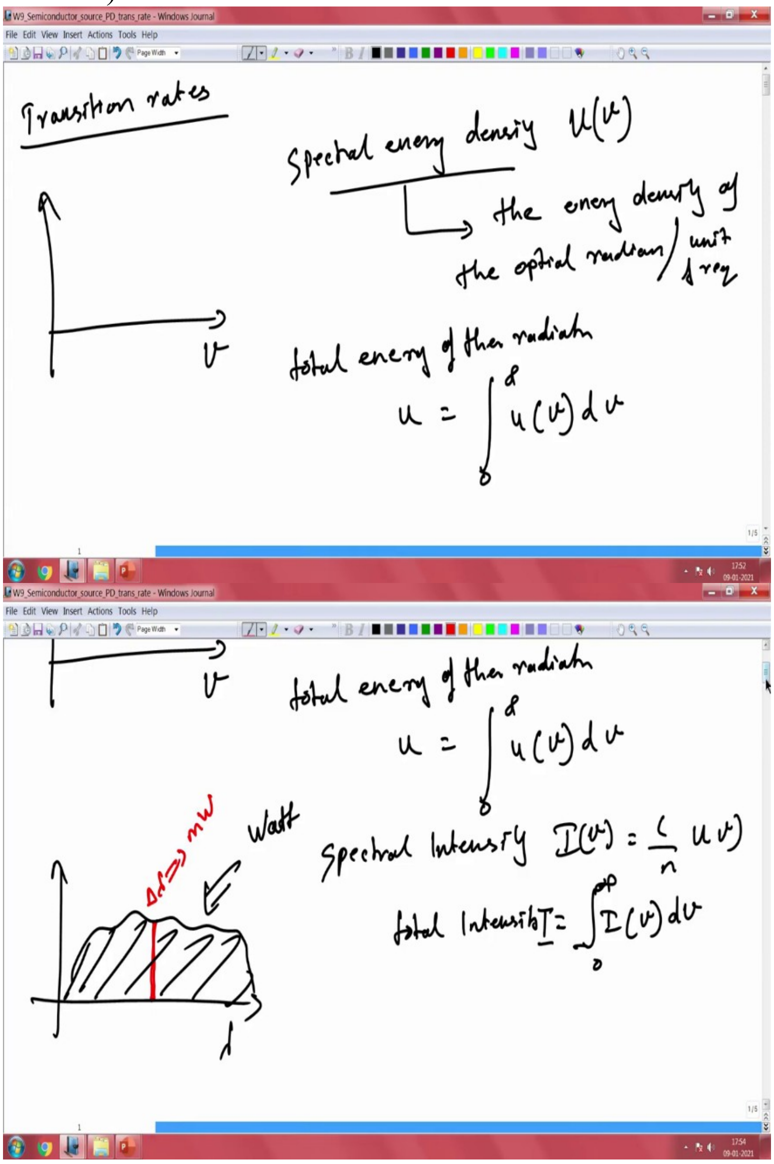
Photonic Integrated Circuit
Professor Shankar Kumar Selvaraja
Centre for Nano Science and Engineering (CeNSE)
Indian Institute of Science, Bengaluru
Lecture-39
Transition Rates

Hello everyone, let us now look at the transition rates. So, we looked at the transition earlier, but let us look at what all the rate constants associated with spontaneous emission, stimulated emission, and absorption, and how these rates are going to affect our light emission and the characteristics of light emission as well. So, let us look at that.

(Refer Slide Time: 00:46)



The transition rates depend on the energy states that we have, and associated rate constants. So, the spectral distribution of any field, so when you, when you talk about light generation over a certain frequency, we should also understand, what is the energy density that is associated with this, or the spectral density that we have, with the spectral distribution across, by the radiating field.

So, the transition rate, it is actually, is induced by this spectral distribution of optical radiation. And the spectral characteristic of the, of the resonant transitions as well. So, let us look at the spectral distribution of a field. So, the spectral distribution of field is given by this, the spectral energy density is given by some u(ν) let us say. So, the energy density of the radiating field per unit frequency that is what, this spectral energy density is.

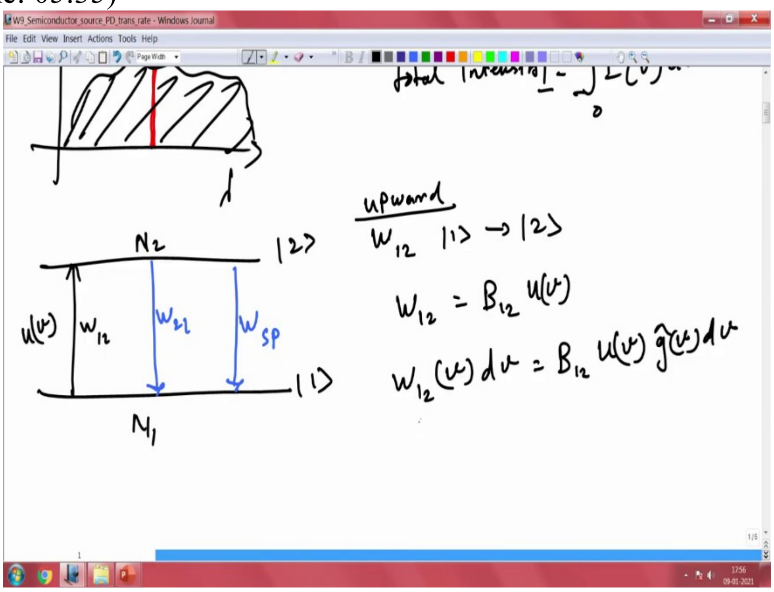
This is nothing but the energy density, energy density of the optical radiation per unit frequency or the interval of frequency or whatever. So, this is how we define it, the total energy is nothing but an integral of this, so total energy of the radiation is given and say it is 'u', its integral of what we have here. So, that is the total energy that we are, so that is important to note that when we talk about the total power carry, carried by an optical wave or optical beam, we have to qualify that with respect to the frequency or the wavelength that you have.

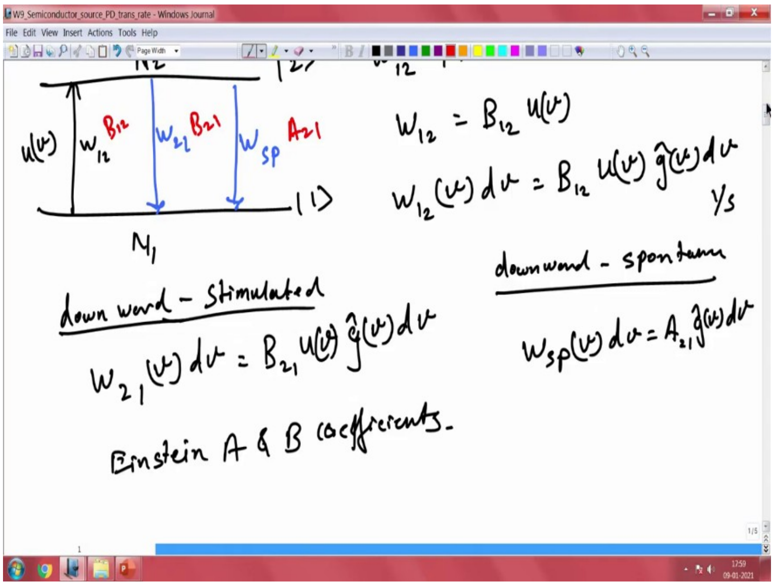
So, the power could be couple of watts. So, the, the light, the light bulb or whatever that we have the energy that is put out can be measured as a, as a total energy of radiation. So, it could be, few milli watts, let us say. So, but, then the spectral energy, spectral density is nothing but the energy of individual frequencies or individual wavelengths. So, here let us say if you want to put the spectral intensity, so the spectral intensity is given by, $\frac{c}{n} u \nu$.

So, now, I want the total intensity, will be equal to the, again we are going to integrate this over the frequency range. So, the intensity over that frequency range will give you the total intensity 'I'. So, it is important to differentiate between the total intensity that we have and also your spectral intensity. So, the way that you can visualize this is, if you have a wavelength, it will have a certain spectrum.

And you, when you, when you measure the power or the total power, you have to integrate this whole region. So, this whole region integrated will be in some watts, let us say, some watt power. But then the energy spectrum or the spectral energy here is in this particular Δλ, let us say. So, how much of power within this. So, this will be some milli watt. So, the total power is different from the spectral intensity or the spectral energy that this particular wave carries. So, that is the reason why the spectral relation is very important to understand.

(Refer Slide Time: 05:35)





So, let us look at the transition rate. So, all these emission happens through transition between the two levels, that we already know of. So, let us take this, the two levels here. So, you have N_1 and you have N_2 and you have transition from here. So, we call this as transition rate w_{12}, which is equal to B_{12} u times ν. So, let me only write this, so that I can write it in a separate way. And then you can go from top to down. So, I will pick a different color. So, you can go from top down that is w_{21}.

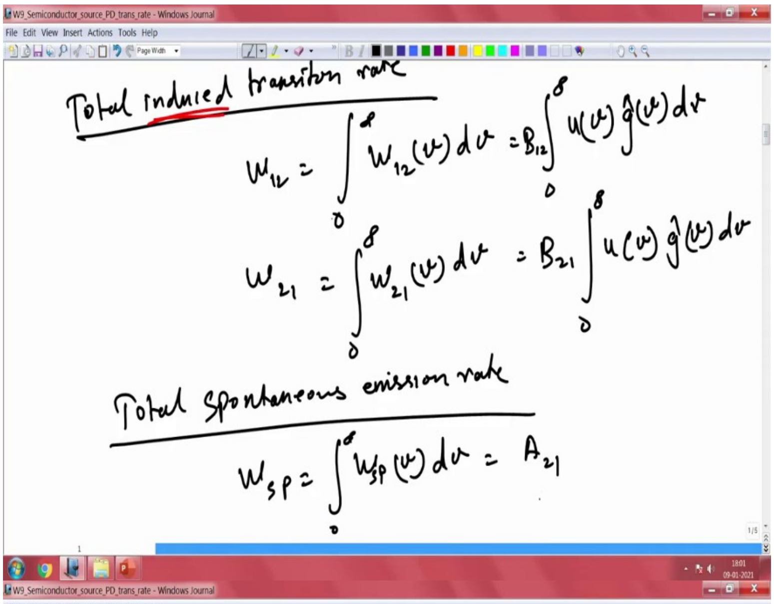
And then you could have another transition that is $w_{spontaneous}$. So, w_{12} is our upward transition, this is upward transition w_{12}, that is from level 1 to level 2 and that is given by B_{12}, this is absorption parameter, u(ν), or in other form as a function of frequency, $B_{12}u(v)\hat{g}(v)dv$, this is per second. So, this is the, the transition rate. So, that means, this will be per second. The same way you have two downward transitions. So, let us look at the downward transition.

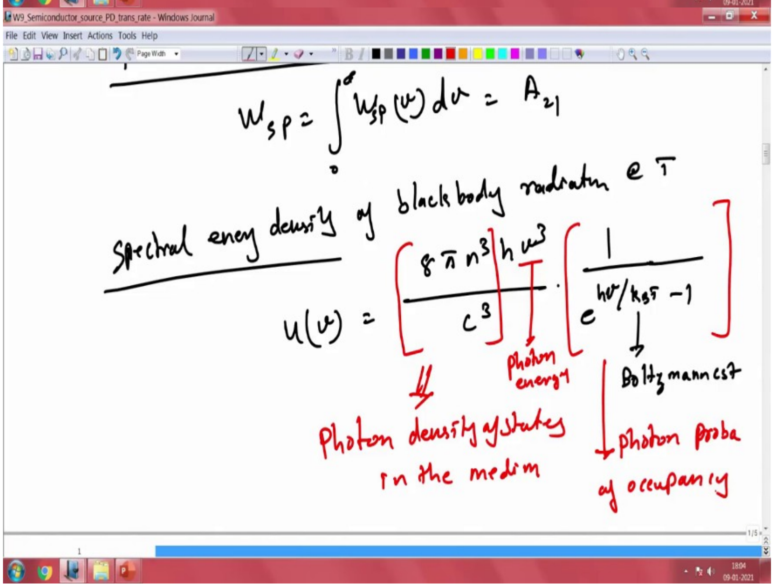
So, the downward transition, we have $w_{21}(v)dv$ is nothing but $B_{21}u(v)\hat{g}(v)dv$. So, this is our stimulated emission. So, this is our stimulated downward, stimulated. And then we have downward spontaneous and that is given by w spontaneous frequency v which is nothing but A_{21} which is something that we already saw, $\hat{g}(v)$. So, let me write it, was $A_{21}\hat{g}(v)$. So, this is our spontaneous emission that is independent of the energy density of the radiation. So, this is the transition function here is the line shape that we have for this.

So, the A and B here are nothing but the Einstein coefficients. So, now we have this process defined, we are, we have upward transition and we have two downward transition. So, the spontaneous emission, there, it is characterized by our emission coefficient A_{21}. So, this A_{21} is the spontaneous transition here. So, this is characterized by A_{21} and B_{21} is characterizing our stimulated emission and then B_{12} is induced absorption, we can call this as induced absorption or absorption. So, these are all the 3 different processes that we have and we want to look at the transition rates. So, how, how quickly this the transition happens.

So, how do we do that? The transition rates or the total induced transition rates are nothing but the integral of this. So, we have defined this as a function of frequency. So, we want to understand what is the total rate? So, that means, we need to integrate this over the frequency.

(Refer Slide Time: 10:47)





So, your total, total induced transition rate is nothing but w_{12}, which is integrating it with all the frequencies that we have within (())(11:10). And that would result in B_{12} 0 to infinity nothing but your line function here. Similarly, we can do this for w_{21} as well. So, w_{21} dv which is B_{21}. So, now the total spontaneous emission now, this is total induced transition, so, induced transition, you should understand, underline here.

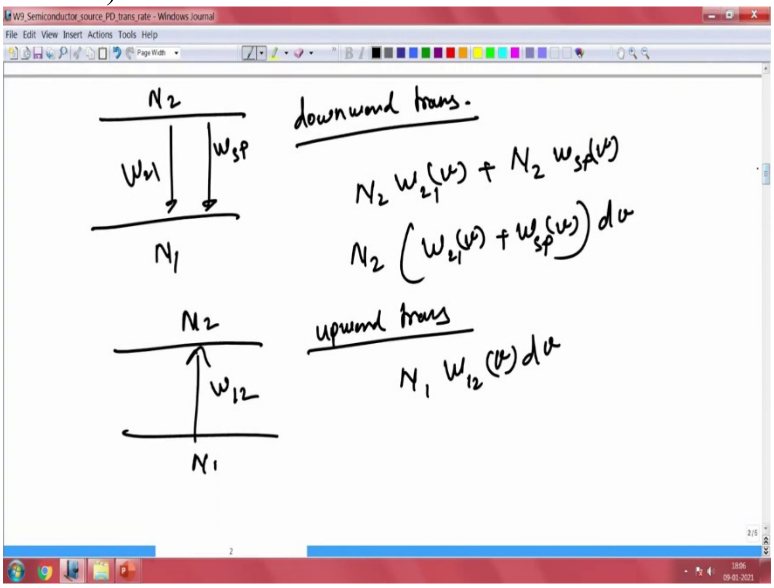
So, what is the total spontaneous emission, the spontaneous emission rate here? That is a, that is 'w_{sp}' which is given by $\int_0^{\infty} w_{sp}(v)\,dv$ which is A21 as simple as that. So, these are all the factors that determine your transition rates between the different levels here, so higher level and lower level. So, the induced and the spontaneous transition rates are now directly proportional to one another.

So, this is coming from our, the line shape that we have and this could be related through our, the line shape itself and that is coming from our spectral energy density of any radiation, for example. And that is given by the Planck formula the spectral energy density, the spectral energy density of blackbody radiation which we could use here, at temperature T is given by this simple formula that we all know of, $hv\,k_B T$ minus 1.

So, here k_B is your Boltzmann constant. So, the rest of the things are straightforward here, where 'n' is the refractive index 'c' is the speed of light here and ν is the frequency that you have. So, your k_BT here is at room temperature 300 Kelvin is about 26 milli electron volts that you could use in here. So, based on this we could clearly see, what are the different contributing factors here.

So, one is the photon density of states in the medium. So, this, this factor that, let me chalk out this, this factor is the photon density of states in the, in the medium. And this is nothing but your photon energy. And this factor that we have is nothing but the photon probability of occupancy, this is photon probability, probability of occupancy. That is giving you this states, this is nothing but the average number of photons in each state. So, given by Bose Einstein distribution. So, this is, this is how you could deconvolute what we have from our Planck's formula here.

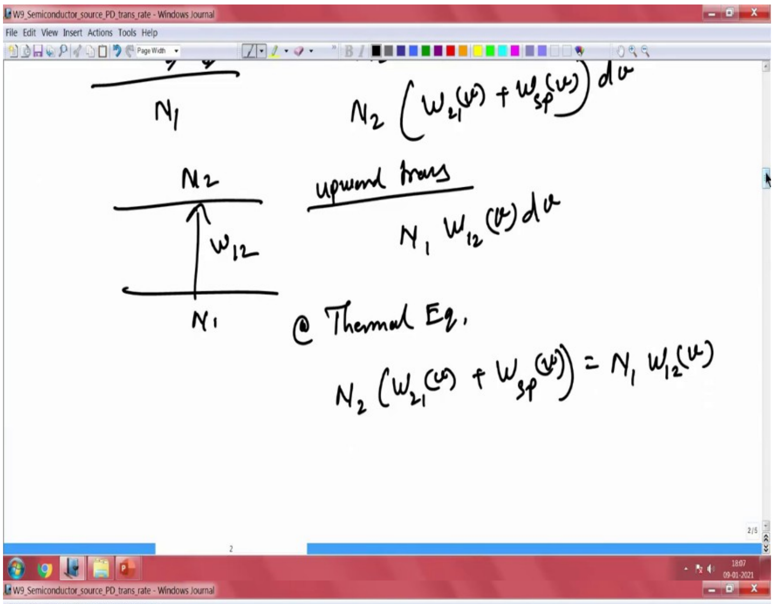
(Refer Slide Time: 16:00)

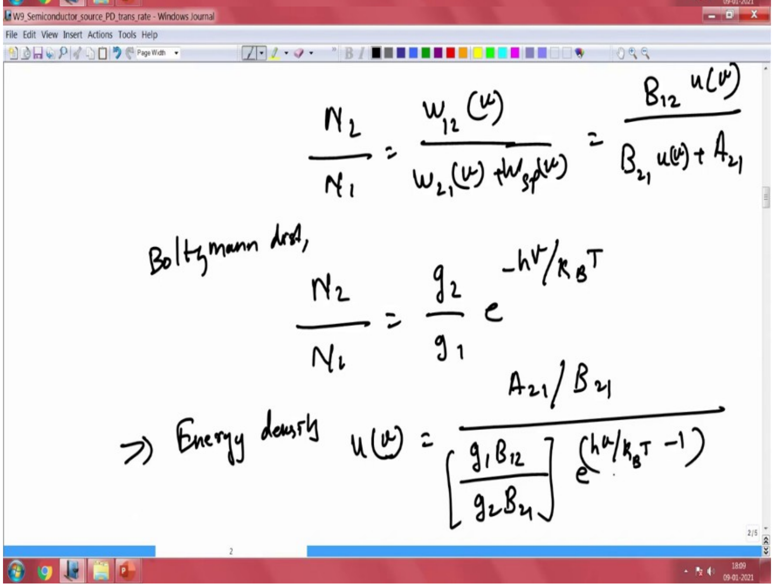


So, we know all these transition, but let us look at the, the rate relation between the two states. Now, so, we want to, at the end we want to derive or relate these time constants and also find the, the intensity and the cross section that we are going to get. So, for that, let us assume that N_1 and N_2 are the states in level 1 and level 2. So, now let us look at these two levels and we have N_1 and N_2. And we also know that your upward transition and downward transition. So, let us say the two downward transitions I have got w_{21} and w_{sp}.

So, now the emission so, let us say the downward transition is given by just the, so downward transition is nothing but $N_2 w_{21}(v) + N_2 w_{sp}(v)$. In other words, N_2 is nothing but $w_{21}(v) + w_{sp}(v)$. So, this is our upward downward transition. So, let us look at the upward transition. So, upward transition is given by w_{12}. So, from N_1 to N_2, and that upward transition is now given by $N_1 w_{12}(v)\, dv$. So, this is our upward and downward transition that we have.

(Refer Slide Time: 17:51)





So, we need to look at the equilibrium, or thermal equilibrium, these two should be equal. So, at thermal equilibrium, your $N_2\left(w_{21}(v)+w_{sp}(v)\right)=N_1 w_{12}(v)$. So, this is in the thermal equilibrium. So, that is, the steady state population now. So, what is the steady state population now? Between this N_1 and N_2 you could easily get this from this ratio there. So, that is $\frac{w_{12}(v)}{w_{21}(v)¿(v)}$ here. And we also know the coefficients here, we can remove this rates and then put the coefficients that we know, $\frac{B_{12u(v)}}{B_{21}u(v)+A_{21}}$.

So, this is the steady state equilibrium condition. So, now, since this transition that happens, they could have degenerate states here. So, whatever we, the transition that we are talking about from different positions here, these are all degenerate states here anyways. So, one could write this as a as a degenerate states, looking at the Boltzmann distribution that we all know. So, the Boltzmann distribution, gives you this relationship from the degenerate mod, $h\nu/k_BT$.

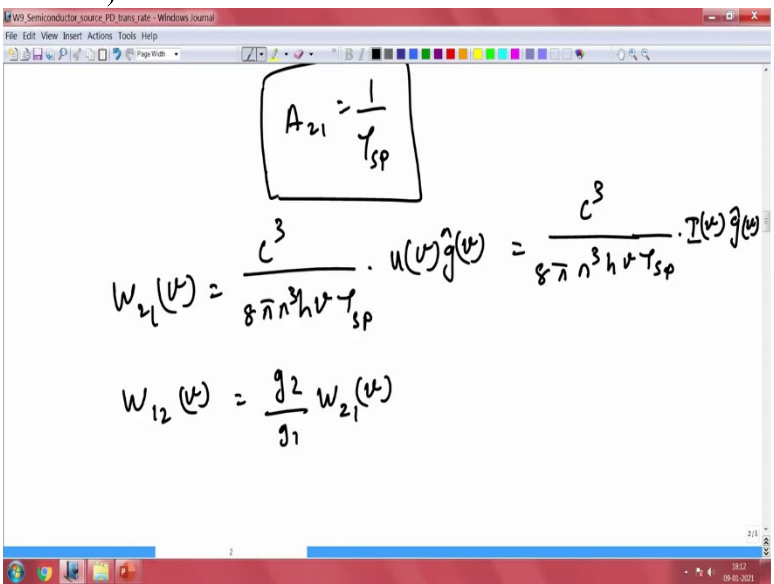
So, now, so, the g is nothing but the degeneracy factor, that you have and based on this relation we could write our energy density. So, based on this, we could write energy density

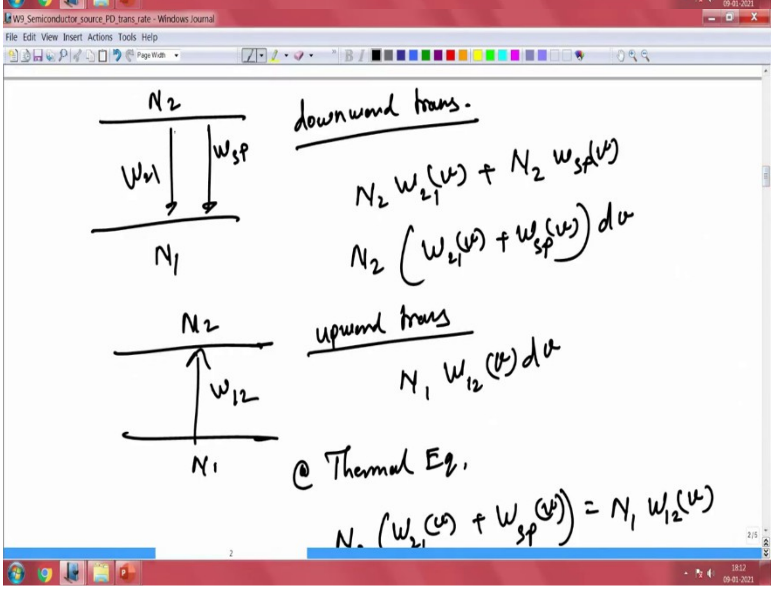
$u(v)=\dfrac{A_{21}/B_{21}}{\left(\dfrac{g_1 B_{12}}{g_2 B_{21}}\right)e^{\left(\frac{hv}{k_B T}-1\right)}}$. So, this is our modified energy density, when you are looking at that transition rates. So, this gives us how the transition is going to be, and you can clearly see this, this transition that we see as the induced process, the absorption and simulate, stimulated emission process is directly related to the spontaneous emission rates.

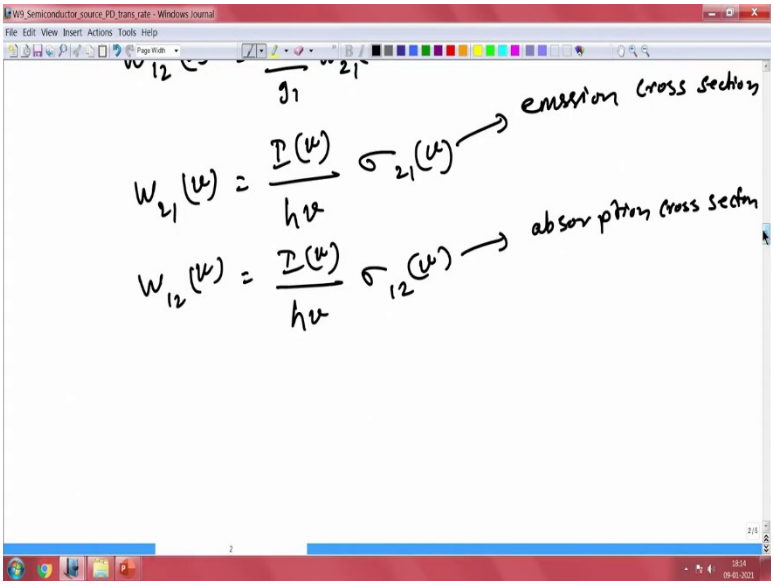
So, there is no escaping from the spontaneous emission here. So, what else do we want to understand? So, we, we want to understand, what is the capture cross section or emission cross section of this.

So, the spontaneous emission factor we all know, but we, let us try to probe a little bit more on this w factor. So, we had this w_{21} and w_{12}. So, what are all those? So, there is those are all transition rates, but we have not, converted those transition rates into something that, that is measurable or something that we can relate to the material property. However, A_{21} we already know.

(Refer Slide Time: 22:22)





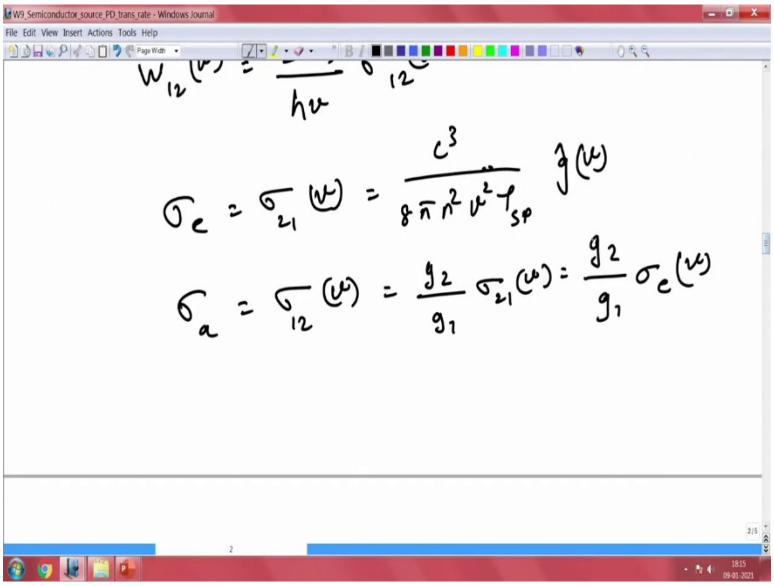


So, A_{21} is nothing but, we all know A_{21} is nothing but our, the transition that we, that we already know of, that is 1 by the decay. So, this is something we know. So, we want to also understand how this, the other factors the B sorry, w_{21} and w_{12} factors are, from that also you can also calculate your B factors.

So, the transition rate now, w_{21} can be written as, $\frac{c^3}{8\pi n^3 h\nu\tau_{sp}}u(\nu)\hat{g}(\nu)$ which is equal to $\frac{c^3}{8\pi n^3 h\nu\tau_{sp}}I(\nu)\hat{g}(\nu)$. So, the $w_{12}(\nu)=\frac{g_2}{g_1}w_{21}(\nu)$. So, this is the, the transition rate of w_{21} that you have, that is going from the, the excited state to the ground state and w_{12} which is our ground state to excited state. So, now, we have, we have written, the rate constant going from ground state to the higher state, and also the higher state to the ground state.

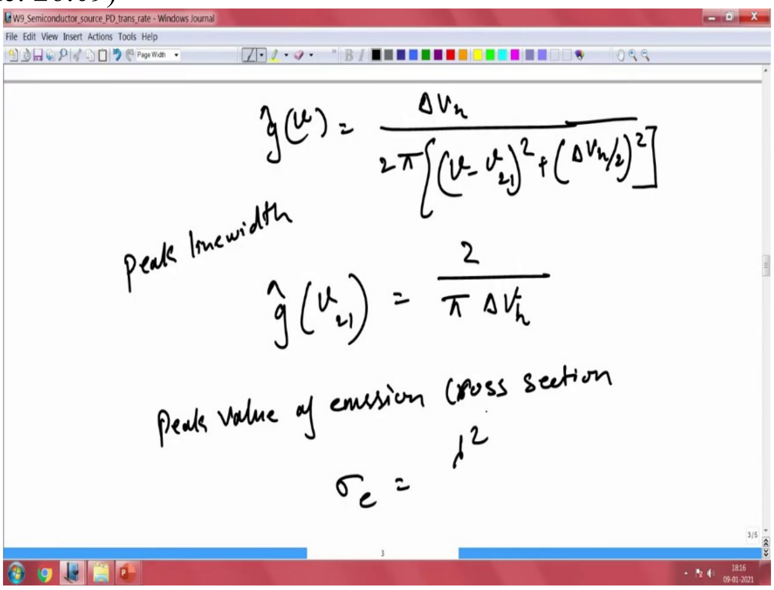
So, so now, we can express this, the transition probability as a function of optical radiation now. So, now we can write this as a function of intensity here. So, intensity of this (()) (24:34), this is nothing but our photon flux times your cross section. So, this is what we call emission cross section. And similarly, we can write w_{12}, which is our absorption cross section. So, what is this absorption cross section and emission cross section?

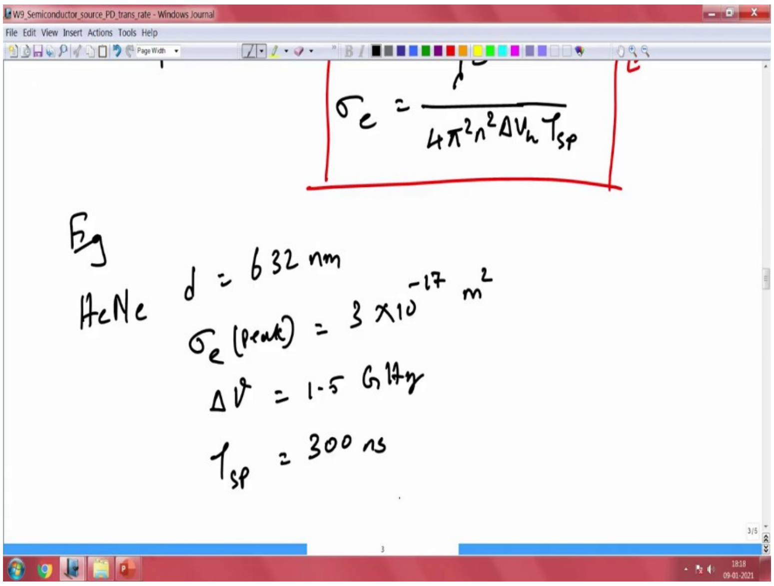
(Refer Slide Time: 25:19)

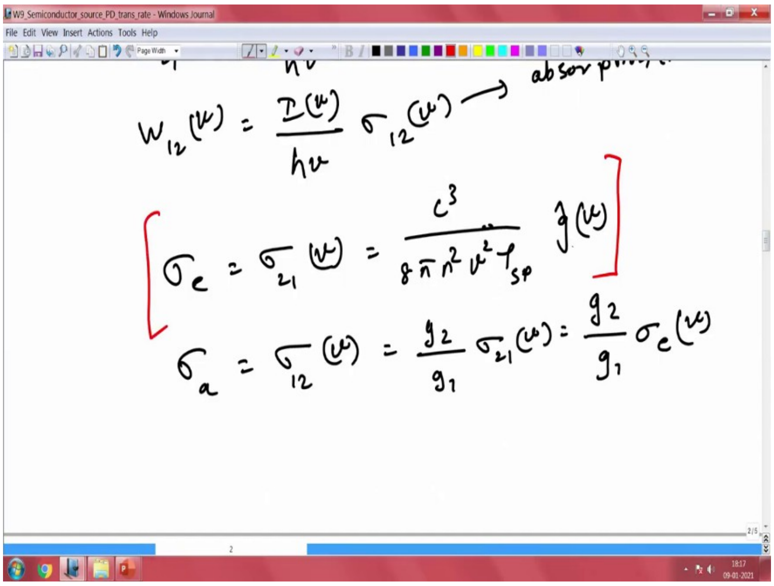


They are related though. So, they are nothing but what we already saw. [τ] spontaneous. So, this is our emission cross section and our absorption cross section is given by $\frac{g_2}{g_1}\sigma_{21}(v)=\frac{g_2}{g_1}\sigma_e(v)$ (())(25:58). So, this is the emission absorption factors and we already know the g here.

(Refer Slide Time: 26:09)





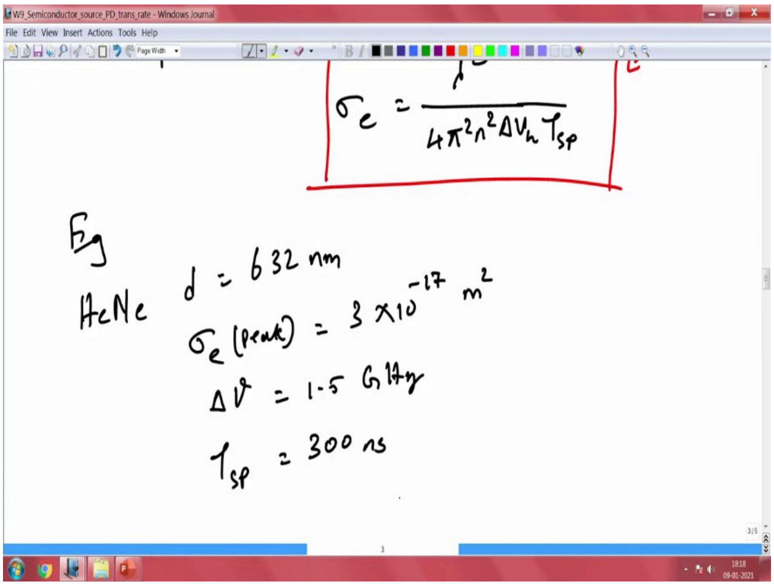


So, the line shape that we already saw, I am just going to write this again from our earlier equation, which is $(\nu-\nu_{21})^2$ plus the bandwidth here. So, the peak of this, so, where is the peak that appears in the line width? So, the peak line width, the peak of the line width so, where will it appear?

So, when the denominator is the lowest. So, that means you are your, you are in resonance. So, ν should be equal to ν_{21}. So, if that is the case, then your ν_{21}. So, when ν is ν_{21} that is where you have the, the peak of this resonance is going to be there and the peak value of emission cross section. The peak value of emission cross section that is, that is given by, you just substitute this. So, that is all I am going to do.

I am going to substitute this there and you will get. So, this is our peak value of the cross section at the central wavelength λ So, at this particular wavelength in the spectrum, so what is your peak emission cross section. So, these are all factors that are related to the material now. So, we know that this, this is what you get. And more importantly, we can also do the emission cross section, in this case and also calculate it from here. So, this, this is something we saw and we [substitute], we know where the peak is and by substituting this we got the actual emission cross section. So, this is something that we can actually measure and then calculate. So, based on this calculation, we can find out how much is our emission cross section.

(Refer Slide Time: 28:50)



So, for example, I will give you a very simple example here, for a material. So, let us take what is the material, we look let us say helium neon laser. So, that is something that we, we all know. Your, your wavelength of emission here is going to be let us say 632 nanometers and the peak here, peak at this wavelength is, is about $3 \times 10^{-17} m^2$.

So, this is your peak cross section. And the spectral width of this one is about 1.5 gigahertz, that, that results in. And the spontaneous lifetime is 300 nanoseconds, is what we have. And the stimulated one, the stimulated one, let us say the stimulated one is about 30 nanoseconds. So, you can see here the stimulated lifetime is rather small compared to the spontaneous lifetime, which is natural. The spontaneous lifetime is natural life time but this stimulated one is from an external one, which can be always be shorter.

So, this is what is going to dictate your line width. So, whatever comes on top is, is something that, that you have to live with. So, similarly you can, find these kinds of parameters for various laser materials. So, this is for helium, helium neon. So, you could also have for semiconductors. So, for a semiconductor the values are, pretty broad. So, your wavelength here could be anywhere between, 370 nanometers to about 1550 or 1650 nanometers. So, your capture cross section here, the peak cross section is going to be much larger.

So, 1 - 5, 10^{-20}. So, this is rather small cross section area and that is a reason why you have the spectral width which is also pretty large. And this results in our spontaneous lifetime, if it is larger than your lifetime has to be smaller. So, you are talking about nanoseconds and you are stimulated processes also in nanosecond. So, you can see here the time constants. So, semiconductors are much better in handling the spontaneous decay rate. So that we could have, a better spectrum when it comes to the generation of wavelengths that we have.

So, with that, we have now understood, the transition, different transitions we have. And how to calculate this line widths, and what is the implication of this transition rates on our final line width. So, in summary, it is, it is the line width that is important and also your capture cross section your capture cross section, make sure that, your capture cross section has to be larger while your band width or the line width that is required to be shorter.

So, this is the relation that you have, I mean, even when you look at the, the spontaneous lifetime if you have longer lifetime, your capture cross section will be smaller. And that is the reason why, in a semiconductor, your spontaneous emission rate is in few nanoseconds. While in, in gas lasers and other things, it is much larger, in few tens or hundreds of nanoseconds, which makes the capture cross section reasonably large.

So, these are all the relations that you could, find out from a different material and then take a call on whether this material is suitable for your kind of application, when it comes to the spectral shape and the emission rates and so on. So, going forward in the next class, we will look at even deeper into this absorption and emission. So, we understand all these dynamics inside but now, we need to generate photons and this generation, generated photons should act as a, as an amplifier to an incoming signal. So, this means you need to have absorption in the material and then emission in the material in a stimulated fashion. So, let us look at that in the next class. Thank you for your listening.