Photonic Integrated Circuit
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Lecture 32
Micro-Ring Resonators

Hello everyone. So far we have been looking at power handling, so in all the passive devices that we discussed we looked at how do we handle power. So, we had coupling of power between two systems and we also had how to split the power and how to combine the power and how this splitting and combining could be done in spatial way, so you can spatially put the light in one waveguide or the other, you can split it in the ratio that you want, you can do it 1 to 2 splitting or you can split from 1 to n, so all those power handling we had looked at.

But what can we do to handle wavelength? So, I want to have wavelength selectivity let us say, so I want a structure to be very selective to a particular wavelength that I am interested in, so this could be to realize a filter for example. So, I have n number of wavelengths and I want to pick up only one wavelength from that, is it possible to realize in a passive geometry? And the question here is how to achieve that?

So, we have looked at this in a different device but did not emphasis on the wavelength. So, if you remember we discussed about Mach-Zehnder interferometer. So, in the interferometer the phase change causes change in the intensity, so when there is a change in the phase in one of the arms you looked at the intensity change, it is a cosine function.

But then instead of changing the phase we fix everything and when we change the wavelength, the input wavelength, so changing input wavelength is same as changing the phase for a fixed wavelength. So, the phase change can be now replaced with the wavelength. So, the wavelength change through a Mach-Zehnder interferometer will give you the same cosine type transfer function.

So, we do not have to think too much, so we already discussed this, so I am going to ask you to go back and look at the Mach-Zehnder discussion that we had, how did we built the Mach-Zehnder using two y junctions and so on. So, we keep the wavelength structure intact, so we are not changing anything but we will change the wavelength, so when we change the wavelength you will get the spectrum.

And if you are still not sure about it, we will discuss some of this practical device and measurement in the later part when we talk about the functional devices and demonstrations that you can do with photonic circuit. So, now let us look at a slightly different way of implementing this wavelength selectivity.

In your introductory classes in photonics you might have studied something called Fabry-Perot cavity, so Fabry-Perot cavity is a simple cavity where you have two mirrors both the sides and there is a space between these two mirrors and the reflectivity of one mirror could be less than the other in order to take the light out but nonetheless they are mirrors and the mirror is going to reflect light and the light is going to bounce back and forth in this cavity.

So, not all the wavelength can fit into this cavity. So, that is something you also saw, so that is the way of bringing wavelength selectivity. Let us briefly discuss this Fabry-Perot cavity and then build on top of this Fabry-Perot cavity how we can realize a wavelength selective cavity filter which called a ring resonator in a photonic IC based implementation, in particular waveguide based implementation of a ring resonator, let us look at that.

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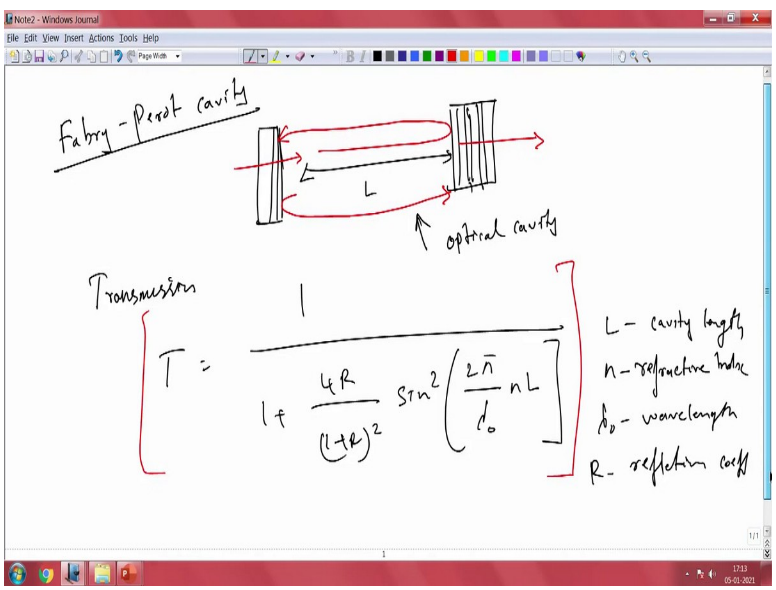

$$T = \frac{1}{1 + \frac{4R}{(1+R)^2} sin^2\left(\frac{2\pi}{\lambda_0} nL\right)}$$

So, let us say you want to build a Fabry-Perot cavity. So, Fabry-Perot cavity is basically having a mirror this side and there is a mirror on the other side, so this is a distributed bragh mirror let us say. And then there is a cavity of length L, so the lights that enters into this cavity will keep bouncing back and forth.

So, it is going to be bouncing back and forth creating what we call optical cavity that you capture the light inside. Now, light is entering so it will also go out. So, what will be the transmission? So, the transmission depends on the reflection that we have and your phase factor. So, L is the length of the cavity and n is the refractive index.

So, L is the cavity length, n is refractive index, lambda naught is wavelength and R is reflection coefficient. So, this is a relation that you must have already familiar with if not you can brush up, so it is already there. So, when you have high mirror reflectivity the smaller is the change in the index required for any change in this particular cavity that you have.

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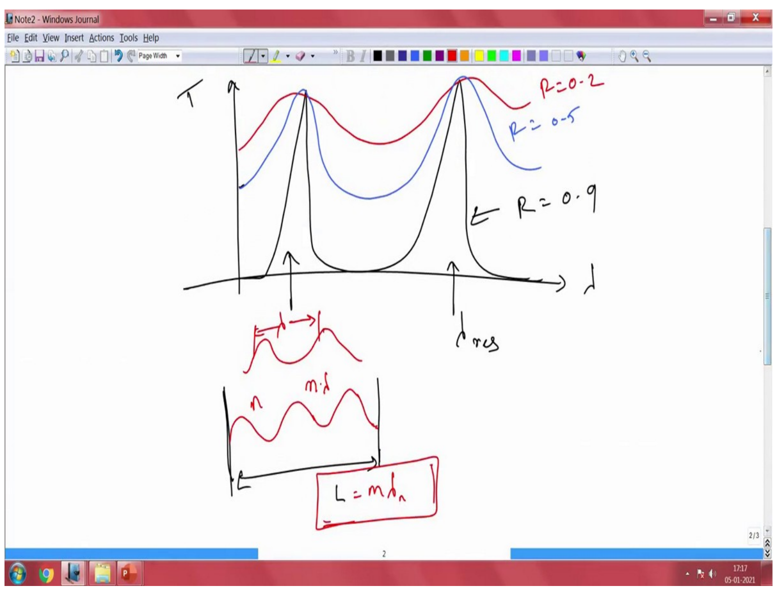

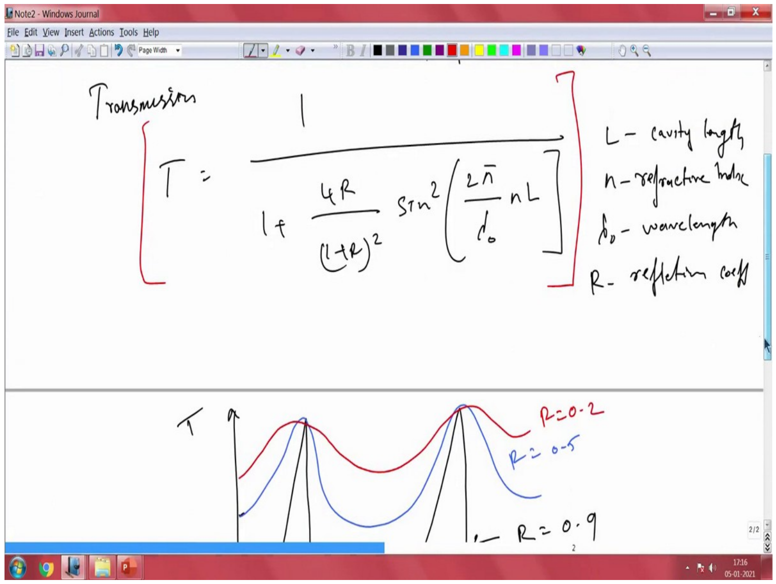

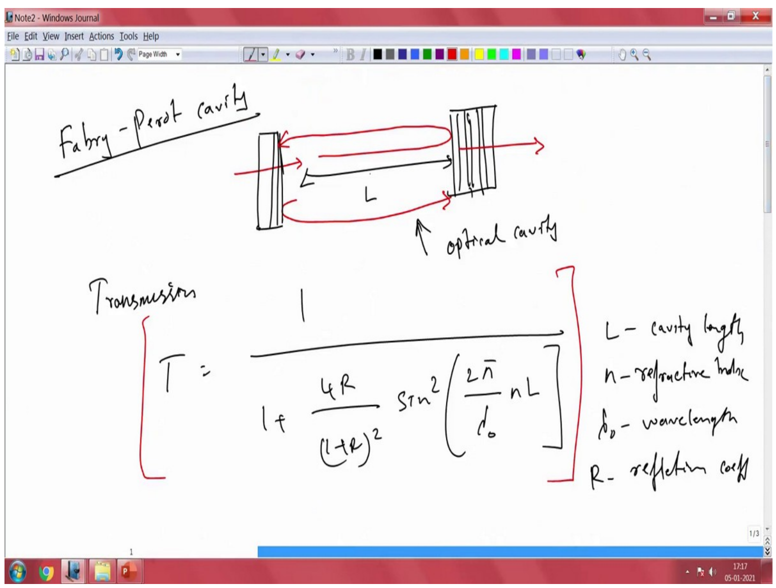

So, let us look at how this the spectrum would look like, the spectrum as a function of lambda let us say, you would have very, this is wavelength selectivity that we are talking about a particular transmission in this case. So, now this is for a certain reflectivity R. So, let us say this is R of 0.9, so when you have very low reflectivity then the cavity would look something like this, this is when R is let us say is 0.5.

And you can even reduce it then R 0.2 so when you reduce the reflectivity you are going to reduce the contrast that you have between the trans the resonant or the cavity wavelength we call this as lambda resonant wavelength to the off resonance or the light that is not captured. So, another way to look at this the light that is captured inside is by looking at this cavity.

So, let us say there is a length to this cavity and you have a certain wavelength of light, so I have a certain wavelength of light lambda, so it has certain space to fit in. So, now the question is how many wavelengths I can fit into this length L, so which is phase match, so that means I should be able to fit this wavelength inside this particular cavity.

And that is an integral multiple of the let us say of the wavelengths that I have and wavelength in the medium basically. So, how many such wavelengths I can fit into this particular lambda? So, in this particular medium which has refractive index n, so that is why I am writing it as lambda n, it is not lambda naught, so the refractive index here is n.

So, based on this we should be able to fit how many lambdas could be sitting in this, so it has to be a whole number, so it has to be very close or equal to the length that you have, and that is the reason why you have this 2 pi coming in into this picture because it has to be phase matched. So, now let us take this whole understanding and then see how we can make a waveguide based cavity. So, if you can look at this you have two reflectors and a cavity here.

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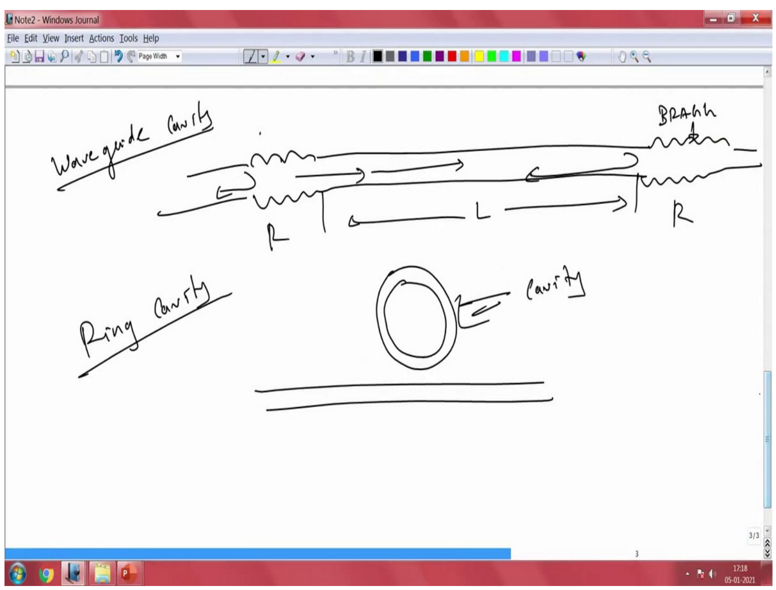

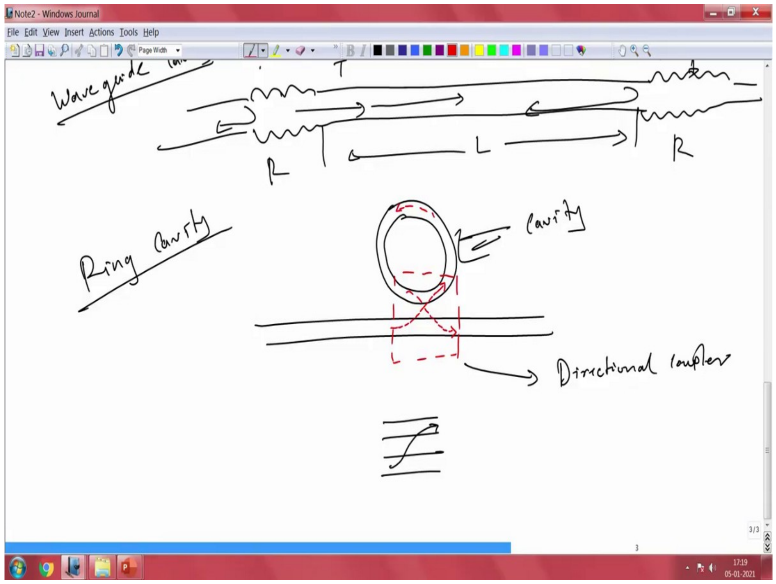

In principle you can also do this by using waveguide cavity a simple waveguide and then you create a grating on the left side and then right the side. So, now this is the cavity length L and then you have the reflectors on both the sides. So, we looked at this bragh, bragh reflectors on this side and this side. So, you can simply corrugate or creates a perturbation where you could have a lot of reflection within the system, so light will go back and forth and that would create a cavity here and the same discussion what we saw with Fabry-Perot applies here.

But what we could do is something called a ring cavity, this is even more interesting. So, instead of making the inline waveguide, what we could do is we can take a waveguide and then make a ring next to it and this will act as a cavity. So, how are we going to couple light in and out? So, we had reflectors here, so two reflectors or two mirrors were there and we were able to launch it through this system because there is a transmission associated with this.

But in this case look at this region, this particular region, this is nothing but a directional coupler, so directional coupler we looked at how the light could be coupled between one waveguide to the other waveguide, so here that is exactly what we are trying to do. So, we take the light and put the light into the cavity and the light would propagate through and then we could take the light out.

So, this is something that one could realize using a simple directional coupler, so you are coupling it in and out, so now we need to look at how this transmission should look like. So, let us that can be given by the transmission and the coupling that we have inside this system. So, the light that you need to couple in, so that will be given by kappa and the light that is coupling from the ring into the waveguide is minus kappa, so that is opposite direction.

And you could have a transmission through the system as t, so E input and then E transmitted is what we are looking at, so now the light that is going through the system is called Ei2 the input to the cavity I can put this as Ei1 and t1 here so then the light is inputted that is input light and then you could have one that is transmitted, so Et2 so E1 is going in and E2 let me do it this way, this is Et sorry Ei2 and then Et2 is what is coming up.

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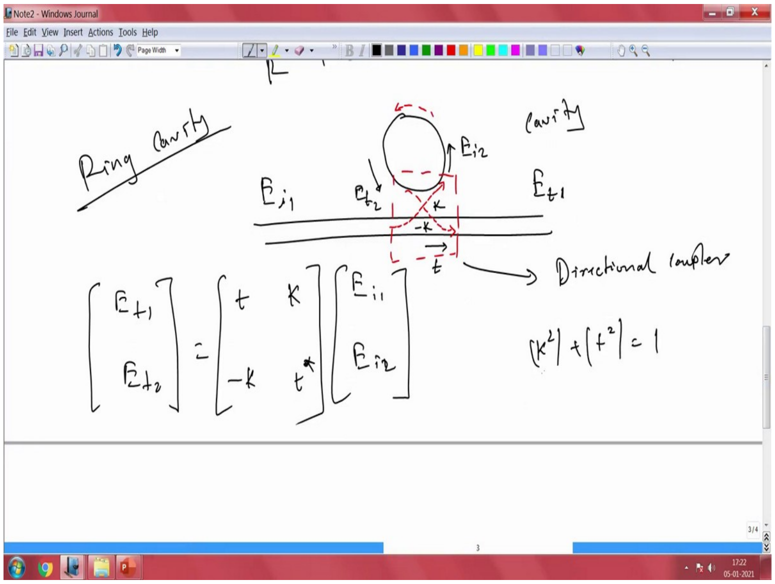

$$\begin{bmatrix} E_{t1} \\ E_{t2} \end{bmatrix} = \begin{bmatrix} t & k \\ -k & t^* \end{bmatrix} \begin{bmatrix} E_{i1} \\ E_{i2} \end{bmatrix}$$

So, now you may want to see how this coupling could be visualized now. so you have Et1, Et2 and what you are looking at is the, what is the transmission as a function of input Ei1, Ei2 and this is coupled through our kappas and t now. So, t is the transmission that we have kappa minus kappa and just this is t star, so this is how you can easily come up with the matrix between or the relation between the input field and the transmitted field. So, kappa square plus t square should be equal to 1. So, that is the relation that we have, so whatever goes through and whatever is coupled into the waveguide.

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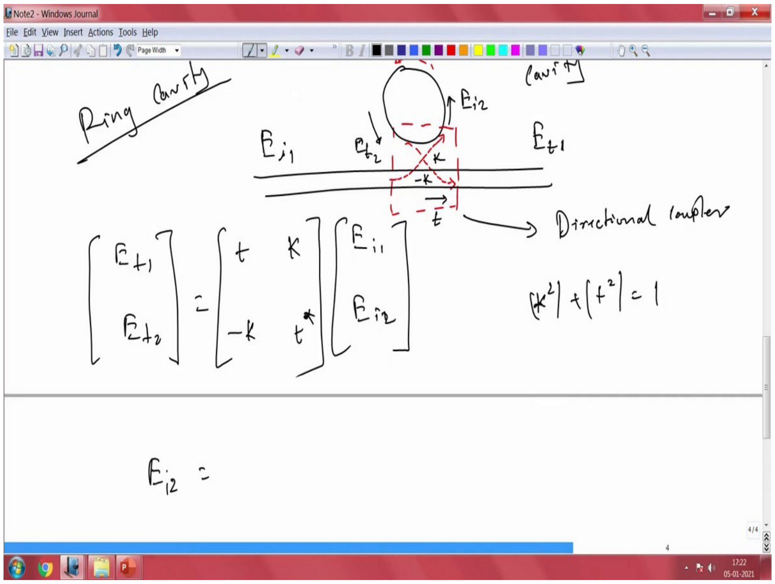

So, that is something that that we just formulated but now we need to bring out how exactly we are going to find out this coupling. So, the way to write this is Ei2 for example depends on Et1 and the phase that is that you are getting out, so let us, let us try to write what should be the phase and what should be the beta before we start with it.

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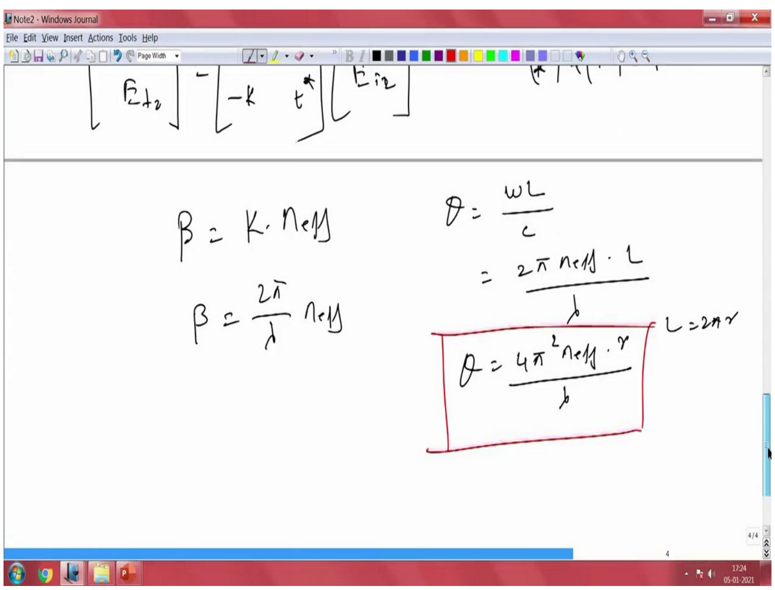
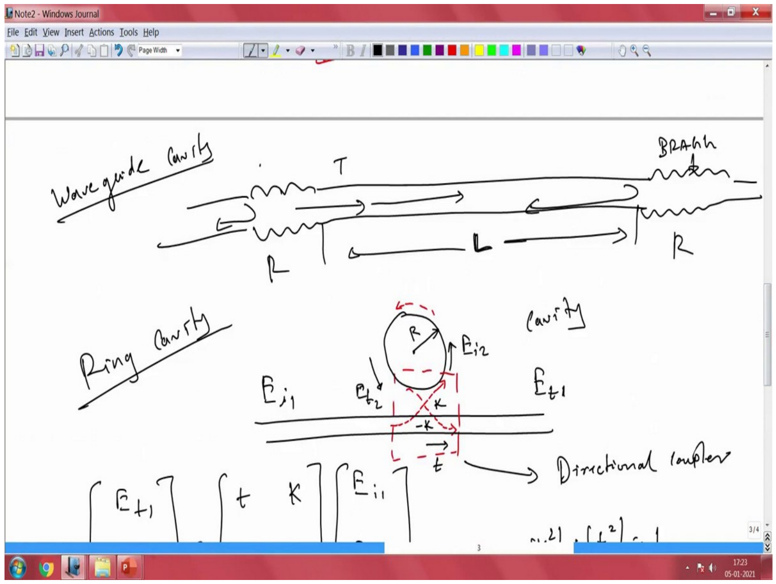


$$\beta = kn_{eff}$$

$$\beta = \frac{2\pi}{\lambda} n_{eff}$$

$$\theta = \frac{\omega L}{c} = \frac{2\pi}{\lambda} n_{eff} L = \frac{4\pi^2}{\lambda} n_{eff} r$$



So, beta is given by K times n effective, so you can expand this, so 2 pi lambda n effective is your beta and now we can talk about the phase that you are going to have and the phase can be given as theta here, so omega L over C or in other words 2 pi n effective times L divided by lambda, so L here is nothing but 2 pi R.

So, what is the cavity length here? So, it has a certain radius R, so in this case L is the length, in this case the whole circumference, so that is 2 pi R. So, then it becomes 4 pi square n effective times R divided by lambda. So, this is the optical path length that light is going to take, so this is the path length you are going to take. So, now you can write all our resulting equation Et1.

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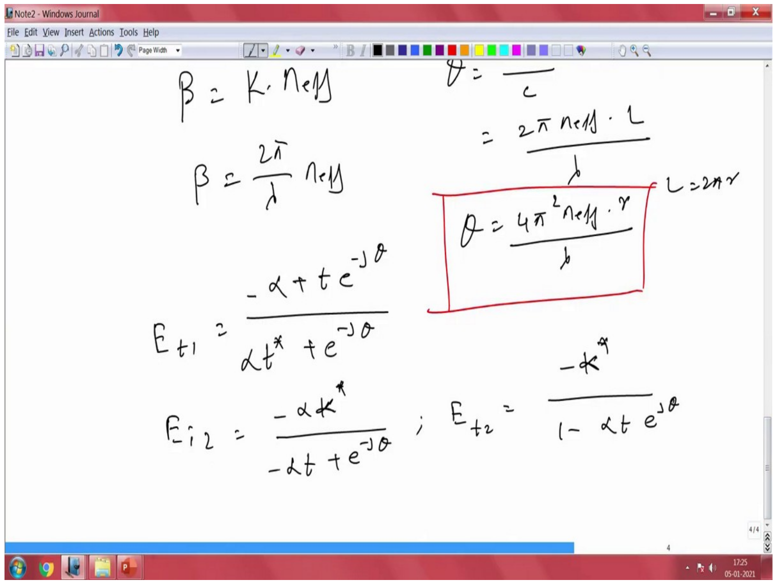

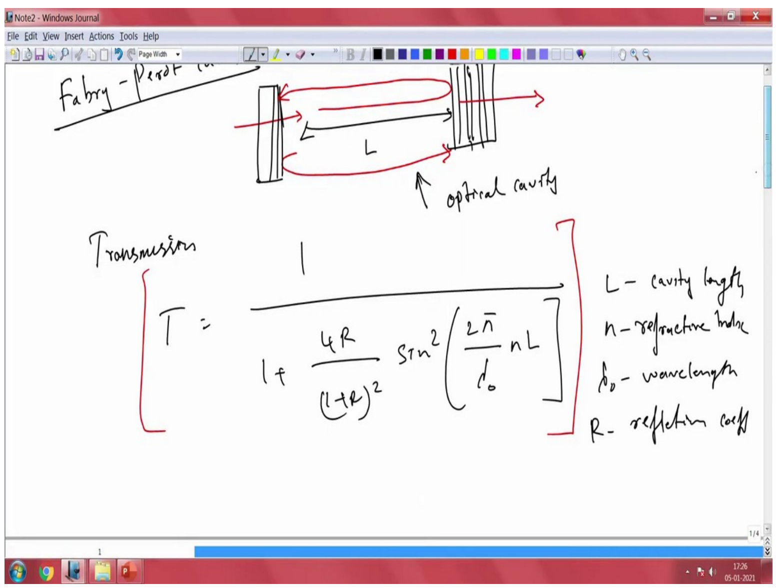

$$E_{t1} = \frac{-\alpha + te^{-j\theta}}{\alpha t^* + e^{-j\theta}}$$

$$E_{i2} = \frac{-\alpha k^*}{-\alpha t + e^{-j\theta}}$$

$$E_{t2} = \frac{-k^*}{1 - \alpha t e^{j\theta}}$$

So, let us say Et1 is given as there is there will be a coupling loss factor here that is your alpha, so alpha will be the loss factor or how much light is uncoupled here plus transmission times e minus j theta divided by alpha t star plus e to the power minus j theta and you can do this for the iE2 that is going in so where you have minus alpha K star divided by minus alpha t plus e to the power of minus j theta and then Et2 which is nothing but minus K minus kappa divided by 1 minus alpha t e to the power j theta.

So, this is how the characteristics are going to look like when you change your theta or rather your wavelength as you go through this, you can clearly see that this is a function of lambda. So, once you have the coupling taken care so this is the transmission that you see and if you work it out your Et1 will exactly follow this particular equation, so inverse of this particular equation, so you can try to do that but this will be the transmission equation of our coupled system here, it will be an inverse of that though.

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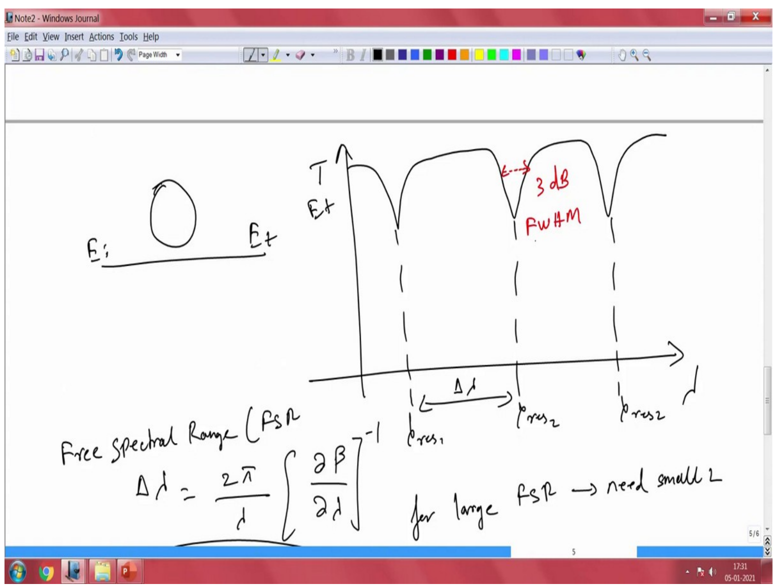

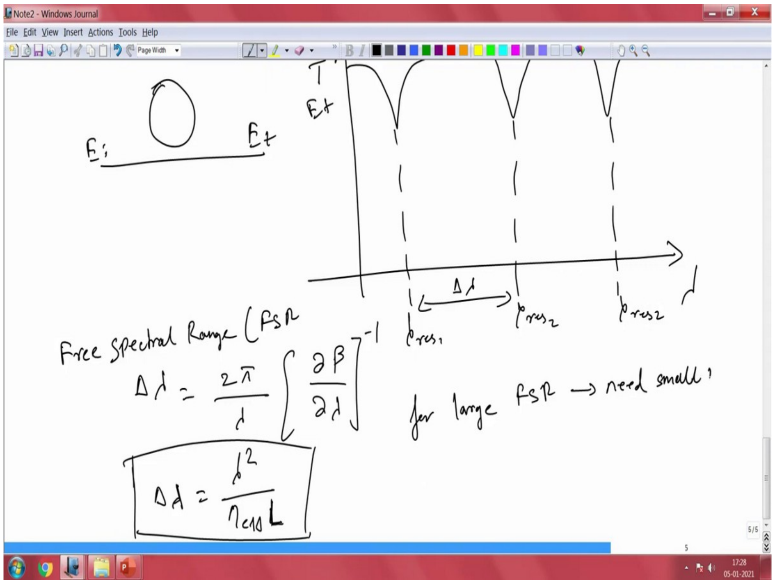

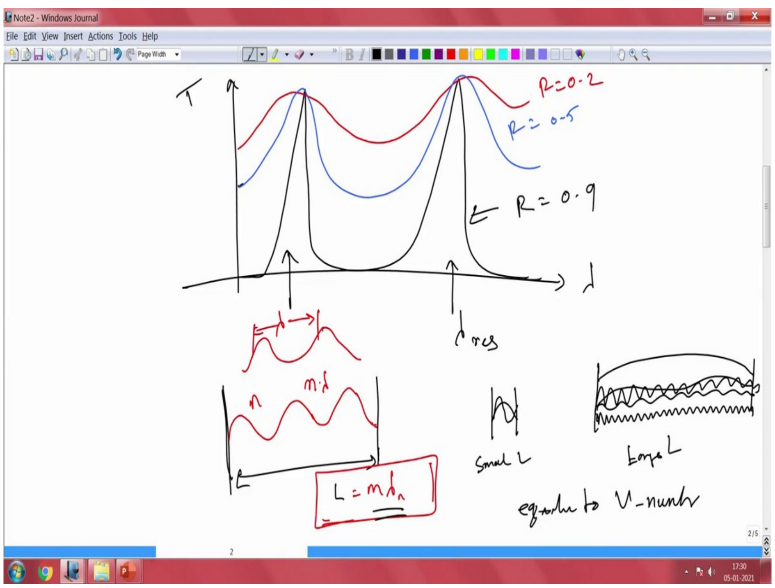

$$\Delta\lambda = \frac{2\pi}{\lambda}\left[\frac{\partial\beta}{\partial\lambda}\right]^{-1}$$

$$\Delta\lambda = \frac{\lambda^2}{n_{eff}L}$$

So, let me draw how that response is going to be, so we had a ring like this, so this is Ei and then let us say Et so the output field. So, as a function of lambda and this is your transmission Et. Well look, looks something like this and this is all the resonance wavelength and so on. So, now the question is we have multiple resonances. Why do we have multiple resonances? Because in

Fabry-Perot cavity that is what we learned that there are multiple longitudinal modes that this particular cavity can support.

So, it will be an integral multiple of a certain wavelength that you have. So, there are various wavelengths that can satisfy this particular condition and that is the reason why you have multiple wavelengths that could be captured inside this cavity. And the distance between these two resonances is called free spectral range and this is called delta lambda and this delta lambda is called free spectral range, in other words FSR and that is given by 2 pi over lambda dou beta over dou lambda plus 1 or in other words lambda square by n effective times L.

So, this is more often used when you know the effective refractive index and if you know the cavity length so we should be able to calculate what should be the free spectral range, so this is how you can design. For example, you want very large free spectral range so that means you need small l, so it means very small cavity. So, for large free spectral range we need small l, so this is again very important to go back and confirm our theory right now. So, we said when you reduce the l your free spectral range will increase so look at this.

So, I have two resonances here coming because there are two wavelengths that are very fit into this particular cavity, so when I reduce this cavity length, so I have a small cavity and I have a large cavity, so in this very small cavity only let us say one cycle could fit in let us say so the other wavelengths that you could fit in is like this.

But then in this case if you look at it there are multiple wavelengths that can be fitted into this particular so there are high, very low wavelengths and also very large wavelengths can fit into this particular cavity so that is the difference between a large cavity and a small cavity. When you take a small cavity the allowed energy levels are lower, this is again going back to our very simple waveguide mode simulations.

So, there we also discussed that when your size is reducing, when the waveguide or the thickness of your slab is reducing then the number of modes allowed is (())(24:28), so here it is longitudinal mode and there it was transverse mode. So, you are looking at, this is equivalent to V number, so the larger the dimension the larger the V number.

So, in this case the smaller the cavity length the smaller the number of longitudinal modes that will allow only few wavelengths to survive inside that system, and that is what your free spectral range here tells you about. So, that is another thing that you can take it out from here. The other factor that we look here is what is the 3 dB bandwidth or full width half max, so FWHM we call it.

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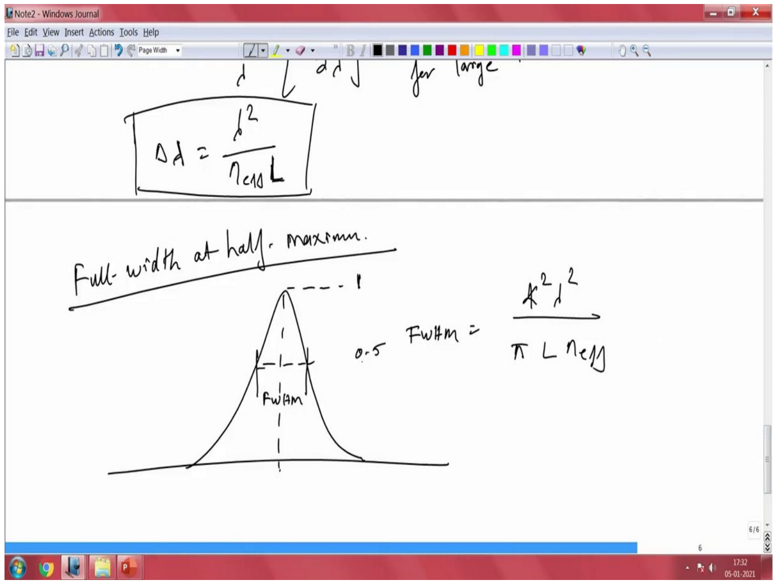

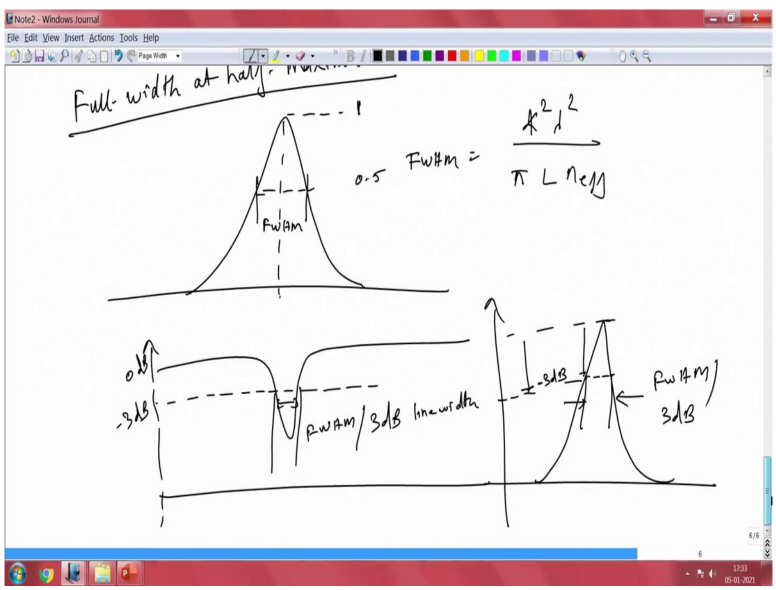

$$FWHM = \frac{k^2\lambda^2}{\pi n_{eff}L}$$

$$Q = \frac{\lambda_{res}}{FWHM}$$

So, let us look at what is that full width at half maximum. So, what is that full width half max means so this is 1 and this is 0.5, so what is the width here at 0.5, this is called full width half max. So, any resonance that you have we will calculate how much is the line width or what is your full width half max of this system and that is given by your coupling and your lambda square.

So, your full width half max is given by kappa square lambda square divided by pi L n effective, so this is full with half max. And the way to calculate this from the measurement is you take it from the top maximum 50 percentage of it or when you are using a dB scale if this is what your spectrum is and this is your max, this is 0 dB let us say and you draw a 3 dB line so this is 3 dB let us say and this is your full width half max or in other words 3 dB line width or bandwidth.

You can also have a scenario where you have the inverse of this, so when you have this again from the max you do 3 dB down so this should be from here it is 3 dB minus 3 dB in this case so this is minus 3 dB and this is what we call full width half max or 3 dB.

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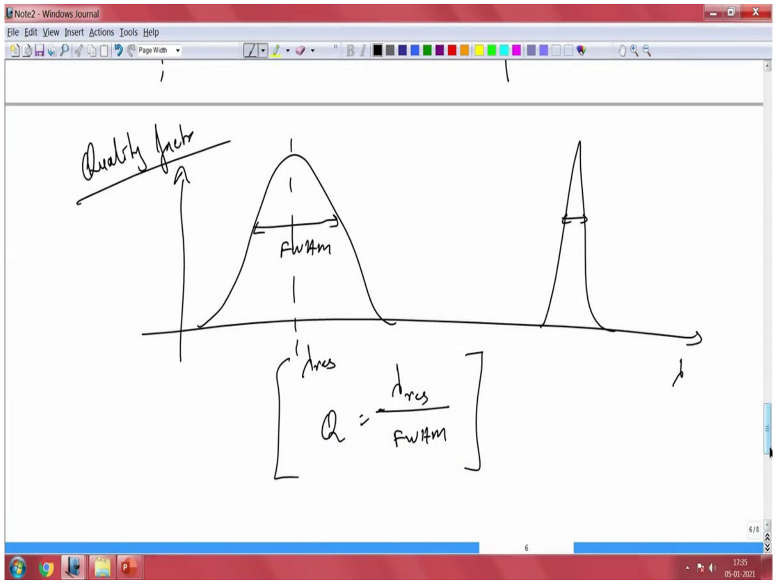

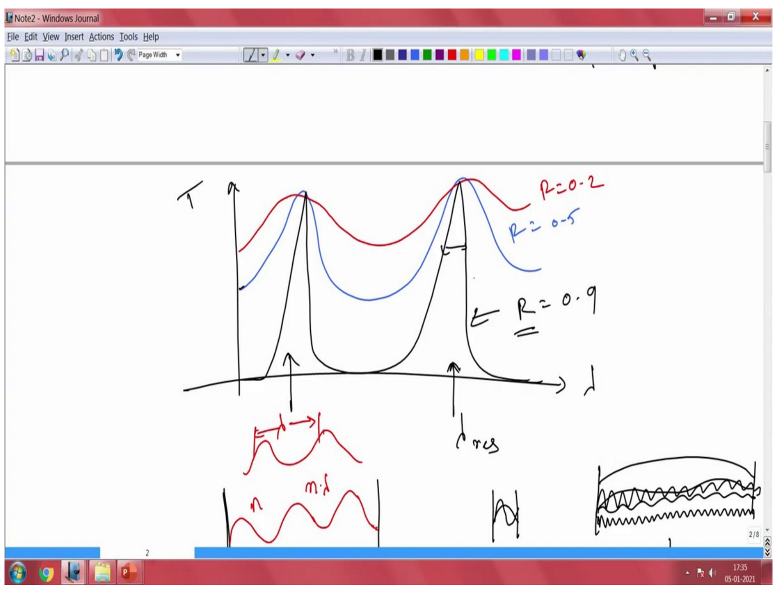

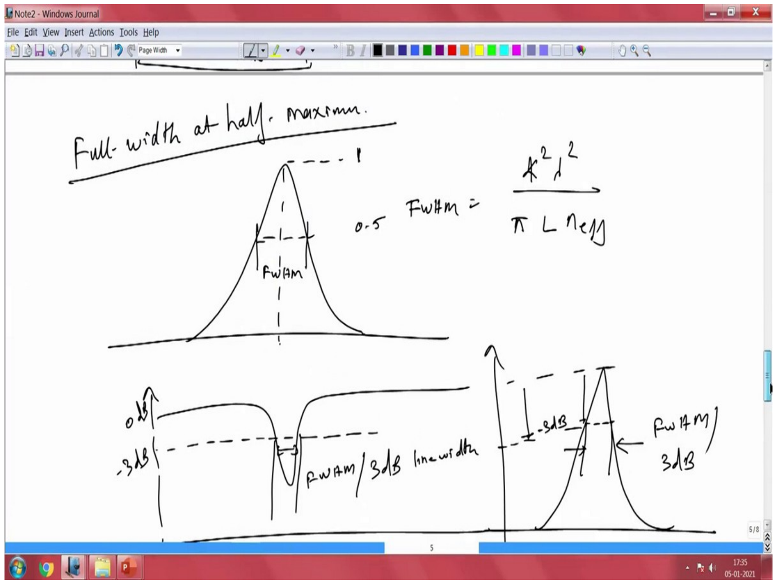

So, in other application we are going to use this full width half max to calculate the q factor what is called quality factor, so what is quality factor? So, there are two resonances, there is resonance like this and there is a resonance like this. So, we look at the full width half max of these two, the one with the lowest full width half max is called a better quality factor or a much narrower resonance we have.

So, we use that to find what is called the quality factor q, quality factor q is nothing but lambda resonance over delta lambda or in other words full width half max here. So, we need to look at what is this full width half max and at what resonance here, this is lambda resonance and this is full width half max, if we do that so this is what the quality factor is, the narrower the resonance the better the Q factor, so that the bottom part goes to smaller values and this is what you get out of this particular calculation. So, why are we interested in this quality factor? That is because of the narrow resonance that we always like.

So, when you look at the Fabry-Perot cavities this reflect, when you have higher reflection coefficient your resonances have become narrower and narrower that means you only take the lambda that you want, you want to be very selective in your lambda. So, if you want to do that you should have higher quality factor, so that is given by this full width half max.

So, you can say full width half max or you can represent it as a quality factor because the quality factor here also depends on the loss factor that you have, that is a kappa that you have, the coupling into the structure as well. So, those are all the parameters that you should understand when we talk about wavelength selective filters.

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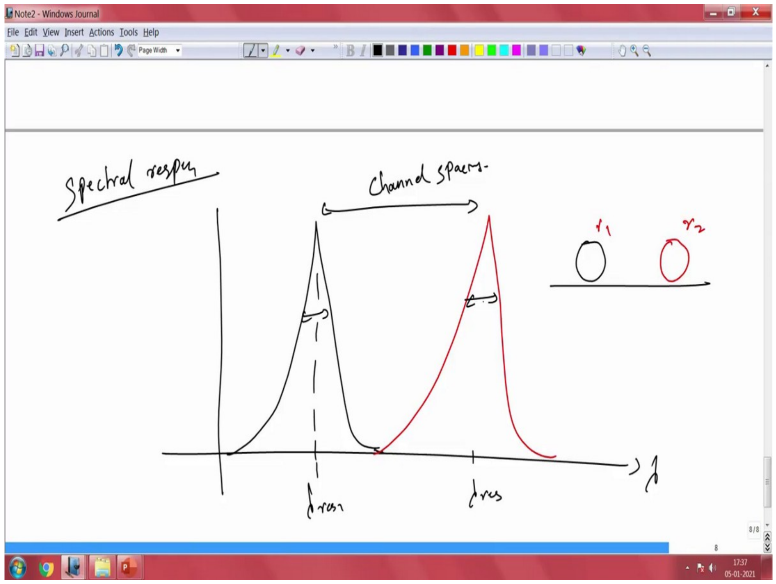

So, now let us take a very simple spectral response, so when you take a spectral response of a filter so I am taking a filter here and then there is one more filter sitting next to it, so the distance between the two different filters is called the channel spacing there are two channels, so this is resonance 1 and this is resonance 2, in this case you have two different resonators, so this has r1 and this has r2, so the distance between this is these two are called channel spacing.

And they should be far enough at the same time the resonance should not overlap with each other. So, by using this you can build very complex filters which is something that we will see in the case studies but just to give you an idea this is how you can implement multi wavelength filters or wavelength selective device as a whole.

So, with that we have understood how to create spectral selectivity, so in the past we only handled power. So, now we have a whole to control the flow of power as a function of wavelength now, a certain wavelength could be captured and directed somewhere else.

So, with that we have completed the whole the passive part of the fundamental elements but then we will revisit all the components that we discussed theoretically here and then practically we will see how they respond in our case studies which will be more interesting, because right now it might just look like relations between different parameters and structures also, but later on we will see how we realize this and actually how the response would look like, you will be surprised that it will look near ideal when you look at this interference and also the spectral response of this wavelength selective devices. With that thank you for listening, see you in our next class.