Photonic Integrated Circuit
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Lecture 9
Dispersion

Hello everyone, welcome to another lecture on photonic integrated circuit. In this lecture we are going to look at material technology for integrated optics but going very specific about integrated optics we need to understand the material and how optical energy interacts with material. So, this is how you can expand your understanding from basic interaction to making complex devices using this interaction, sometimes we exploit these interactions to create new functionalities. So, let us look at what are all the primary properties that one could understand.

So, basically when light propagates through a medium it undergoes a certain change because material is going to interact with optical field and because of this atom that are around and molecules that are around in a material it is going to affect our electric field. So, whether this electric field that we talk about is constant across the frequencies that you are going to use or are there any frequency specific interaction on and the other kind of interaction that you can look at is the from the material itself, is the material isotropic? Is the material anisotropic?

So, based on the anisotropy or isotropic nature of the material whether the light will react to this changes and then the next thing is whether the material is going to absorb the energy. So, if the light is going to absorbed, if the light is going to be absorbed by the medium then your intensity of light is going to reduce as you propagate through the medium all this combination of effects that we just discussed can be of interest to realize various functionalities.

So, in this lecture let us look at the basics of this the first thing I would like to discuss is dispersion. So, you might have heard about this word dispersion in various context and even in our lecture earlier we have seen the origin of refractive index. So, what is the origin of refractive index of a medium.

So, we normally write reflective index as n + i k. So, what is the origin of n and what is the origin of k? And they are related to the material property, very microscopic properties and we know these properties affect light propagation. So, in this lecture let us look at a little bit deep into how this affects light propagation the first thing I want to discuss is dispersion of a medium.

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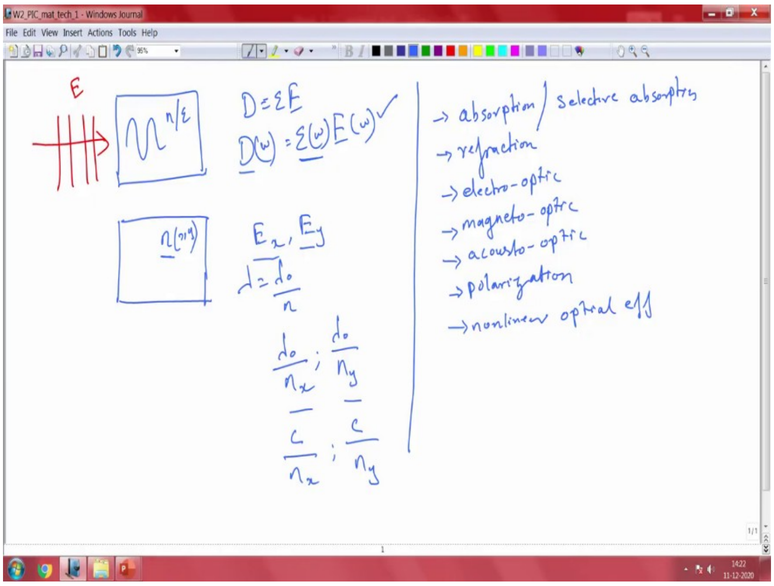

So, when you have a medium of interest and you are going to come in with a plane wave. And now the material is characterized by a certain refractive index as a function of position let us say. So, if it is a homogeneous material and if that is the case then comfortably you can write as a function of r and if it is constant across we can just use a constant here n.

So, that means you have epsilon also a constant. So, once you have electric field interacting with the material here it is going to interact with the dielectric constant and that is going to create a displacement field of $D = \boldsymbol{\varepsilon}E$. Now we all know that this is all a function of frequency. So, for a given electric field of a certain frequency, your epsilon and the displacement field also is a function of frequency.

So, when there is a change in the frequency of incoming field then your dielectric permittivity here will also change as a function of frequency and this would give rise to an interesting electro optic property. And the other kind of dispersion that you could have, let us say if it an anisotropic material because then it is not a constant, it is a function of position let us say if that is the case then your light propagation in x and y direction is now affected by refractive index n.

So, we know this wavelength of light inside the medium is λ_o/n. So, now because you have two refractive indices then your speed along x direction will be represented by λ_o/n_x and along y direction it is going to be λ_o/n_y. So, now you can see that the material strongly affects how light propagates through the medium; one we see the effect of electric field or effect of frequency of this electric field on the displacement field.

So, as the light propagates you are going to create this polarization very specific to your frequency but in the other case you have material that has two different refractive indexes in two different directions. So, that means anisotropic. So, if that is the case the light that is oriented in x or field oriented in x or y direction is going to see difference in the wavelength and also speed as well. So, the speed of light is also going to change. So, that means based on the property of the medium you are going to see interesting interactions with light. So, you can understand all this by using classical electromagnetic theory. So, that is what we would like to see.

So, in solid there are various properties that one can study; one is absorption, sometimes you could have selective absorption and the next property is refraction, you can have electro-optic effect because it is a function of electric field you can see optical effects, magneto-optic so any change in the magnetic field could change the optical field here and then acousto-optic, you could also see polarization effect and you can also see non-linear effect.

So, these are all various effects that one could get from a solid, any material for that matter not necessarily solid but in case of guided wave optics we primarily interact with solid. So, that is the reason why it is appropriate to look at this in solids. So, one of this could be of interest to realize a certain functionality or more than one of this property or phenomena one could exploit and all this could be understood from electromagnetic theory.

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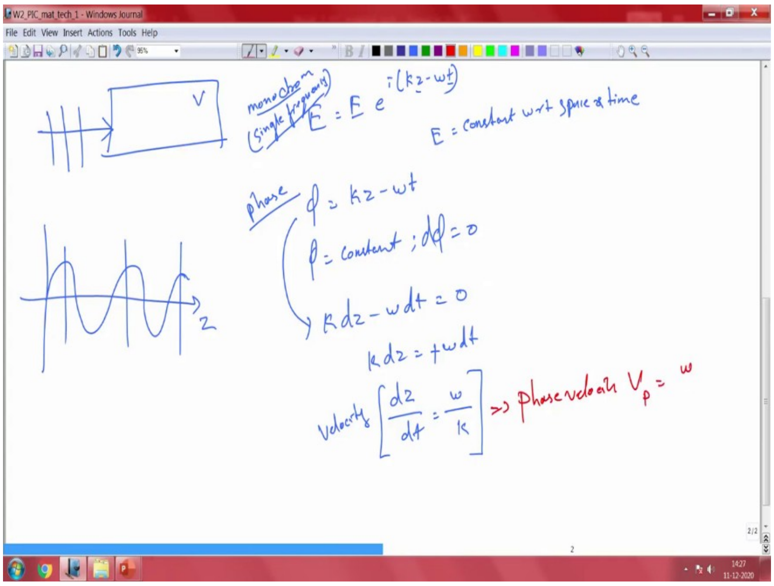

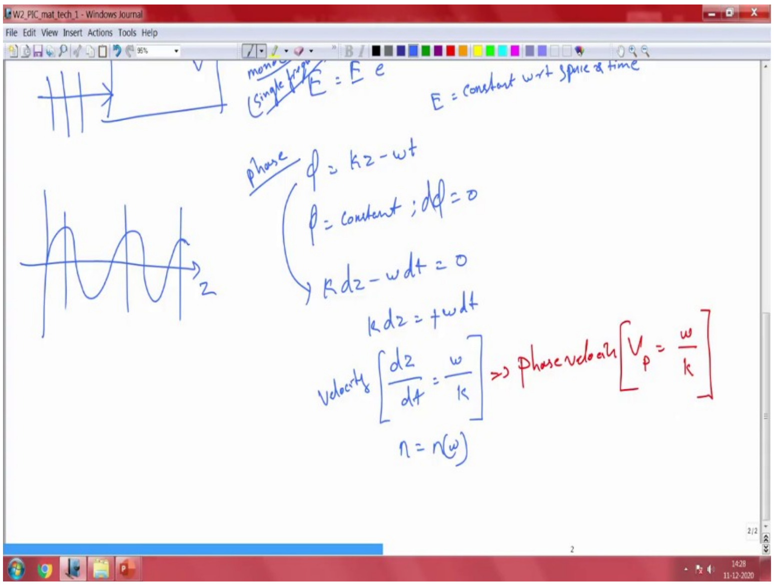

So, let us look at some of the interesting interactions that are very fundamental the first thing is light propagation inside the medium. So, I take a medium and then I would like to propagate this plane wave through this medium and I want to know what is the speed at which light is going to propagate through this medium and let me take a monochromatic light. So, let us take monochromatic light $E=Ee^{(ikz-\omega t)}$. So, this is a very simple monochromatic that means single frequency.

So, we are comfortably choosing restricting our light to have a single frequency because we know that the property of medium is frequency dependent in this case we want to understand what happens if a light wave with a single frequency passes through a medium. So, we have

E and then E is nothing but a constant independent of space and time let us say E is a constant with respect to space and time.

So, now when you look at this a very simple wave it is a sinusoidal wave and it has a phase that is varying along the propagation direction z and also with respect to time. So, the phase here can be written as, ϕ = kz - ωt. So, this is the phase of your very simple propagating wave. So, now if you want to understand how fast this particular wave moves we need to look at how the phase prorogates. So, if you take a very simple wave like this and it is propagating through this and you want to understand how fast it moves the first thing that you do is look at the phase.

So, how the phase or the constant phase moves in space and time. So, that is how you will find out how fast this particular wave is moving. So, in order to do that we need to force this phase to be a constant. So, it should not vary over time. So, this is how you can easily find out what this phase is and to be very specific you want this phase to be 0. So, the phase change as you move along should be 0 you need to force that at all these points.

So, you see a constant phase and then the phase difference between these points the lines that I drew here is all equal or the phase difference is 0. So, if that is the case then one can write sorry this dϕ = 0, kdz – ωdt = 0. So, now you can write kdz – ωdt = 0. So, you just differentiate this. So, what one could do is easily rearrange this. So, this gives you the velocity. So, this is nothing but velocity of the wave specifically we call this as phase velocity, velocity v_p which is nothing but ω/k. So, this is a important relation.

So, now you can see how fast it will move. So, it is wave vector with a propagation constant with respect to frequency. So, when there is a frequency change you do see a change in the phase velocity. So, now this phase velocity is not a constant we already just mentioned that and we also know that your refractive index in the medium is also not constant. So, what I mean by that is your refractive index n is actually a function of frequency. So, that is something that we already know from our earlier discussion.

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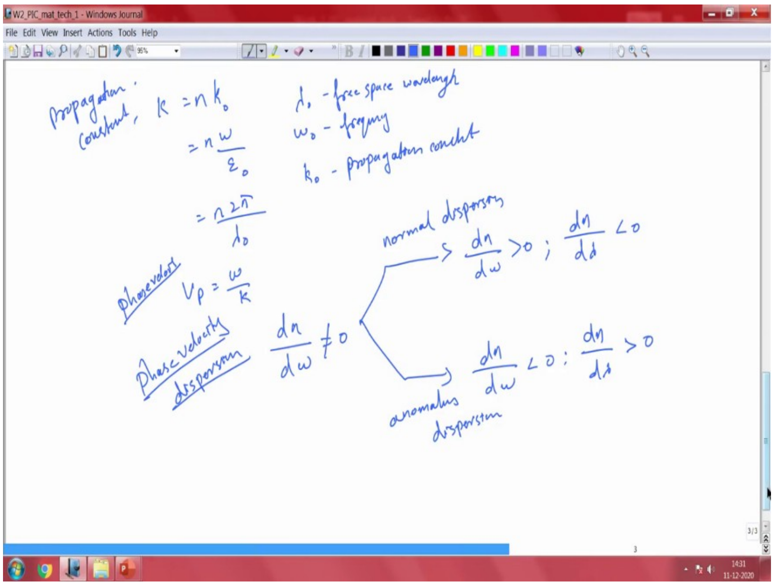

So, let us look at what is the propagation constant of this the propagation. So, propagation constant k is nothing but k = nk_o. So, this is something we have seen earlier as well. So, k can be written as k = $n\omega/\varepsilon_o$ and this is nothing but $2\pi n/\lambda_o$. So, lambda naught is nothing but free space wavelength, ω_o is the frequency, k_0 is propagation constant of light in free space.

So, now the phase velocity is something that we know now. So, the phase velocity is now given by $v_p = \omega/k$. So, this is our phase velocity but then the phase velocity would change as a function of frequency. So, that is what we call phase velocity dispersion. So, phase velocity you have phase velocity dispersion and that is a function of frequency. So, we know that the light is going to travel at different speed based on the frequency that you have because $dn/d\omega \neq 0$.

Because, that is what you have because of the dispersion and there are two type of dispersions that you can have one is normal and the other one is anomalous. So, in normal dispersion your $dn/d\omega$ is positive and here $dn/d\omega$ is negative. In other words, $dn/d\lambda$ is negative, $dn/d\lambda$ is positive. So, we are sure that there will be dispersion we know that and there are two slopes that you can have you can either have positive slope or you can have negative slope.

So, for the positive slope $dn/d\omega$ we call that as normal dispersion and $dn/d\omega$ we call this anomalous dispersion if it is less than 0. So, this is all for a single frequency analysis. So, that is why we comfortably said it is a monochromatic, monochromatic signal here that one could use in order to understand the velocity and the implication of frequency dependency on this velocity. So, let us look at what happens if it is not a single frequency. So, in most of the cases you will have photons of slightly a different energy.

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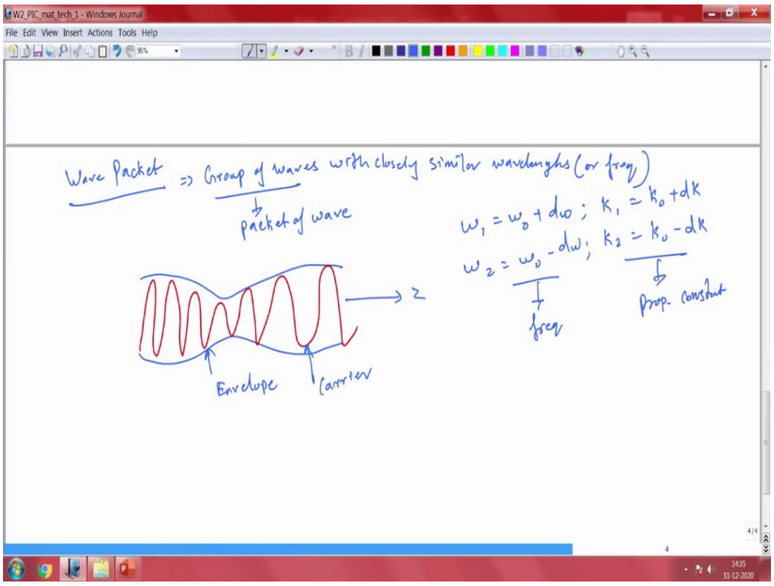

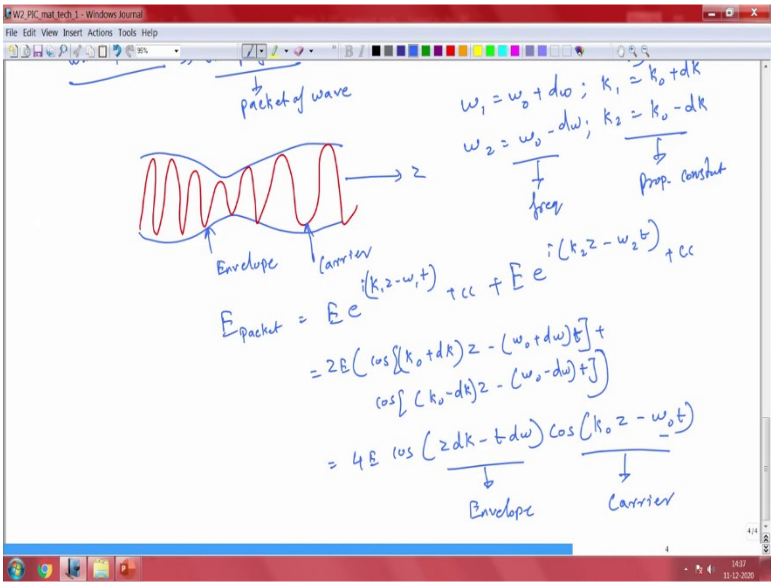

So, if that is the case what will happen and in that case it is not going to be a single monochromatic wave what you will have is a wave packet and wave packets is nothing but group of waves that are closely spaced, this is nothing but group of waves with closely similar wavelength or frequency and they are all combined and they move as a pack of waves.

So, they are packet of waves. So, how would they look like it is for some of you who might have done signal processing it looks like an amplitude modulated signal. So, you will have an envelope and you have the carrier inside this envelope. So, they are moving along the z direction. So, they are of same frequency, pardon me for this frequency change here, but they are all travelling at the same frequency and this is the envelope and this is our carrier.

So, this is how a packet of waves would travel. So, now let us look at how one could represent the propagation of this. So, we will have a frequency let us say there are two closely spaced ones. So, here you have the carrier and you have the envelope. So, that is $\omega 1 = \omega_o + d\omega$ let us say a small difference there and then you have $\omega 2 = \omega_o - d\omega$ which is $-d\omega$ and the corresponding propagation constant is $k1 = k_o + dk$ and $k2 = k_o - dk$.

So, this is our frequency and propagation constant. So, this is our frequency and this is our propagation constant. So, how can we represent this wave in a very simple equation form. So, let us say this packet the wave packet could be represented as we will just pick up what we already saw earlier, $E_{packet} = Ee^{i(k_1 z - \omega_1 t)}$ 1 + c.c because they are going together $E_{packet} = Ee^{i(k_1 z - \omega_1 t)}$ 1 + c.c + $E_{packet} = Ee^{i(k_2 z - \omega_2 t)}$ 1 + c.c and by expanding this into trigonometric form.

So, this would become $2E\{\cos [(k_o + dk)z - (\omega_o + d\omega)t] + \cos [(k_o - dk)z - (\omega_o - d\omega)]t\}$ within this. So, now one can rearrange this and once you rearrange you will get $4E\{\cos [(zdk - td\omega) \cos [k_o z - \omega_{ot}]\}$. So, what see here this is the envelope and this is our carrier.

So, the resultant wave packets as a carrier which has a frequency ω_o. So, this is the carrier that has a frequency ω_o and it has a propagation constant k_o and an envelope that is $\cos(zdk- td\omega)$. So, now we have an envelope. So, we have a wave packet and it is propagating through a medium and we can now calculate the speed at which this wave packet moves.

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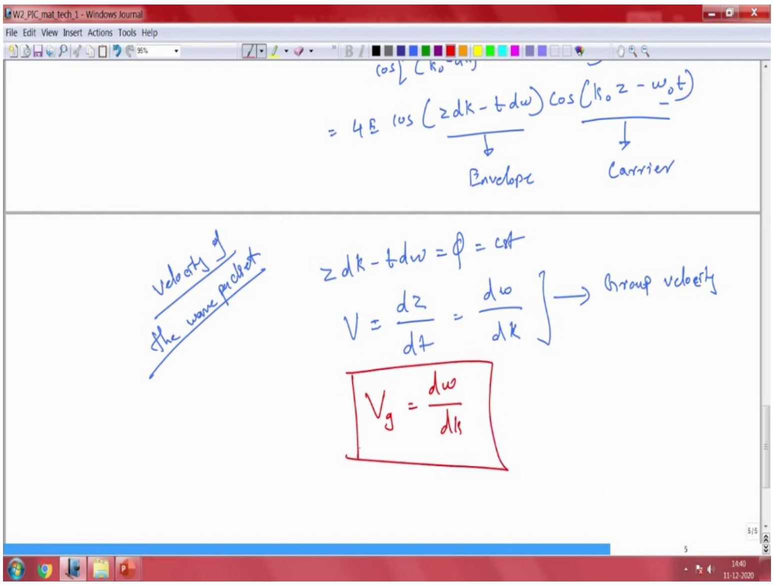

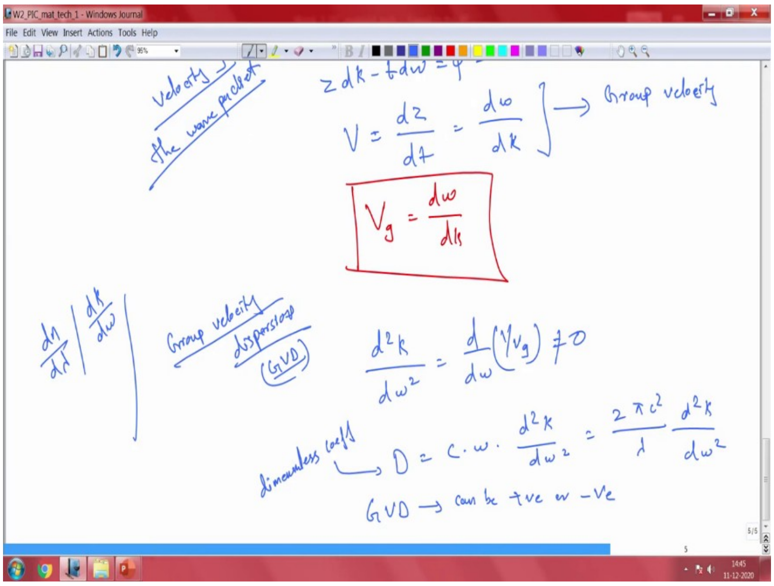

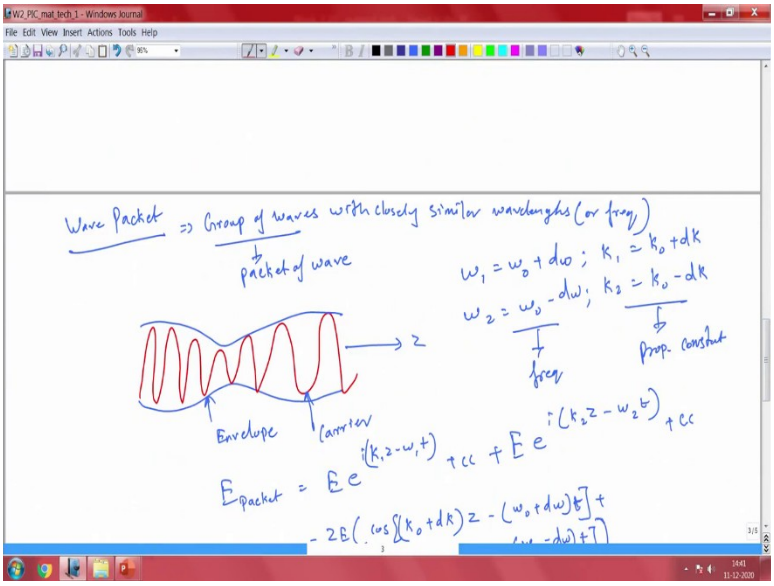

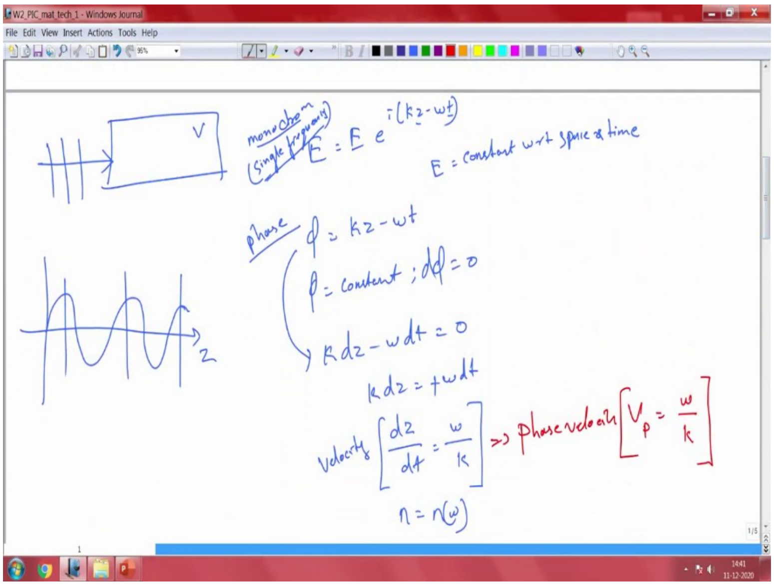

So, let us look at the speed at which this moves. So, we have already seen this let us look this envelope how fast this envelope can move. So, that is taken from the phase. So, let us look at this velocity of the wave packet. So, let us look at the velocity of this wave packet and that is given by $zdk-td\omega = \phi$ should be equal to a constant phase. So, we have seen that this should be a constant.

So, now let us find the value the velocity here the group velocity in this case. So, because the waves are moving as a group we can this as group velocity and thus that velocity is nothing, it is given by dz/dt and that is nothing but the, this is what the velocity is. So, if we look the velocity here it is going to give you $d\omega/dk$. So, this is our group velocity.

So, $d\omega/dk$ is our group velocity this is nothing but group velocity because it is a group of waves and that is the reason why we call it as group velocity in order to differentiate this from the phase velocity we put $v_g = d\omega/dk$ which is nothing but $d\omega/dk$. So, z/t is what you have and the velocity here is represented this way. The phase velocity and group velocity are both velocities but then the phase velocity represents a single wave while group velocity represents group of waves.

So, this is how a group of waves are going to move which may or may not be equal to the phase velocity we will come to that shortly why this group velocity and phase velocities are slightly different. So, the constant phase front that travels at group velocity but the group velocity is the velocity at which the energy travels it is slightly different here. So, here if you look at the wave packet. So, the energy at which this envelope moves is the group velocity.

So, this whole uniform envelope that we have created because of multiple wavelengths are very closely spaced frequencies are present and this is what the group velocity is all about but in case of a phase velocity it is a single wave. So, the all the energy is propagated this particular velocity all the energy there but in this case it is not about the carrier. So, you have a carrier

ω_o but the energy is not carried at the speed of this wave of frequency ω_o it is carried at the speed of the envelope itself. So, that is going to be a difference here.

Why it is, it is critical to understand this packet of waves with different frequencies compared to a single frequency in most of the communication networks we use lasers with slightly wider line widths that means you will have photons of slightly different energy and at the same time we are going to use light with different wavelengths altogether. So, when you are sending information through a channel, the channel will have its own dispersion and because of that your single frequency or this multiple pulses with different wavelengths are going to travel at different speed and it has implication on the information carrying capacity or the speed of transport.

So, this is our group velocity as I mentioned you we also have dispersion associated with this. So, we call that as group velocity dispersion. So, similar to what we saw, so dn/dλ. So, that is our dispersion, in the other words it is dk/dω. So, we knew that the refractive index is a function of lambda, here again we have the same issue. So, your refractive index or your propagation constant is your dielectric medium is going to respond to this way.

So, this is nothing but and this is not equal to 0 and this is the reason why we have dispersion and this is what we call group velocity dispersion and this group velocity dispersion is represented by a dimensionless coefficient and this dimensionless coefficient is called D which is given as D = $c\omega\frac{d^2k}{d\omega^2}$. group velocity dispersion or in popularly called GVD group velocity dispersion and you can expand this as D = $\frac{2\pi c^2}{\lambda}\frac{d^2k}{d\omega^2}$.

So, this group velocity dispersion is an important consideration in propagation of optical pulses for the same reason that we just discussed it can cause broadening of individual pulses. So, when you have a pulse with different frequencies it can result in broadening of the pulse and it could change the time delay between different pulses of different frequencies and you could have the group velocity dispersion, this could be, the group velocity dispersion this can be positive or it can be negative, a similar way we saw the normal dispersion and anomalous dispersion that we saw for the wave propagation, the group velocity dispersion will also have positive and negative implications.

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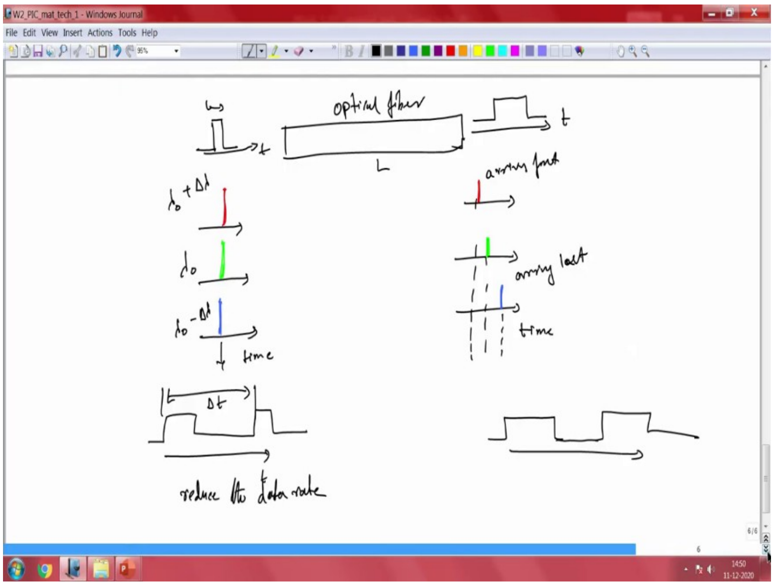

Let us, look at how this can affect your pulse propagation through a medium. So, let us take a popular medium here, let us take an optical fiber. We take an optical fiber I am going to put a signal here so there is an input signal as a function of time I am putting a very simple signal and this signal has let us say three different wavelengths. So, I have red and I have green and then I have blue. So, look at it all at the same time. So, I am putting at the same time.

So, this is important. So, this is my λ_o and this is $\lambda_o + \Delta\lambda$ and this $\lambda_o - \Delta\lambda$. So, this is as a function of wavelength but as a function of time they are at the same time we are putting it and I am sending it through an optical fiber and let us see what happens at the output of this individual colors.

So, how are these colors going to come out. So, if you look at the shorter wavelength, the shorter wavelengths are going to come much early. So, they travel faster. So, they will arrive first and then comes green and then is our red. So, you can see here already they come arrive at different period of time as a function of wavelength. So, this is arriving, sorry this is arriving first, it is a other way around this is arriving last.

So, because of this you will have pulse broadening. So, the pulse are coming at different point of time from different wavelengths and as a result you will have a broadened pulse. So, when you have this difference in time of arrival of different wavelength you will have change in the pulse width that you send. So, why are we interested in this pulse width. So, when you are sending a signal you are going to send on, off signals.

So, 0s and 1s are going to go through the system and when you have this kind of pulse broadening the first pulse itself will take a while in order to arrive and it will broaden while the second pulse will take an overlap here and the third pulse perhaps it will overlap like this. So, when this overlaps happen there is no way to know where is 1 and where is 0 and because of this of interference between the different symbols that we have here you will not be able to capture the data. So, you will not be able to deduce anything from this received signal. So, what you ideally want is just reproduction of whatever you have.

So, in order to do that because they are going to broaden, we have to reduce the number of bits we send. So, even when the broadening happens you will have enough dead time here to get the next pulse. So, because of this you have to increase this time and what that means is you can only send few bits at a time. So, you have to reduce the data rate because of this group velocity dispersion. So, this is the main implication of having a dispersion affect your transmission here.

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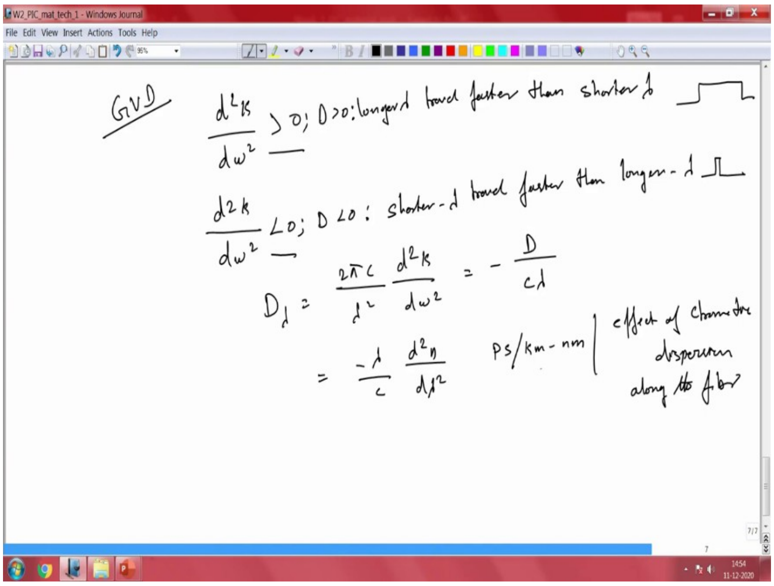

So, let us look at some of the interesting aspect of this group velocity dispersion. You can have $\frac{d^2k}{d\omega^2}>0$. So, if it is positive long lambda wavelength travel faster than shorter lambdas and your D > 0 here and now the other condition could be $\frac{d^2k}{d\omega^2}<0$ where your D < 0, then your shorter wavelengths travel faster than longer wavelengths.

So, you can have both. So, you can have a broadening of a pulse you can also have narrowing of pulse which is very handy to do in some of the interesting optical signal processing experiments and some optical signal processing functionality but you have to make sure that you achieve positive and negative group velocity dispersion. So, that is very important.

So, the group velocity dispersion is different from phase velocity dispersion. So, we are not going to discuss about phase velocity dispersion at this point of time, we are more interested in group velocity dispersion and implication of that when you propagate. And another way of representing this group velocity dispersion quantifying it when it is travelling through a medium is through this particular relation which is given as $D_\lambda = \frac{2\pi c^2}{\lambda}\frac{d^2k}{d\omega^2} = -\frac{D}{c\lambda} = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$ and this is represented as picosecond per kilometer nanometer.

So, this coefficient is what we use to measure the chromatic pulse transmission over a fiber or any transmission length for that matter. So, this gives us the effect of chromatic dispersion along the fiber. So, it is given as picosecond per kilometer nanometer. So, for a given length one can easily find out what this delay is. So, let us look at what are all the implication of that.

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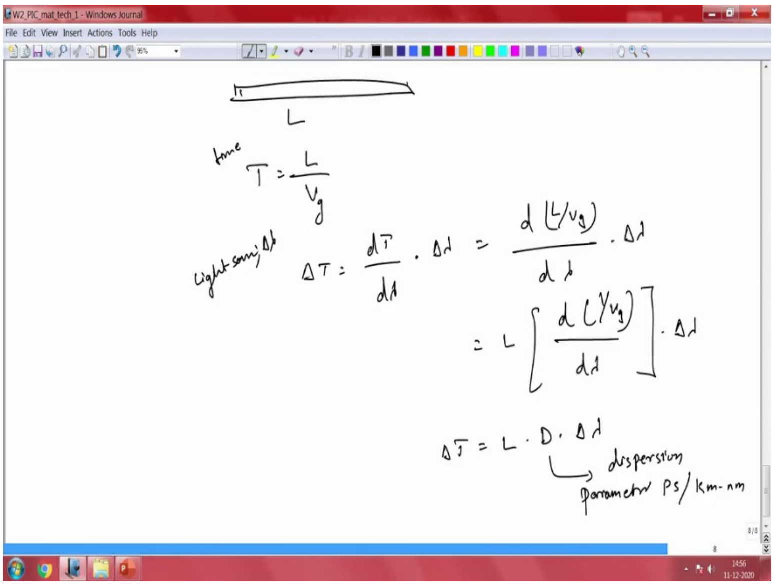

So, if you take a certain length of optical fiber L and you are putting a pulse. So, you are putting a pulse and you want to know how long does it take. So, its T is the time it takes for the pulse to reach there. So, T is the time it takes and it is easily given by $T=\frac{L}{v_g}$ length over the group velocity and let us say you are using a light source of certain spectral width.

So, light source as a certain spectral width $\boldsymbol{\Delta}\lambda$ and now you can write ΔT that is the delay that you have could be represented as $\Delta T=\frac{dT}{d\lambda}\Delta\lambda$ which one can write simply as

$\Delta T=\frac{dT}{d\lambda}\Delta\lambda$ = $\frac{d(\frac{L}{v_g})}{d\lambda}\Delta\lambda$ in other words $\Delta T=L(\frac{d(\frac{1}{v_g})}{d\lambda})\Delta\lambda$, *we can already see that* ΔT = L.D.$\Delta\lambda$ where D is nothing but your the dispersion parameter and which we have already saw seen that meter per nanometer.

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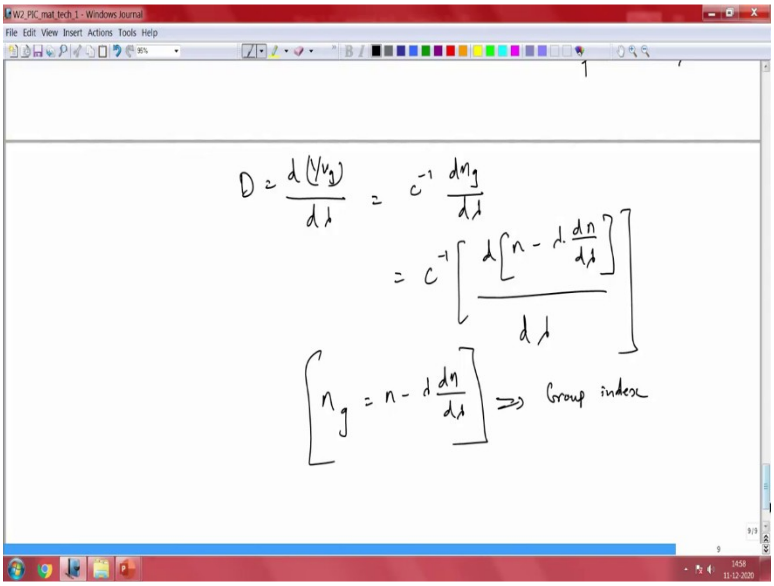

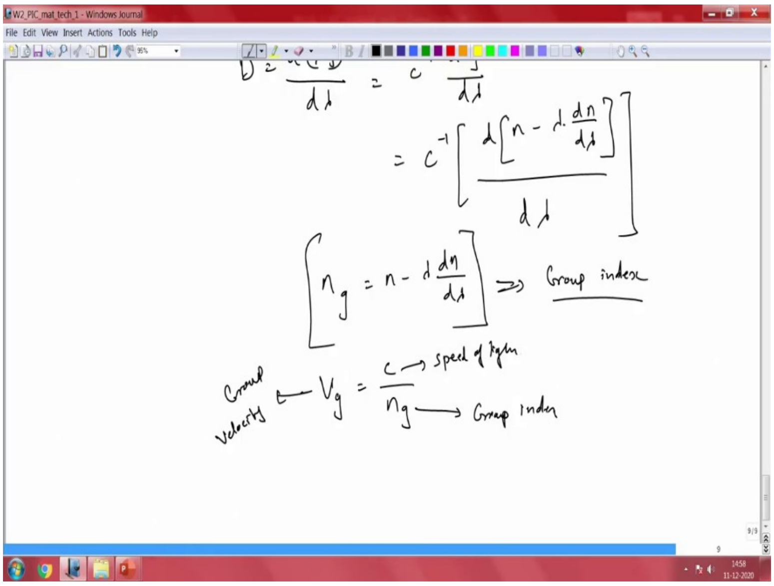

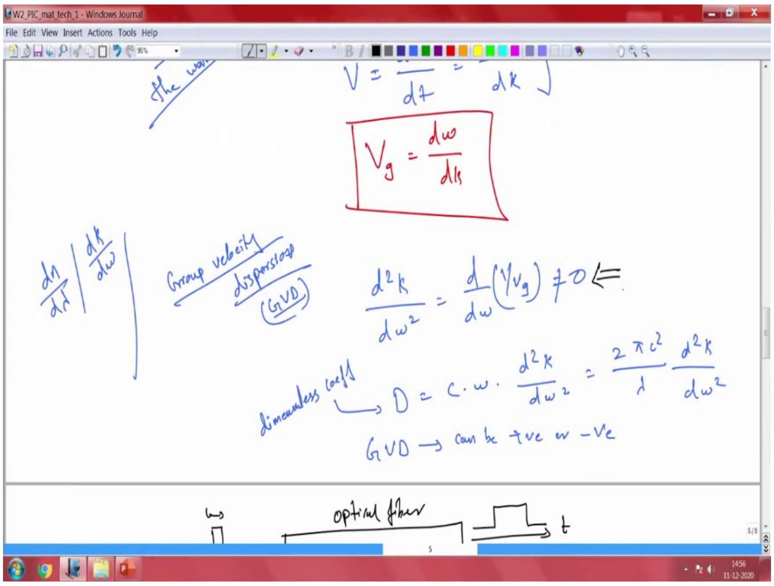

So, now we can write this d we already saw this D is represented as let me pull to our earlier thing. So, you remember this. So, let us find out what this group velocity that we have could be used to find out the change in the refractive index now. So, we are trying to find out from the group velocity dispersion here how one can as a function of refractive index that we have.

So, we already know that this can be given as $D=(\frac{d(\frac{1}{v_g})}{d\lambda})=c^{-1}\frac{dn_g}{d\lambda}=c^{-1}(\frac{d(n-\lambda\frac{dn}{d\lambda})}{d\lambda})$.

So, the group index is nothing but $n_g=(n-\lambda\frac{dn}{d\lambda})$ which is something we have already seen and once you navigate this group index is nothing but $n_g=(n-\lambda\frac{dn}{d\lambda})$. So, this is what we call the group index. So, we know the refractive index n. So, that is for a single frequency and now we have a group of frequencies and this group of frequencies will feel a slightly different refractive index compared to a single frequency wave and that is the reason we represent this as a group index and the $v_g = \frac{c}{n_g}$.

So, this is our group index and this is our group velocity and c is our speed of light. So, now what we have converted from a single frequency propagation through a group of waves that are propagating through a system. So, with that we have now understood how a single wave propagating though a medium experiences the property of the medium, in this particular case the change in the refractive index and this change in refractive index has an effect on a single frequency of a wave that is propagating but in reality you always have packet of waves.

So, when you take a packet of waves with slightly different frequency then it is not anymore that single frequency concept you are going to see a combination of this multiple frequencies going through. So, it is not a single frequency that dictates this is the speed at which we are going to propagate, no it is the whole bunch. So, the energy that is being transported is not a single frequency velocity it is going to be a group velocity. So, now all these group of waves are moving at certain speed and also the speed at which they are moving will be affected by the dispersion there.

So, each frequency will have its own associated dispersion. So, all of this combined is what we call group velocity dispersion. So, once you have the velocity dispersion you also have refractive index change or refractive index dispersion we call this as group index. So, group index, group velocity, group velocity dispersion all these are important concepts that you should keep in mind with this we will end this particular session in the next session we will see actually how an optical fiber manifests its dispersion. Thank you very much.