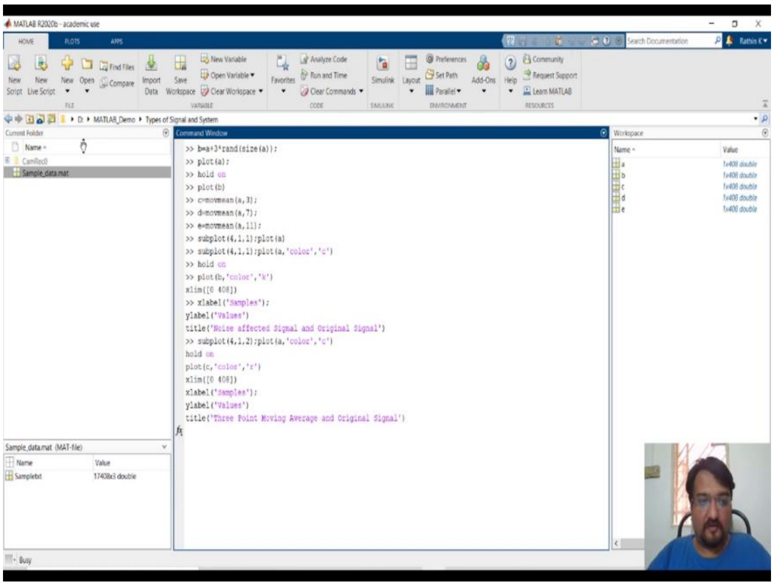

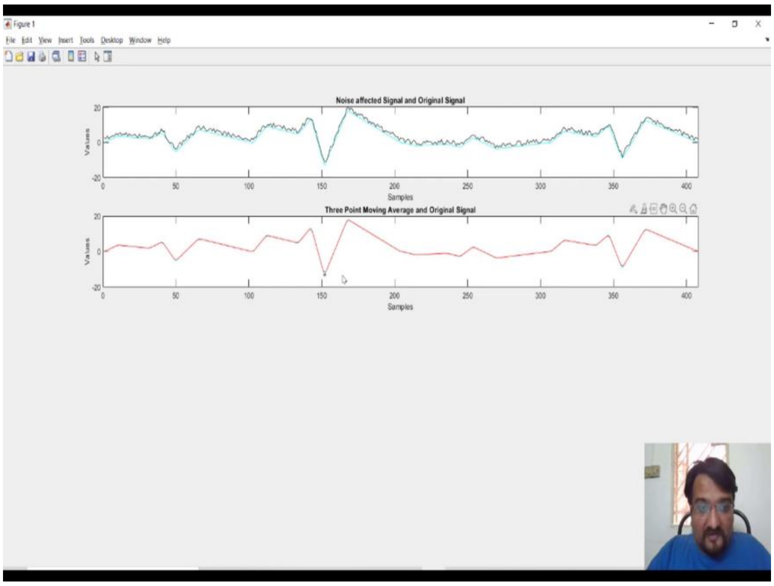

Now, let us see the moving average part with three samples. So, which is nothing but your c. So, this things will remain as it is till this point we will make the essential change wherever it is required. So, first I want to plot a in cyan colour only then I want to plot this as a second subplot. So, this should be, then I want to plot c not b we have already used black colour. So, use blue colour for this or red colour for this, sample values will remain as it is, this would be 3 point average, 3 point moving average.

When I write moving average, there are different types of moving average known as weighted moving average, exponential moving average, but this is a simple moving average where we are not multiplying it by any number I mentioned that MAC thing Multiply And Accumulate, here we are just using a simple moving average with 3 point. So, let us quickly see how that graph looks like. So, if you see this red one, is your moving average, whereas this new one is your original data.

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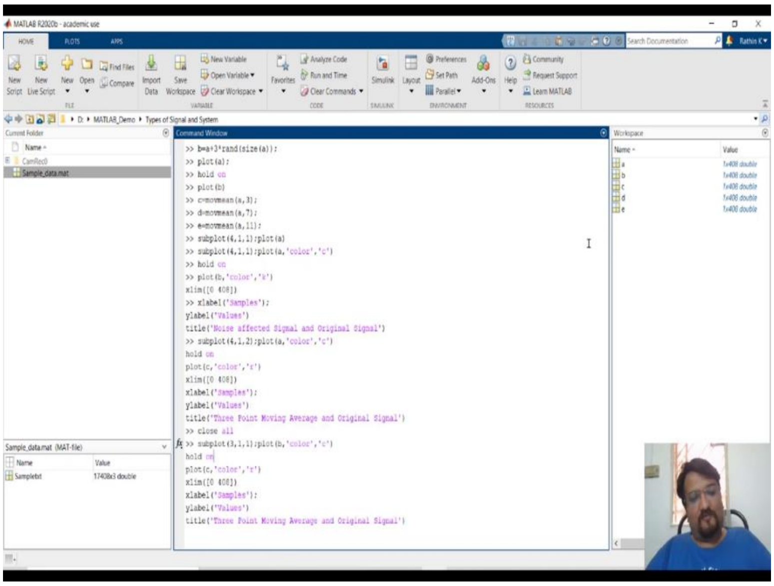

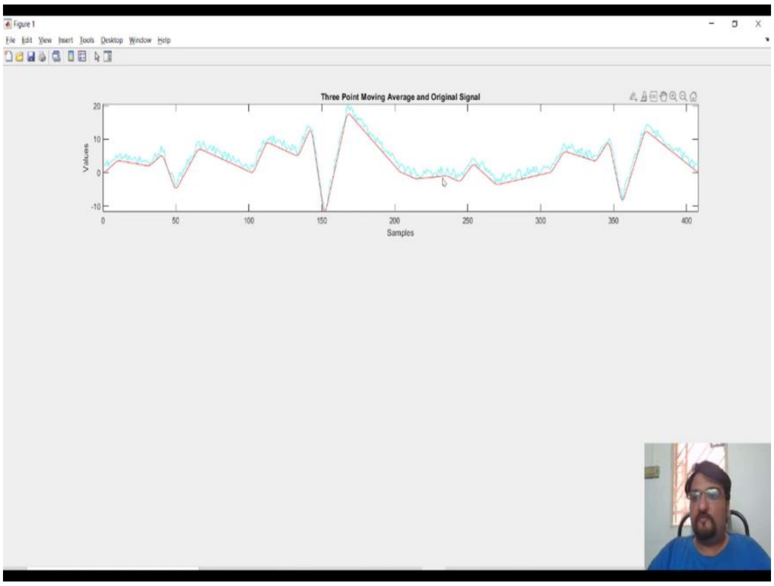

Better if I can show you with respect to the noise affected version, why because the change would be evident okay, so let us in that case just will plot first we will see it in all three versions. So, what I will do is, I want to plot b, so I will store this for now. I will close all the existing figures and then I give the same command.

So, now we will see three plots, because the earlier one input data set was almost like straight without any change. So, better if we can see with respect to the one which has noise, it will give more clear idea. So, let us see this first then. So, this is your 3 point moving average, you can see your input will have all the variations whereas your 3 point moving average it still have some, it is not completely smooth, but it is better than what is there in cyan colour your input race.

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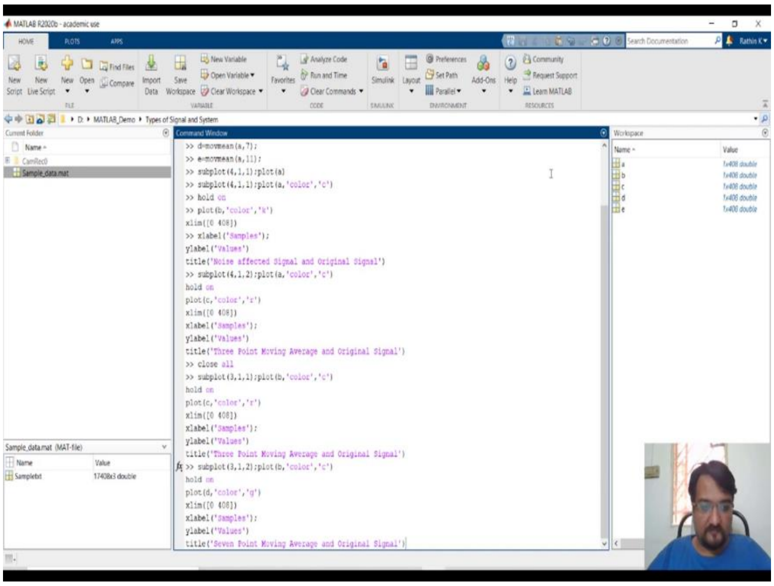

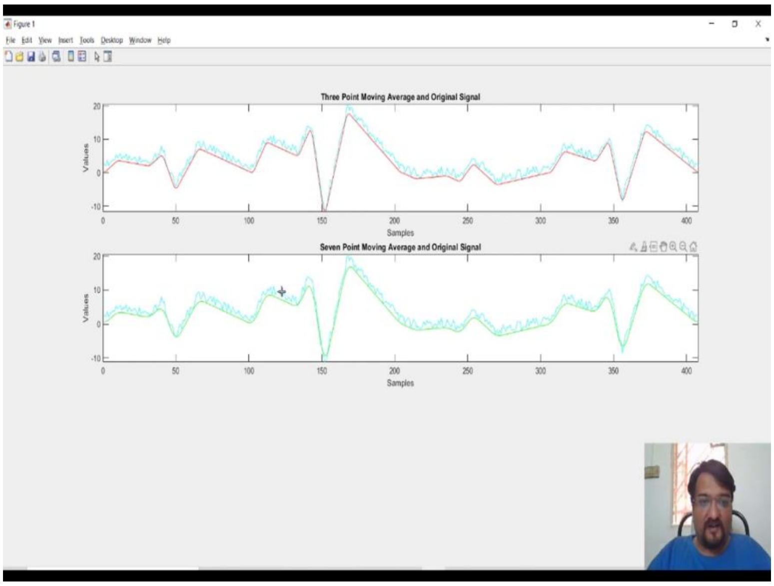

Similarly, we will plot for 7 points again. We already did it once. So, we will not only do work we will do smart work, this is your most effective version. Consider that as your original signal because most of the biomedical signals are originally noise affected let us plot it in green colour the next one, next one is nothing but your 7 sample, 7 point moving average.

So, and that is your d which I have already changed, this is your 7 point moving average. Let us see this how it is looking like. So, it is much better, you can see the smoothness compared to this point and this point is much more smoother and some of the variations are been approximated or nullified. So, high frequency changes are nullified.

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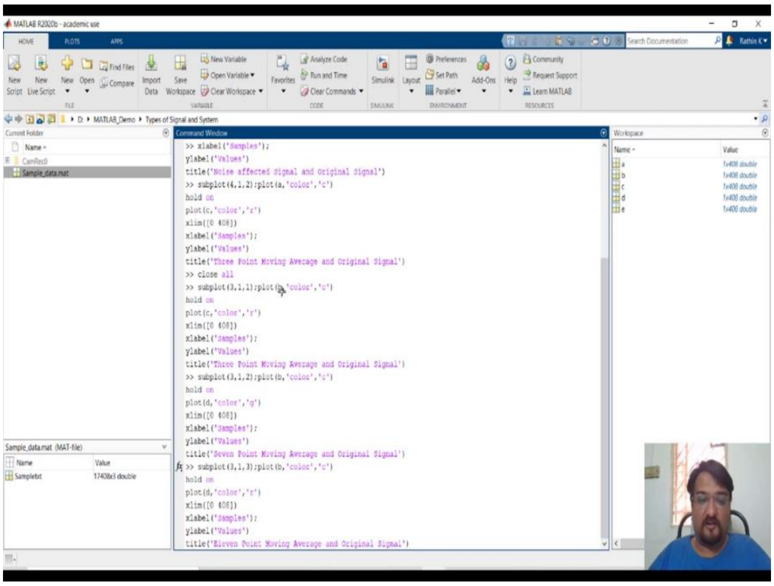

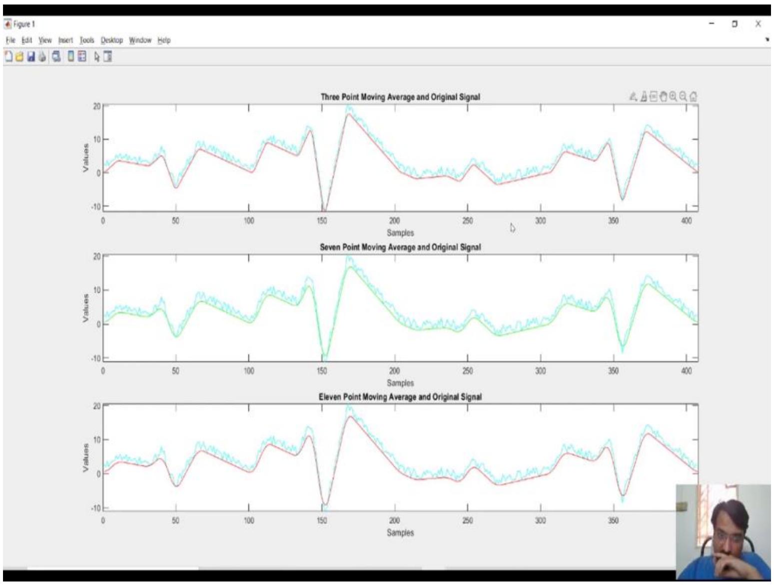

Similarly, let us see the next 11 point version as well. A lot of applications are there, it is just the smoothening or low pass filtering, but you are adding more number of samples which means you are adding more number of terms in your impulse response which means you are adding, if you want to realise you need more hardware to implement that. So, you are getting a better waveform, but at the cost of additional hardware or more computations.

3 point version will have simplest computation less hardware, whereas 11 point will have a lot of hardware but it will give you a better smoothening, it depends, it is a trade-off between your, it is you have to decide, how much data fidelity or you know quality of the data you want. So, let us see the third one, this is 11 point it is even much better you can observe this point, here it is almost like minus 10, here it is gone down, here it is even become further.

Smoothness increases when you go from 3 point to 11 point very important point and with respect to the trend remains the same. Further if you go it will even become smoother if you go up below 3 2 sample it will more sharp. So, this is what basically I wanted you people to visualise or show in terms of 3 point, 7 point and 11 point simple moving average with respect to your noise affected signal.

Again, we will come back to this when we will learn FFT and with respect to this on this side, I will show you the FFT of this, it will give you an idea how the role of the frequency response is getting tuned or modified with respect to the number of sample what you are using, let us quickly go back to the present.

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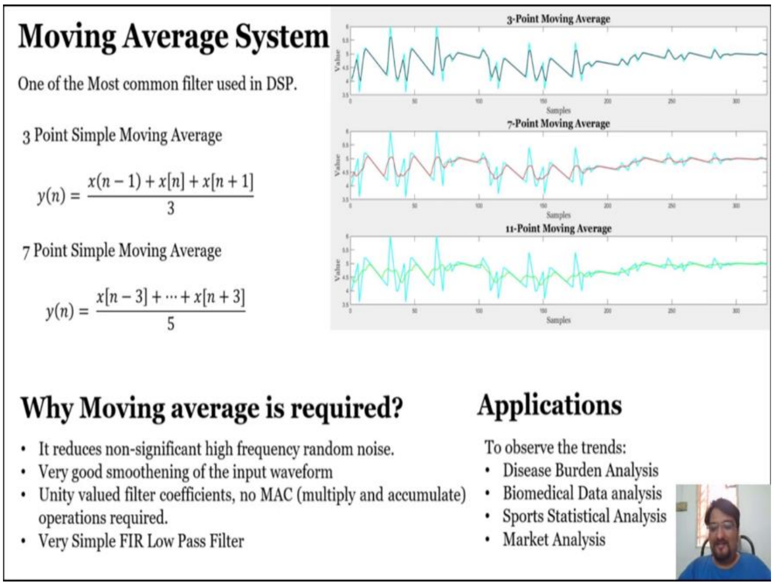

So, you have seen the moving average demonstration. So, this is something like it is looking like something like this. This is another example for another data set, which I have used, but I would encourage you people to use it, you can use it even using Microsoft Excel or any other simple data computation software, you can generate your data, it is just a random data.

So otherwise, if there is some in the future, when I will be talking about biomedical data set, I will share the sample data set. I would comment it or I would include it in the description of this video when this video will be uploaded. So yeah, you can try us using this small illustration understand the moving average we will see the impulse response and frequency response and how it changes with respect to FFT in the upcoming lectures.

I will see you again with some more demonstration and other topics. If you have any doubts, you can feel free to write in the forum. I would be happy to answer and once the video is uploaded, you can ask in the like YouTube comment section as well, up to you but yeah. For this course, students, it is better if you ask us on the forum because we have to knowingly replying in that forum. I will be checking forum more frequently than when it is uploaded YouTube or anywhere. So I will meet you again till then all of you please take care. Stay safe. Bye.