Photonic Integrated Circuit
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Lecture 37
Waveguide Modulator

Hello everyone. Let us look at, how we can exploit our understanding of this anisotropy on the relation between the dielectric constant change to our electric field which is related by our electro-optic tensor. So, let us take one material system and develop the relation between the electric field and the change in the refractive index and also how much field is required, how much power would you need in order to create a certain amount of intensity change or phase change that you are looking at. Let us look at that.

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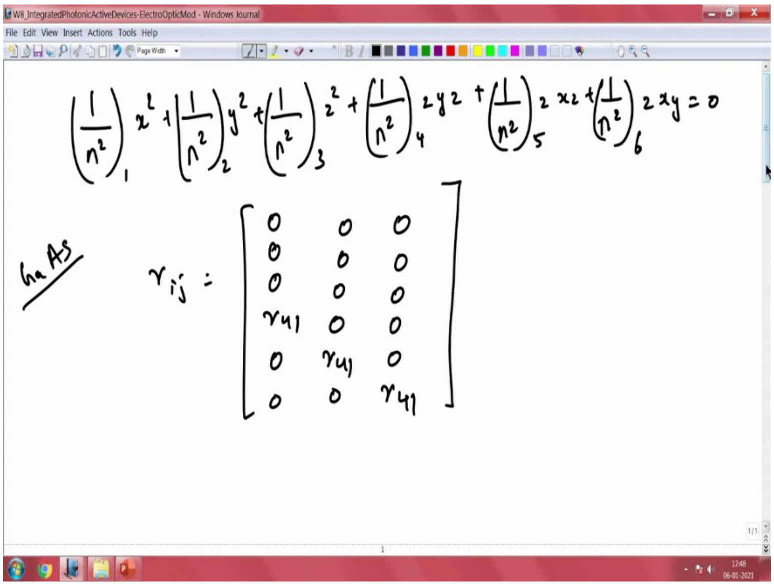

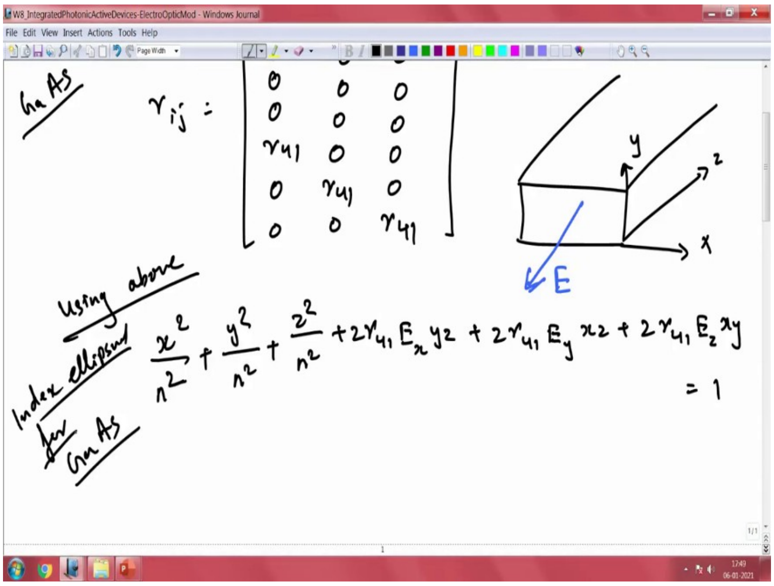

So, I would like to start from our two equations, the two important equation that we had in our Pockel's discussion which is given by the modified index ellipsoid here. So, that is the index ellipsoid with the various factors here. So, let us write the factors there. So, this is of the primary equation and the next equation for a material in this case for gallium arsenide is what we are looking at. So, here the tensor looked 1, 2, 3, r_{41}, 0, 0, 0, 0, 0, 0, r_{41}, 0, 0. So, this is for the gallium arsenide as a material. So, let us take this gallium arsenide as an example and then see how we could understand the electro-optic effect in this.

So, let us take a simple waveguide made out of gallium arsenide in this case. So, the axis here are this is x, and this is z, and this is y, and here the electric field, the electric field is along the propagation direction. So, this is where your electric field is. So, using these two equations that we have, using above equations we can modify it and write it in this form. So $\frac{x^2}{n^2}+\frac{y^2}{n^2}+\frac{z^2}{n^2}+2r_{41}E_x yz+2r_{41}E_y xz+2r_{41}E_z xy=1$.

So, this is our index ellipsoid. So, this is index ellipsoid for gallium arsenide material. So, we have already seen what is the 'r' coefficient and also ordinary and extraordinary refractive index is something that we know of. So, the first three terms are independent of applied field you can see that. So, those three fields are independent of the applied any field.

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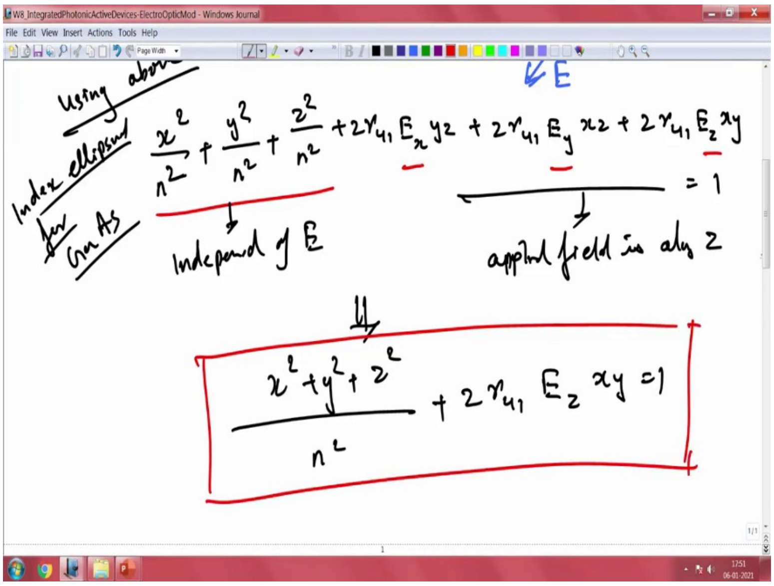

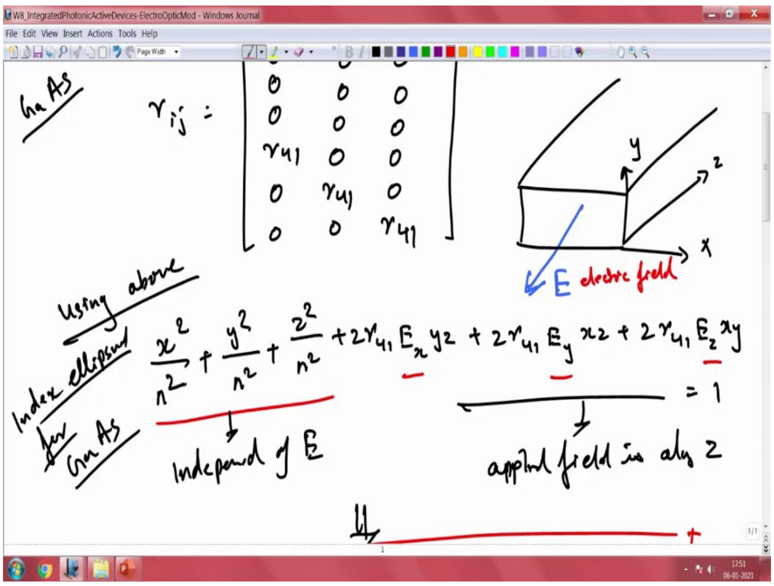

So, these three, there is no electric field involved. So, while the other three factors got electric field. So, now the field is in z direction. So, since the field is in z direction, we can actually deduce this to even simpler form. So, now there are two things, this is independent of this field and then the applied field, applied field is along z. So, that is something that we know. So, because of these two conditions the equation will become even simpler now.

So, we can simplify this $\frac{x^2+y^2+z^2}{n^2}+2r_{41}E_z xy=1$. So, this is our index modified, index ellipsoid when the field is along z direction. So, that is very important. So, we are propagating light along z direction but, then your field is also along z direction. So, this is your electric field, the applied electric field is along this direction. So, this is something that we have.

So, you can see here it has the cross term, the mixed term here that is present that means presence of an electric field along z direction is in the Cartesian coordinate system x, y and z are no longer any principal axis here. So, it is not going to be affected. So, if that is the case let us look at how our index ellipsoid will have when there is no field and when there is field.

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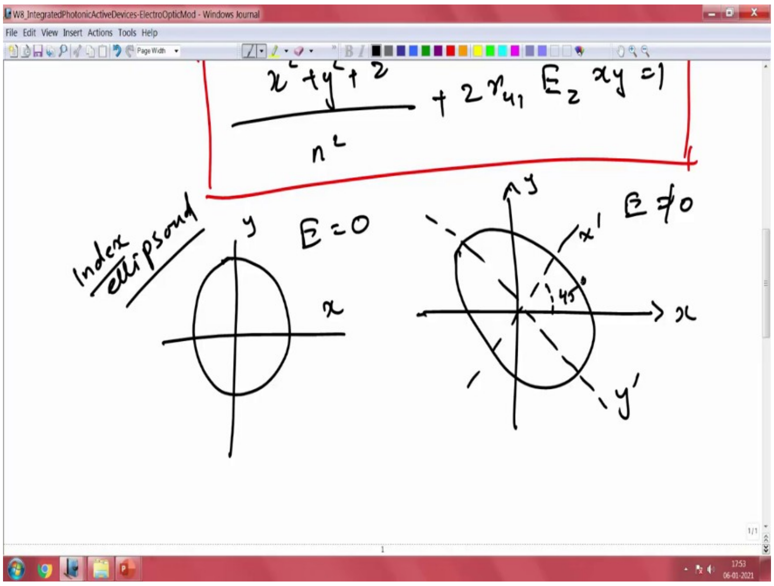

So, when there is no field you will have a circular one. So, that means along x and y when the field is 0. So, even there is no field you will not have any skewness or so on. So, this is nothing but index ellipsoid. So, now when you apply a field, that is very interesting thing is going to happen, when there in the field is non-zero the circular or the equal refractive index or the field is going to be seeing equal interaction, is going to be skewed now.

So, what you will see is a elliptical one with a modified x and y. So, you have x and y here. So, you will have x' and now you have x'. So, now the coupling term can be removed. So, by using this diagonalization. So, when you do this diagonalization, when you apply a field then you have this new coordinates, principle axes that is x' and y' and this is how it looks like, its rotates about 45 degrees. So, this is how you get this elliptical from this circle which is something we have seen earlier as well.

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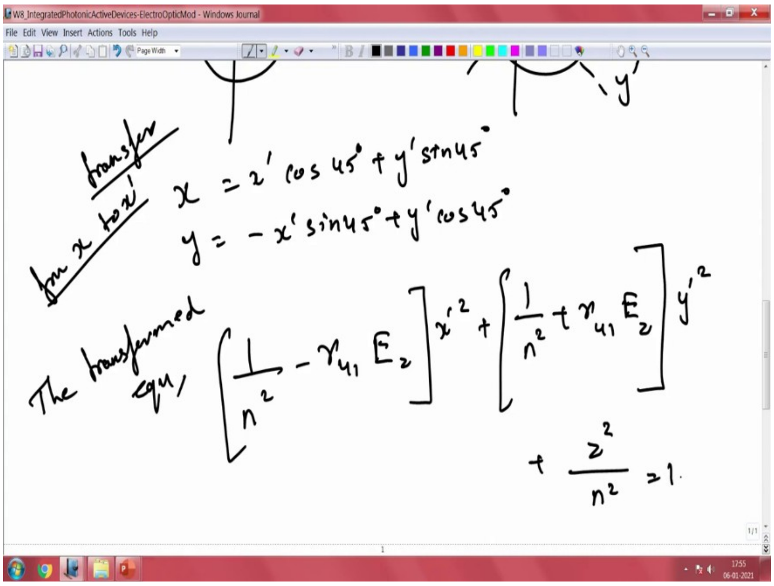

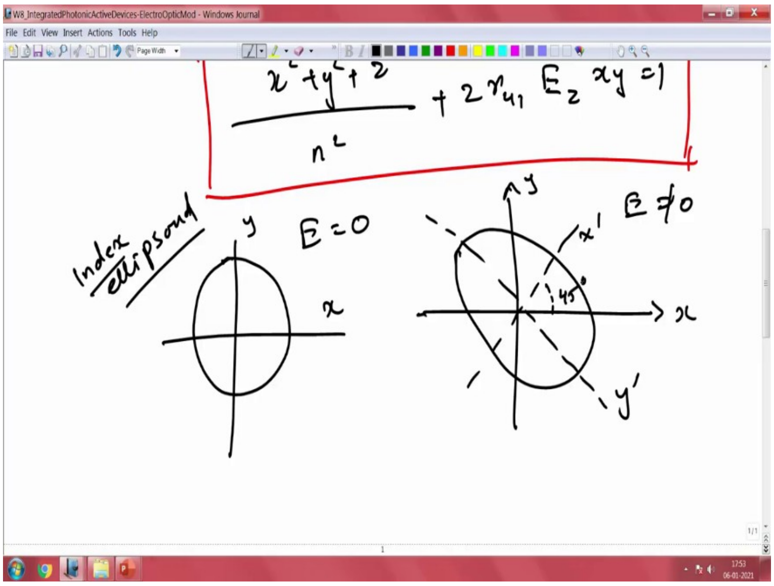

So, now you can substitute this and then transform x. So, you need to transform from x to x'. So, how are we doing that. So, x is nothing but $x' \cos(45^0) + y' \sin(45^0)$. Similarly, y is nothing but $-x' \cos(45^0) + y' \sin(45^0)$. So, this is the transformation. So, transformation of x and x' or x from x'. So, now you can, we can apply this transformation to our ellipsoid here.

So, we have written the ellipsoid equation there. So, we can apply this. Now, the transformed from. So, now the transformed equation will be $\left[\frac{1}{n^2} - r_{41} E_z\right] x'^2 + \left[\frac{1}{n^2} - r_{41} E_z\right] y'^2 + \frac{z^2}{n^2} = 1$. So, this is our transformed equation now. So, from this one, so, we should be able to write what is our refractive index change here.

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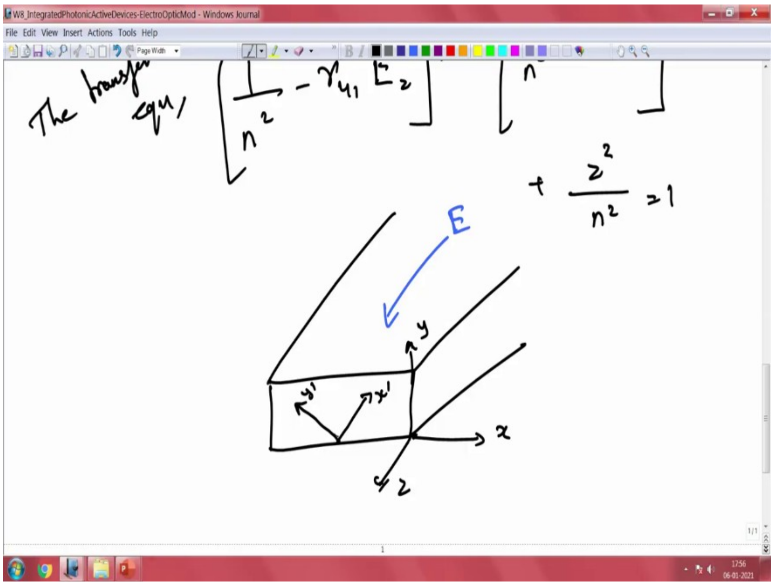

So, let us draw the waveguide again that we had. So, this was x and, this was z, and this was y. So, our electric field again is along this direction. So, this was E. So, now this was x and y. So, we have rotated these things. So, that means your x' here and this is y'. So, this is the change that we have right now and now the question is how our refractive index along this x' and y' is going to be.

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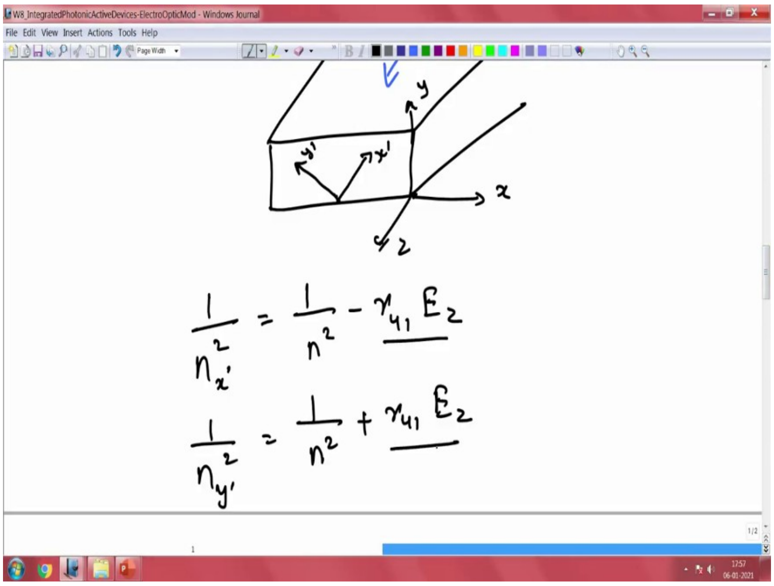

So, this is what we are interested in. So, $\frac{1}{n_{x'}^2} = \frac{1}{n^2} - r_{41}E_z$. So, this is our refractive index or the effective index of refraction along x axis and then we can do the same for y' as well. So, what is our along the modified or transformed y is 1 $\frac{1}{n_{y'}^2} = \frac{1}{n^2} + r_{41}E_z$.

So, this is interesting. So, we have seen that the refractive index is now modified based on the field that you apply to. So, these two are the factors that is changing our refractive index but then whether this particular term is large or small is the question. So, most of the time this $r_{41}E_z$ component is rather small compared to 1/n.

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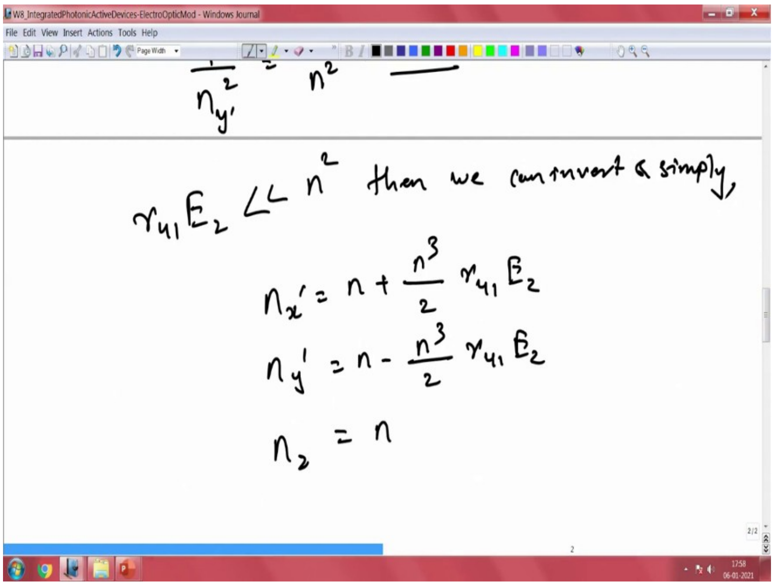

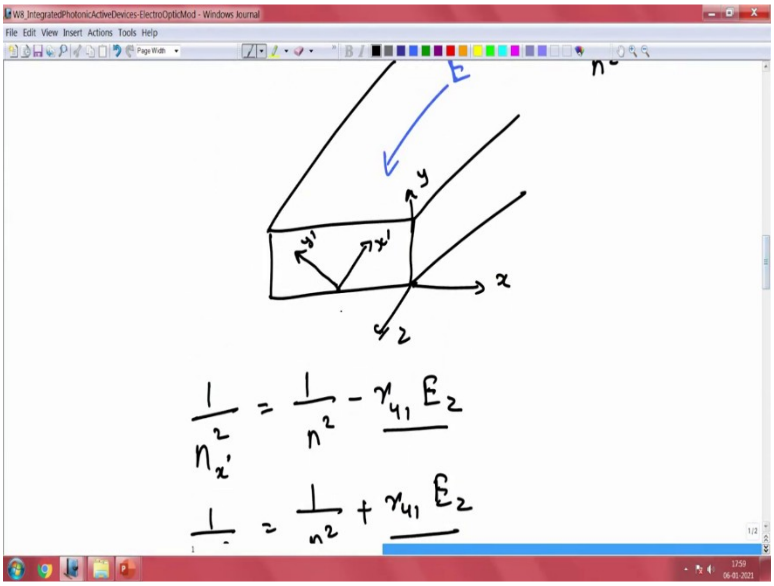

So, this $r_{41}E_z$ is smaller than n , let us say. So, if this is the case then we can invert and simplify the above equation. So, we can simplify this as $n_{x'} = n + \frac{n^3}{2} r_{41} E_z$ and $n_{y'} = n - \frac{n^3}{2} r_{41} E_z$ and finally $n_{z'}$ or in this case n_z we are not transforming it, it will just stay as n.

So, now we see that the principle indices here are linearly modified by the applied field. So, that is an interesting thing here. So, we are when we look at this, there is nothing much difference between these two when you have an electric field then it in the same proportion it will change along x and it will also change along y.

So, we need to understand how this particular index change is going to affect to the propagation so far we just looked at the cross section here what will be the change in the refractive index is something that we have understood now, but then how is this going to affect the propagation. So, that is something that we should try understand, let us look at what happens when the light starts propagating.

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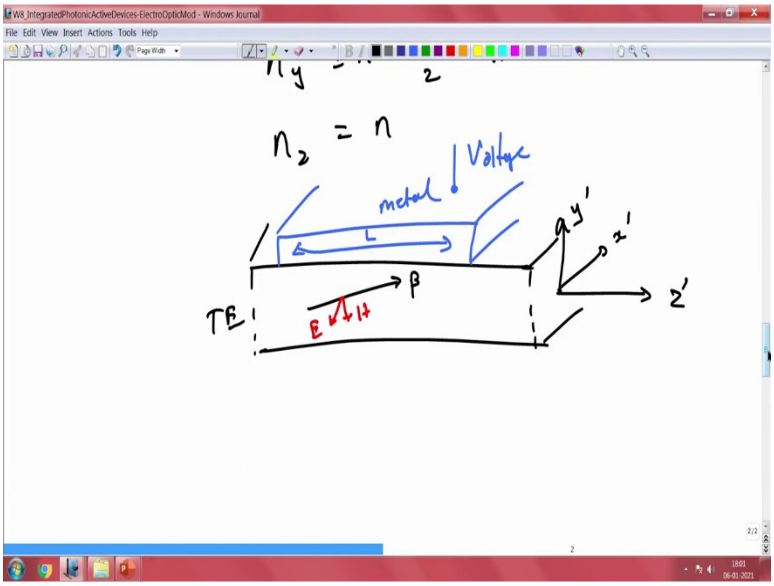

So, for that we should see a cross section of this let us take the film here, the light would propagate along this direction and we have y here and this is our x. So, this is basically x' and z' in this case. So, now we have a light that is propagating through this medium and it has a certain β. So, light is propagating through this and it has certain β and we have our H field and then we have our E field defined here. So, now this is all fine but, then we need to apply electric field.

So, this is your wave guide. So, we will apply an electric field here. So, you can extend these things that means they are it looks like they are thin films, this is propagating along this direction. So, now this is our metal. So, this is our metal electrode and here we are giving some voltage. So, we are applying some voltage and there is a length associated with this, there is a length L where you have this one.

And we are launching a mode in this case, let us say, TE, TE mode is launched into the system which is going through. So, now the question is what will happen to this particular mode when I apply an electric field. So, we know what is going to happen, when the light is propagating through the medium it will have a certain beta and whether this beta is going to change if, we want that β to change the phase shift that we expect it to change.

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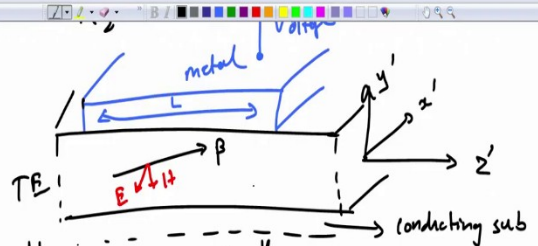
→ conducting sub

phase shift → $\Delta\beta = \Delta n_{x'} k_0$

$$\Delta\phi = \Delta\beta L = k_0 L \Delta n_{x'}$$

$$= \frac{2\pi}{\lambda} \frac{L n^3 r_{41} E_y}{2}$$

$$\Delta\phi = \frac{\pi L n^3 r_{41}}{\lambda} E_y$$



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$$= \frac{2\pi}{\lambda} \frac{L n^3 r_{41} E_y}{2}$$

$$\frac{1}{n_{y'}^2} = \frac{1}{n^2} - \ldots$$

$r_{41} E_z \ll n^2$ then we can invert & simplify,

$$n_{x'} = n + \frac{n^3}{2} r_{41} E_z$$

$$n_{y'} = n - \frac{n^3}{2} r_{41} E_z$$

$$n_z = n$$

So, the phase shift, the phase shift the Δβ is nothing but $\Delta n_{x'} k_0$ is our change in the propagation constant. So, now what will be the phase difference that you expect. So, this is your propagation constants difference what is should your phase change it is nothing but Δβ as a function of L.

So, now we know that this is nothing but $k_0 L \Delta n_{x'}$ and we know this $n_{x'}$ what it is here. So, we know what is $n_{x'}$. So, let us bring that $n_{x'}$ here. So, this will become $\frac{2\pi}{\lambda}\frac{L n^3 r_{41} E_y}{2}$. So, y is Ey here, because the field is vertical. So, you are having a top electrode here and then the field and then you also have. Let us say, at the bottom you have a conducting substrate here.

So, that is why you have the field in the transverse direction. So, that is why you have Ey here. So, once you have this, this field here then we know that this is the phase shift that we are getting. So, you can try to simplify this. So, once you simplify, the 2 will be gone, it is basically $\frac{\pi L n^3 r_{41}}{\lambda} E_y$.

So, you can see here the phase shift is directly proportional to the length that you have. So, what is the how long it is. So, if you want to have larger phase shift you can increase the length and you can also increase your field that you have. So, for a longer device you need lesser voltage. So, to get the same amount of phase shift.

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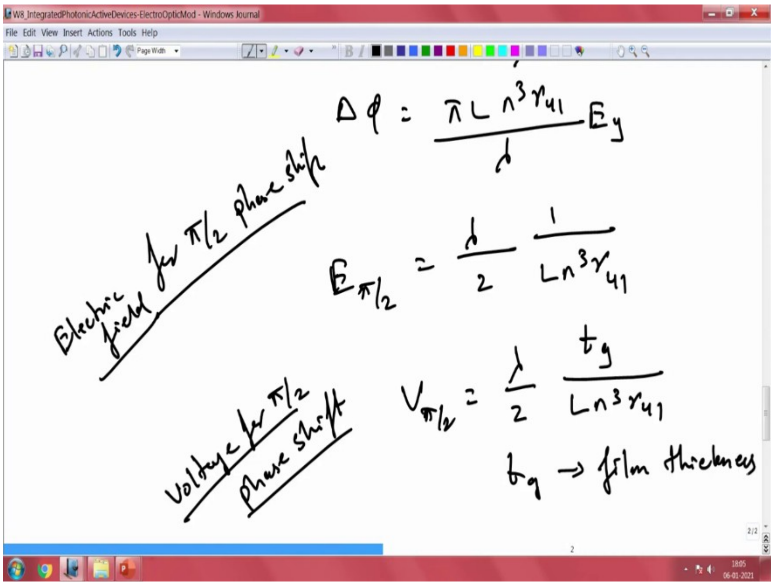

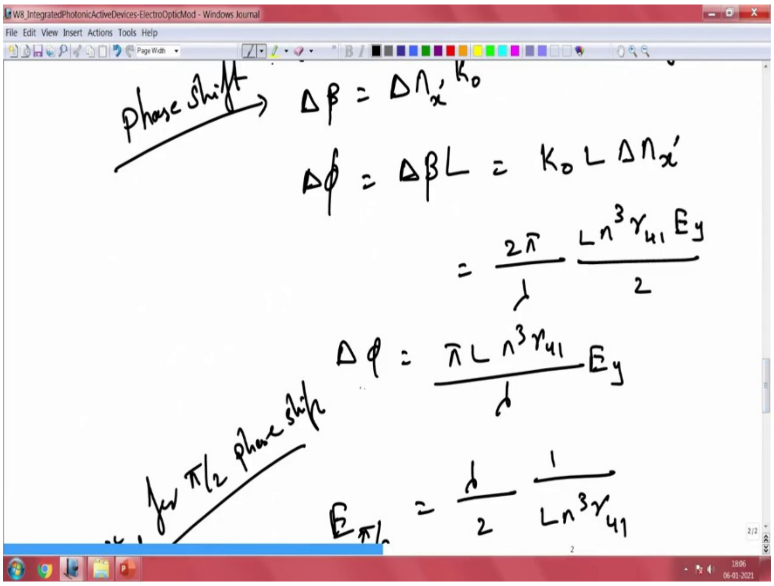

So, what is required to get $\frac{\pi}{2}$ phase shift. So, for $\frac{\pi}{2}$ phase shift how much is required. So, the voltage required to make $\frac{\pi}{2}$ phase shift or let us say here, electric field for $\frac{\pi}{2}$ phase shift is nothing but $\frac{\lambda}{2}\frac{1}{Ln^3 r_{41}}$ and this is the electric field. So, this is electric field. So, what is the voltage required, voltage required for $\frac{\pi}{2}$ phase shift. What is the voltage required here?

So, that is $V_{\frac{\pi}{2}}$ which is given by again $\frac{\lambda}{2}$ the only difference is there will be a thickness factor $\frac{t_g}{Ln^3 r_{41}}$. So, this t_g is nothing but film thickness. So, this thickness will help you to do this. So, now let us just assist this scenario here, where the phase shift is directly proportional to the length and the electric field we have.

So, if you want to increase your phase shift, more often that is what you are trying to do, there are two ways to do it. So, you can up, you can make a very long interaction, or you can increase your field. So, most of the time, we do, we try to keep the device power efficient and also small in size. So, if you make it L longer you will have large device once you have large device then your E_y goes down the requirement for voltage goes down.

So, you can make it power efficient, but then the device will be larger, then if you have enough power to dissipate then you can have this shorter length. But another important thing is r_{41} here. So, this tensor that is sitting here in this case it is r_{41} in some other material it could be r_{13} or r_{33} depends on the material that you work with.

For example, if you take lithium niobate for example. So, this will be r_{33} or r_{13} depends on whether you are in ordinary or extraordinary axis. So, in that case the factor here, the r_{41} or r_{21} factor will decide the phase shift that you get based on L and E_y. So, that other factor that you can choose in this r_{41}. So, r_{41} is the strength or the degree of the electro-optic coefficient or electro-optic strength if the material has high electro-optic strength, then you need shorter modulators with very low power.

So, that is the reason why we always look for material that has larger r coefficients. So, larger r coefficients are always preferable and that is one of the reason why we use lithium niobate which is, which has larger and there are other materials as well. But lithium niobate is a relatively widely used compared to other material for exactly this particular reason.

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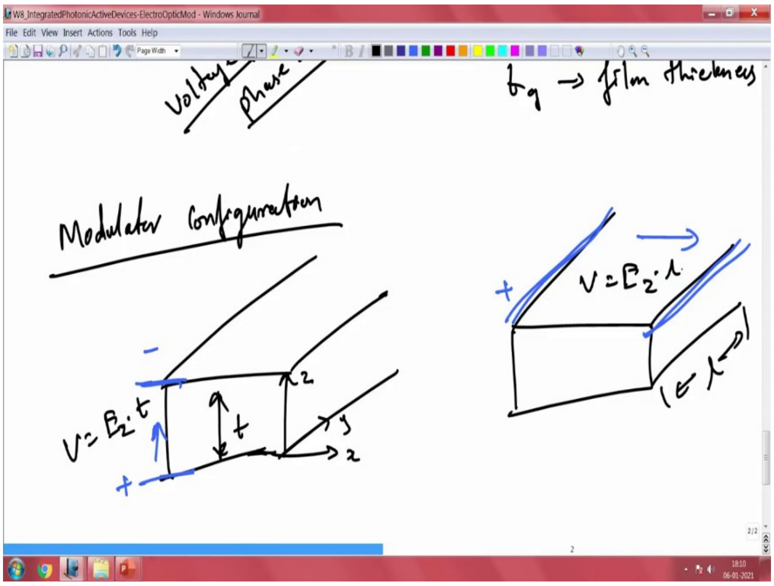

And looking at the configuration itself. So, there are two ways as I mentioned you can excite this. So, let us look at the modulator configuration. So, one way to look at is you could take a piece of the anisotropic material and you can apply a field in transverse. So, you can have minus, plus. So, in this case your volt, let us say this is your x and this is your z and this is your y let us say, sorry you need to, this is your y. So, in this case your voltage that you apply is nothing but E_z times let us say, thickness t. And you can do, something different here.

So, you can take the piece and then apply field between these two levels, between these two. So, you have positive and negative here. And now, the field is in this way in this case the field was in this, this direction. So, this is your E field and in this case you will have a field along the length of this r along the breadth of this. So, here you have a complete length that is available to you. So, this is the total length you have and in this case your voltage will be E_z times l. So, you can already see which is the better configuration when it comes to the efficient use of voltage in order to get what you exploit.

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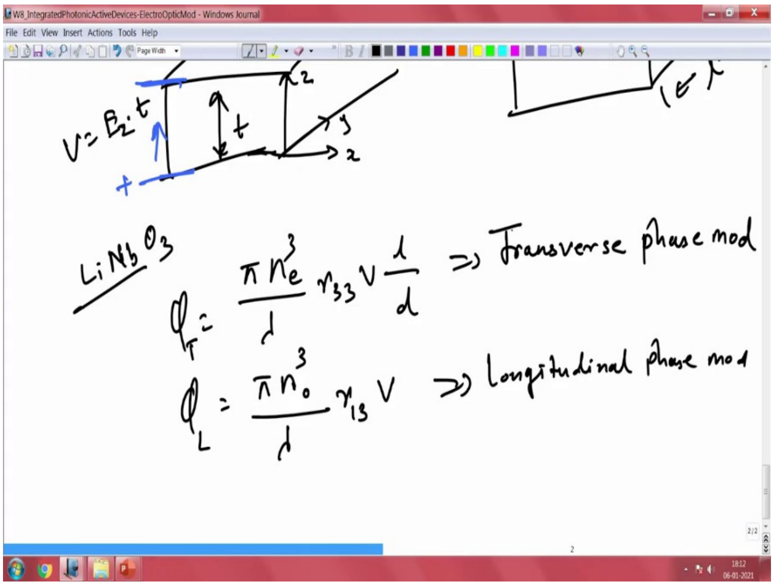

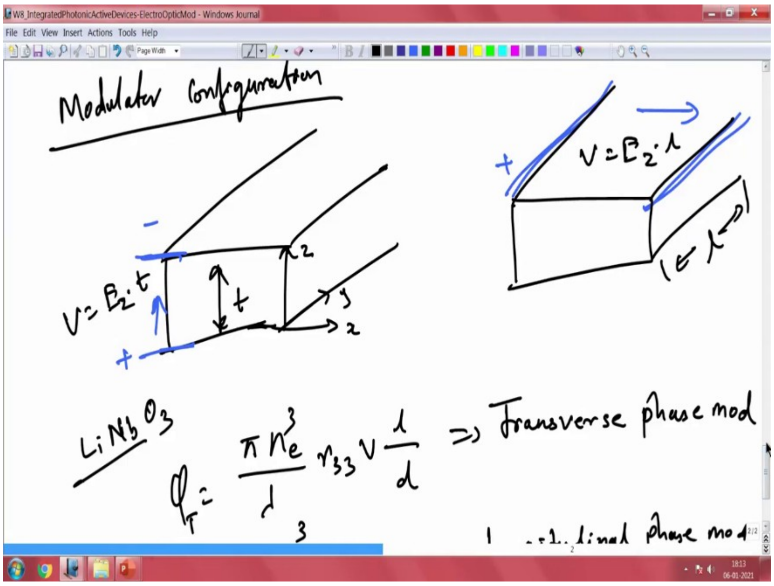

So, for lithium niobate in this particular configuration, for lithium niobate you could have the phase shift as follows; so $\frac{\pi n^3}{\lambda} r_{33} V \frac{l}{d}$. So, this is your phase shift that you get when you have transverse phase modulation. So, this is for transverse phase modulation. And when you have longitudinal, so, your longitudinal phase, so, I can put φ_T and φ_L, the $\varphi_L = \frac{\pi n^3}{\lambda} r_{33} V$.

So, the important thing to notice here is this n. So, for longitudinal component it should be n_o and in your transverse component it will be n_e. So, this is something that you should keep in mind when you are choosing the right orientation, whether it is ordinary axis aligned or extraordinary axis aligned. So, that is something that you should be careful when you are choosing the right orientation. So, this is for your longitudinal, this is transverse modulator. So, so far we have looked at these modulator configurations.

So, how to use or exploit the anisotropy that we have in the material and we can make a thin layer of this and either you can put the electrodes on top or you can put the electrodes on the side, it is all about exploiting your electro-optic tensor here. So, make sure that you we select the right orientation of your electro-optic coefficient, so that you can exploit it.

And also applying the field in the right orientation, both would work, it is not that one will not work than the other. So, both would work but then their efficiencies are going to be different because of the magnitude of the r coefficients we have and also the voltage requirement that you have. So, when you look at the thickness it is basically field times thickness. But then when you have electrodes in the field times length that you have.

So, we have larger or longer interaction length. So, that is the reason why when you have a longer device the voltage requirement goes down. So, with this understanding I think we are ready to design any modulator, any guided wave modulator based on these coefficients that we have here. But we will look at a little bit more later on in the course about exploiting these properties and how they look at. For the moment, thank you very much for listening.