

**Photonic Integrated Circuit**  
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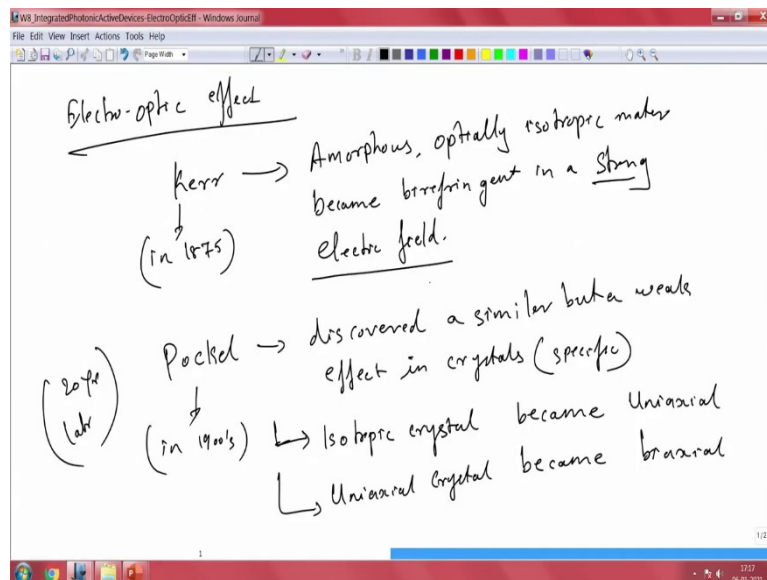
**Lecture 36**  
**Electro-Optic Effect**

Hello everyone. So, let us look at the light interaction with the medium that would create Electro-Optic Effect. So, we have briefly discussed this in the early part of the lectures, when we talked about light interaction in anisotropic medium. So, where we looked at the index ellipsoid, how the refractive index in different coordinate system is going to influence the light flow, particularly, when the light is either linearly polarized or circularly polarized.

So, based on this anisotropy you could make light propagate or change its propagation vector and also change the polarization as it propagates through. So, this is relatively passive process as it propagates through the material, based on the anisotropy you can go from circularly polarized to an elliptically polarized light or you start from an elliptically polarized light and then you can make a linearly polarized then elliptical again.

So, these are all the different polarization states you can achieve by using anisotropic medium. But now, the question is, is it possible to have external electric field influence the change here? So, we are going to, we are not going to make it long enough and then just passive. So, passive meaning your actions or your function is fixed based on the length and the dimension. And now, I am not going to change the length. So, length is fixed but, I am going to apply an external field, whether this field will help me to make this conversion possible. So, that is the idea here but, this idea is not very new, let us say.

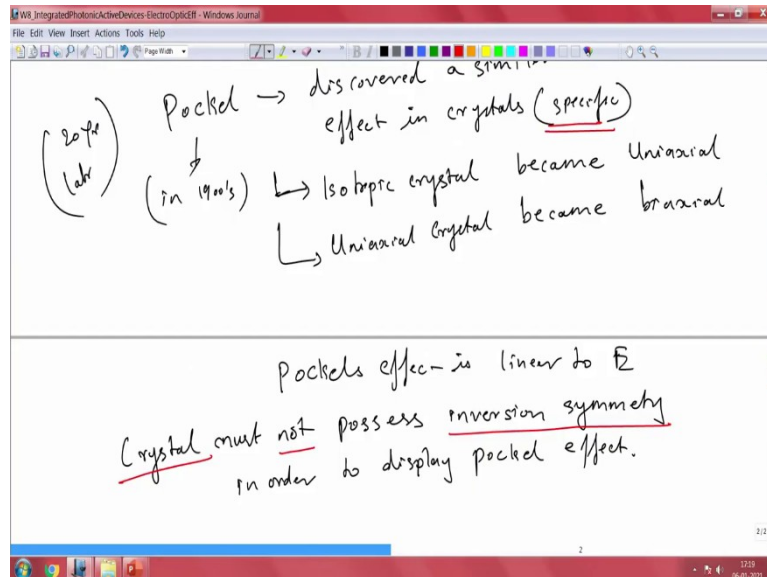
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So, this was already discussed a long time ago, this whole electro-optic effect by Kerr, who discovered that even amorphous material, an amorphous optically isotropic material became birefringent in a strong electric field. So, this was already something that we know a long time ago, this was in 1875. So, this whole idea of having this property change when you have an electric field was known. Whether it was not long that Pockel discovered a similar kind of effect. But, a weak effect in crystals, specific, in specific crystals. What was the discovery here? This discovery, this is 20 years later let us say.

So, it was around the 1900s. So, what he found was an isotropic crystal became a uniaxial crystal. So, there was one finding and then the next finding was if you take a uniaxial crystal it became a biaxial. So, this is all by applying an electric field or by inducing changes in the material by applying a field and this Pockel's effect unlike Kerr. So, in Kerr effect you need very strong optical field where it was quadratic relation. But, in this case Pockel's effect is linear. So, that is an interesting thing here.

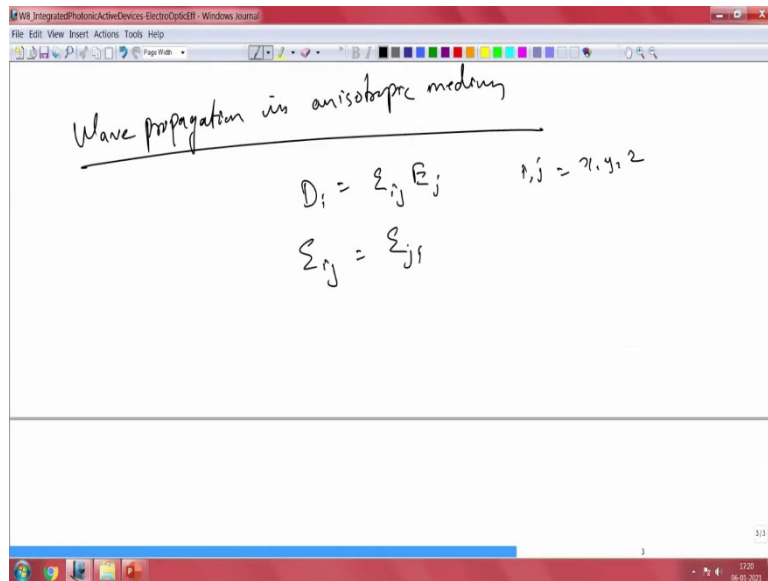
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So, Pockel's effect is linear to the electric field that you have and this is not universally found. So, that is why I mentioned here it is only on specific crystals not on all the crystals that was that is available. So, we have many different type of crystals available. But, this particular property is not available in all these crystals. So, these are only a certain class of crystals that shows this Pockel's effect or this electro-optic effect. And what all those, what is the condition here? The condition is that the crystal must not possess inverse symmetry or inversion symmetry in order to display Pockel's effect.

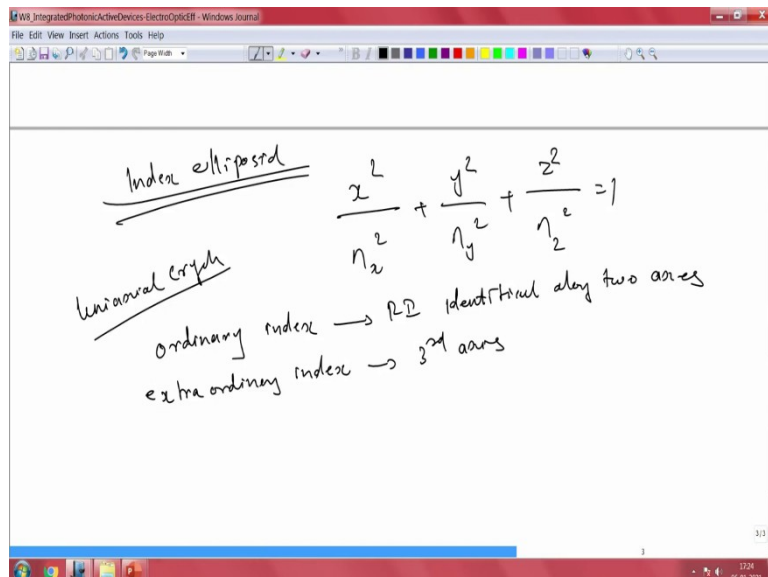
So, the crystal, it should be a crystal should not possess inverse symmetry. So, the material that we know most of them they are symmetric in nature for example silicon is one good example, it has cubic symmetry. So, you will not have Pockel's effect in a crystalline silicon. So, you need to have an anisotropy or in this case you should have a non-inversion symmetry to this material. So, we have already seen this earlier. So, what this anisotropy is going to bring.

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So, if you look at the wave propagation in an anisotropic medium, your displacement vector is a function of the dielectric constant that you have. So, your  $i, j$  is nothing but your coordinates  $x, y, z$ . So, one of this. So, your  $\epsilon_{ij}$  is nothing but  $\epsilon_{ji}$ . So, this is from the conservation theory. So, this is all just a recap of what we have already read and seen and so on. Now, we need to understand how this vector is going to look like when you say this is anisotropic, and that is something we all know, the matrix or the index ellipsoid that we already looked at in the earlier lectures.

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So, that is the index ellipsoid, when you have this anisotropy can be given by  $\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$

So, this is our index ellipsoid. So, we can always find the effective index here. So, what is the effective index of this medium and depending on the propagation direction and so on, we should be able to find the two dominant directions that the wave is going to propagate.

So, we have two axes here. So, we have ordinary index and then you have extraordinary index. So, what is ordinary index, ordinary index is nothing but, where you have in a

uniaxial a crystal, where you have two axes with identical refractive index. So, the refractive index identical along two axes here and the third axis, this is the third axis is called the extraordinary axis.

So, this is primarily in the uniaxial crystals. So, in biaxial crystals you will have three unique refractive indices. So, that is something that we should take care of. So, in this case the ordinary index is where you have two indices that are identical while in the extraordinary which is a different one. So, you have two set of refractive indices. Then if you have all of it (identity), unique then we call that as a biaxial crystal. So, when you take these crystals we have drawn this index ellipsoid and so on.

So, based on the wave propagation along one of these axes, based on the magnitude of  $n_x$  and  $n_y$  you will see the polarization influences the propagation here. So, now the question is how do we bring in the electrical component here. So, because we do not know how the electric field is going to influence this. So, what is the relation between electric field and the material property that we have. So, the way that one could easily understand this is by expanding the index ellipsoid that we have here. So, let us look at the Pockel's here.

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Pockel's effect

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_2 yz + \left(\frac{1}{n^2}\right)_2 xz + \left(\frac{1}{n^2}\right)_{2xy} = 1$$

$\left(\frac{1}{n^2}\right)_i \Rightarrow$  dielectric tensor terms

Crystal must not pass in order to display pockel effect.

Wave propagation in anisotropic medium

$$D_i = \epsilon_{ij} E_j \quad i, j = 1, 2, 3$$

$$\epsilon_{ij} = \underline{\underline{\epsilon_{ji}}}$$

1. ellipsoid 1 2 z<sup>2</sup>

So, the Pockel's effect is nothing but, change in the refractive index when you apply a field. So, that is what it is, but let us look, at the index ellipsoid and slightly modify it to our liking because, if you look at the  $\epsilon$  here. So, the  $\epsilon$  can take six different values here. So, there are six possible values I should not say six different, there are six possible terms that you can have here. Similarly, you can write our index ellipsoid with a different orientation here. So,

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 yz + \left(\frac{1}{n^2}\right)_5 zx + \left(\frac{1}{n^2}\right)_6 xy = 1.$$

So, the  $\left(\frac{1}{n^2}\right)_i$ , so this is something that you can represent the, what we call the dielectric tensor along the different direction here x, y and z here. So, this is all nothing but the dielectric tensor, is nothing but the dielectric tensors. So, this is the dielectric tensor terms and if x, y and z are the three different Cartesian coordinates here. And now, given the fact that these are the coordinates that we have. Now, we can write our electro-optic coefficient here.

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Electro-optic tensor

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \vdots \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

Pockel's effect

$$\left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + \left(\frac{1}{n^2}\right)_4 yz + \left(\frac{1}{n^2}\right)_5 zx + \left(\frac{1}{n^2}\right)_6 xy = 1$$

$\left(\frac{1}{n^2}\right)_i \Rightarrow$  dielectric tensor terms

So, what is that electro-optic coefficient? So, now this is the refractive index the six refractive indices there along different directions and there could be a change in this refractive index.

So, there is  $\Delta\left(\frac{1}{n^2}\right)_1$ ,  $\Delta\left(\frac{1}{n^2}\right)_2$ . Similarly, you can write  $\Delta\left(\frac{1}{n^2}\right)_6$ . So, these are all the terms that will be influenced. What we are trying to understand is what is the change in this terms when you apply a field. And that coupling term is nothing but our electro-optic tensor.

So, there is dielectric constant and you have your electro-optic tensor here. So, we represent it by this matrix where you have  $r_{11}$ ,  $r_{21}$ ,  $r_{31}$ ,  $r_{41}$ ,  $r_{51}$  and  $r_{61}$  and then we have  $r_{12}$  and  $r_{13}$  and you can write rest of the factors this will be  $r_{63}$ . So, this is electro-optic tensor and you are operating it with a field. So, the field is  $E_1$ ,  $E_2$  and  $E_3$  along the three different axes. So, the unlike the dielectric tensors, even if the axes are misaligned or aligned along the principle axes the cross product terms, the cross product terms here, the 4, 5 and 6. So, there three are the cross product terms may not, will not be 0.

So, what that means is when you align it to the right optical axes that means when the light is propagating through that axis you will not find any difference to the propagating wave when you apply electric field or it will not be affected. So, those factors become 0 but when you have the cross product in this case even when you have alignment in the principle axis the factors here are 4, 5 and 6 will not be 0.

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Cubic sym  
crystal

$$r_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{51} & 0 \\ 0 & 0 & r_{61} \end{bmatrix}$$

$$\begin{bmatrix} \Delta \left( \frac{1}{n^2} \right)_1 \\ \Delta \left( \frac{1}{n^2} \right)_2 \\ \vdots \\ \Delta \left( \frac{1}{n^2} \right)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{32} & r_{34} \\ r_{31} & r_{42} & r_{43} \\ r_{41} & r_{52} & r_{53} \\ r_{51} & r_{62} & r_{63} \\ r_{61} & r_{62} & r_{63} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

So, one could write, for example, even with a crystal of cubic symmetry. A cubic symmetry crystal you will have a matrix like this. So, you will have 0 0 0, 0 0 0, 0 0 0. So, now you have  $r_{41}$  0 0, 0  $r_{41}$  0, 0 0  $r_{41}$ . So, now with this tensor, so this is your electro-optic tensor, we find this it is non-zero that means this particular material should be able to respond to an applied electric field. So, when you have a 0 diagonal then whatever direction you apply this field to, the resultant will be 0.

So, you will not have resultant change. So, your  $\Delta$  here you have will be 0. So, what you are looking at is the scenarios where your  $\Delta$  here, so the dielectric tensor here should be non-zero. So, that means you should have non-zero electro-optic tensors here. And there are ways to achieve this by also picking the right orientation and so on. But this is some material property, so you cannot do much to these 'r' coefficients.

But, what you can do is you can align your field accordingly. So, we have  $E_1$ ,  $E_2$  and  $E_3$ . So, based on this orientation of this field you can take advantage of the non-zero factors that you may have within this. So, this is  $r_{32}$ , ( $r_{34}$ ) ( $r_{34}$ ). So, based on their position you should be able to exploit it and as I mentioned this is a material property. So, I am going to take two materials of that are of interest that we will you do some examples with it. So, that you understand it well.

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LiNbO<sub>3</sub> (10<sup>12</sup> mV)

$r_{13} = 9.6$	"	$n_o = 1.8830$
$r_{22} = 6.8$	"	$n_e = 1.7367$
$r_{33} = 30.9$	"	
$r_{51} = 32.6$	"	

GrAs

$r_{13} = 4.1$	$n_o = 1.8830$
$r_{41} = 1.4$	$n_e = 1.7376$

Electro-optic tensor

$$\begin{bmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \Delta\left(\frac{1}{n^2}\right)_2 \\ \vdots \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{32} & \\ r_{31} & r_{34} & \\ r_{41} & & \\ r_{51} & & \\ r_{61} & & r_{63} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

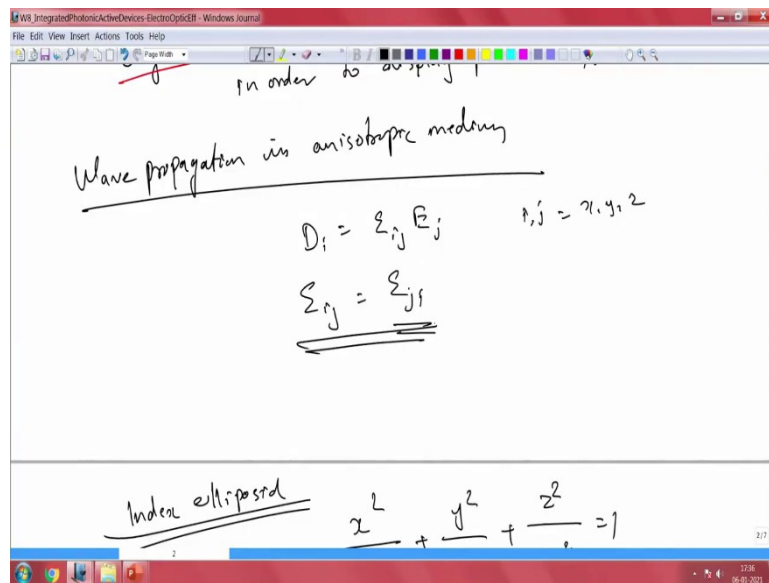
Crystals with Inversion Symm will have all  $r$  coeff identical to zero

Cubic Sym Crystals

$$r_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{41} \end{bmatrix}$$

⇒ Unlike dielectric tensor, even if the axes are aligned along the principal axes, the cross terms ( $r_{41}, r_{51}, r_{61}$ ) are not necessarily zero





So, let us look at those materials, one is a well-known material called lithium niobate. So, lithium niobate is a well-known electro-optic material. So, let us look at its tensor. So, that the tensor for this particular material this is all the non-zero tensors that it has. So,  $r_{13}$ ,  $r_{22}$  here is 6.8 and  $r_{33}$  is 30.9 and then  $r_{51}$  is 32.6. And the ordinary refractive index is 1.8830 this is uniaxial one and this is 1.7367 and this  $r_3$  coefficients the unit is  $10^{-12}$  millivolts. So, for all of this.

So, this is for lithium niobate let us look at another famous material which is gallium arsenide. So, gallium arsenide has  $r_{13}$  of 4.1,  $r_{41}$  is 1.4 again  $n_o$  is 1.8830 and  $n_e$  is 1.7376. So, this is all done using experimental techniques. So, we use experimental methods in order to measure this electro-optic tensor. So, once you have these values you can take these values and try to exploit this. So, I will go back here and then show you that this is the matrix here.

So, the electro-optic tensor matrix now you can populate it based on the data we have and then, electric field is what you apply and based on this we should be able to calculate what will be the change in the refractive index when I apply a certain field. So, this is about the basic understanding of the material system we have already seen this in a different form. But now, we are going towards utilizing this material property for functional application.

So, let me just write the note here that I just mentioned that, so, crystals with inversion symmetry let us say, crystals with inversion symmetry will have all 'r' coefficients identical to 0, this is a very important understanding from this when for a crystal with inversion symmetry this 'r' matrix becomes 0. So, that means you can apply any field that you want but there will not be any change in the refractive index. So, that is one factor. So, the next one that I mentioned is about the non-zero values.

So, then for the non-zero values let us make a statement here, unlike dielectric tensors even if you change, if the axes are aligned along the principle axes the cross products, the cross terms rather, this case 4, 5 and 6 are not necessarily 0. So, that is an example that I just showed you here. So, you can diagonalize it. So, that you can get most of this factors 0. But this is true when you are doing a dielectric tensor. So, we normally do this, for this dielectric tensor where you could (where is that, yeah), in the dielectric tensor because of this symmetry you can actually make the cross products or cross terms 0, because,  $\epsilon_{ij} = \epsilon_{ji}$ .

But, in this case we cannot. So, they do not. So, this cross product, cross terms will not be 0. So, that is another important note that I would like to mention here. So, with this

understanding of the material properties that we had an index ellipsoid and then how do we compare this index ellipsoid with a system where you have an external field, so, for that we need to bring in our electro-optic tensor.

So, I have the dielectric tensor. So, the dielectric tensor is going to change, but how it is going to change? By applying a electric field. So, what is the relation between these two? This is coming from our electro-optic tensor. So, the electro-optic tensor is the strength of the coupling between the electric field and the dielectric.

So, if the electro-optic tensor is 0 there are materials where this tensor is all 0 as I mentioned some material you will not have this inverse symmetry, so, they will have inverse symmetry and when you have inverse symmetry all these values goes to 0. So, if that is that case you do not have any effect of electric field. But, for materials that has non-inversion symmetry the factors, the electro-optic coefficients here the electro-optic tensor matrix is non-zero.

So, when you have non-zero values even when you tilt the principle axes you can try to tilt it whichever way we you want but, still it will be non-zero and because of this non-zero electro-optic tensor, now I have non-zero values with the electric field. So, that means my dielectric tensor now will have a non-zero value when I have a electric or change in the dielectric tensor will have a non-zero value when I operate it with a field.

So, the change in dielectric constant is what we are trying to look at. So, now we have a fair understanding of how the coupling happens between electric field and our dielectric tensor. So, now let us put this to into use by using this understanding in a guided wave system. So, now we are going to make a waveguide and then understand it which is something that we would continue in the next class.