# Photonic Integrated Circuit
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Lecture 30
## Multi-mode Interference Coupler

So, hello everyone, let us continue our discussion on the passive devices. So, in this section we are going to look at multi-mode interference based devices. So, in particular multi-mode interference based power splitter or power combiner, so in this in the same line of power handling, so this is slightly different way of doing power splitting and power combine combining. So, we looked at this briefly, but not in detail, when we discussed about Y-splitters. So, when we looked at the Y-splitters, so the Y-splitters got a multi-mode section, when they combine. So, you see a multi-mode section.

So, this multi-mode section is broad; it is broader than the single mode section that we have. Because it is broad, you are able to generate till the second order mode; so that is the condition. You want to make sure that you can accommodate the second order mode. But, can we go even beyond that? So, the question here is instead of just having it to the second order mode that is v number of 3. Can we make very large v number? That means very broad waveguide. And again exploit this kind of properties of multi-mode interaction. So, it is a little challenging, but then it comes with lot of advantage.

To start with, let us look at the two cases that we discussed earlier; so one is directional coupler, the other one is y splitter. In both the cases there was one input and then two output. But let say in directional couplers, you could have two inputs on one side and two outputs. Best case scenario and then let us look at y splitter. So, Y-splitter is even straightforward, you have one input, two outputs or you have you can have two inputs and one output. That is all you can have. But, then what if I want to have some m inputs and n outputs, where m and n are more than 2? There is large number of inputs I want to give or I want to split into large number of outputs.

How do I realize this? So, one way of doing this is again by using directional couplers and also you can do Y-splitters. But, it becomes reasonably hard to manage things, you have multiple devices of that kind. For example, directional couplers again two input two outputs, so I can cascade this. So, I can cascade number of devices and based on that I could build it, similarly for y junction as well.

But, then is it one device possible? One single unit, one single device whether one can realize such a function. So, that is where our discussion is leading to having one device that is called multi-mode interference device. It is a is a very interesting device, but then one need to understand how it operates, so that we can control the design here. So, let us look at the multi-mode interference device.

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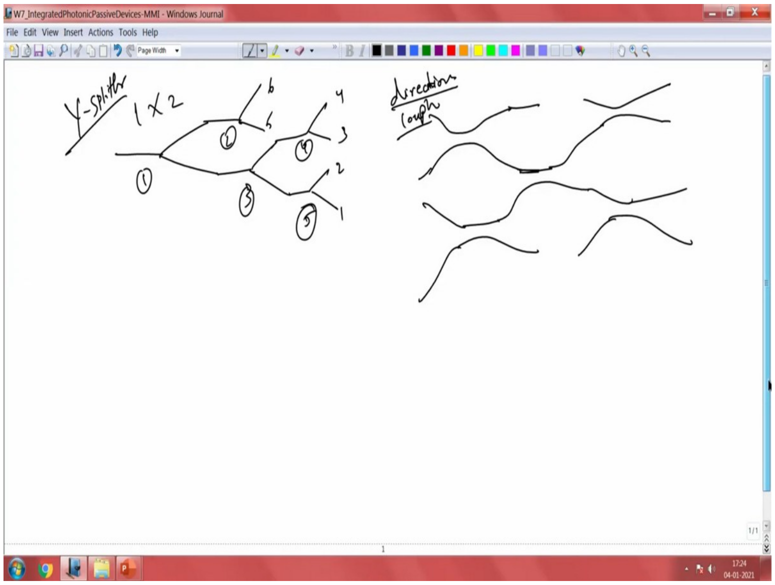

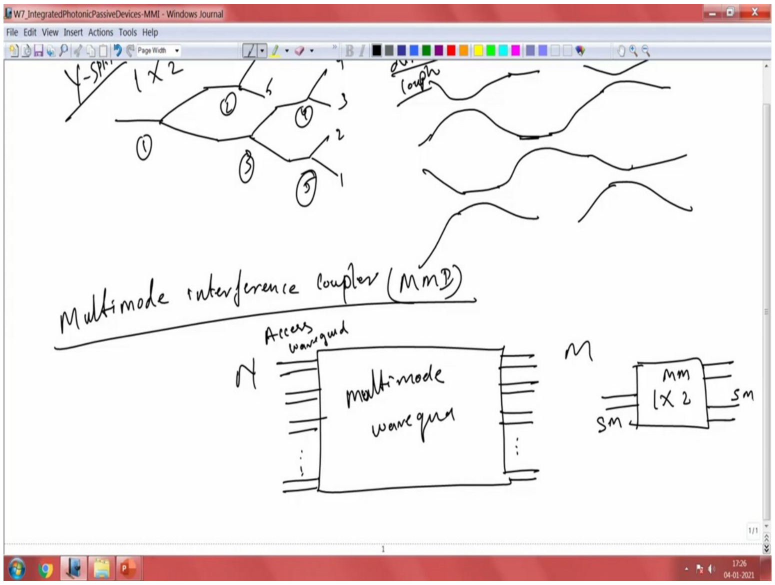

So, before looking at the device, let us look at the requirement. For example, you have Y-splitters like this and if you want to do, this is 1 by 2. So, if you have want to have multiple outputs, so what we do is we again split this. And even more you can split it this way. So, you have to have multiple units here, so this is 1, 2, 3, 4, 5 and so on. You have multiple devices here, so with this 5 device, you are able to get 1, 2, 3, 4, 5, 6 six outputs. So, it will be always device plus 1, so that is the best you can do the number of chains that you have. So, you are not going to get any better.

So, again in directional couplers, you could have something like this and you can put something like this. So, you could have, you can have multiple inputs and multiple outputs in this case. So, again we need to have multiple this is directional coupler and this is our Y-splitter. But, then we want a single device to do that, one way of achieving is is called multi-mode interference coupler, is called multi-mode interference coupler MMI. So, the device looks reasonably straightforward, it is a multi-mode section. What we call this is the multi-mode waveguide with inputs single mode inputs like you wanted.

So, this is input waveguide or axis waveguide, could be input or output; similarly, you could have output waveguides as well here. So, there are N here and there could be M here. So, you should be able to split the power this way. A simplest form could be similar to a 1 by 2 splitter; so this is 1 by 2 MMI splitter. So, this section again is multi-mode section and this is single mode, and this is also single mode. So, this is a very simple construction of a multi-mode waveguide. So, let us again let us look at what happens in this multi-mode section.

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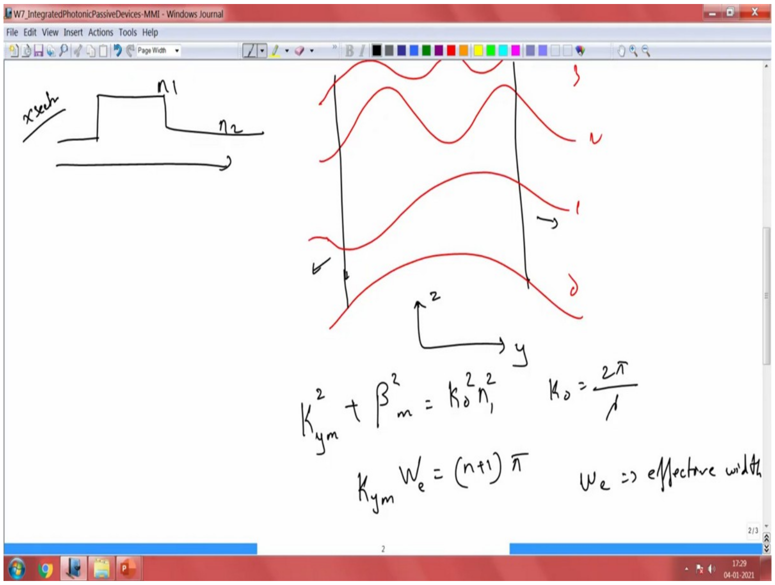

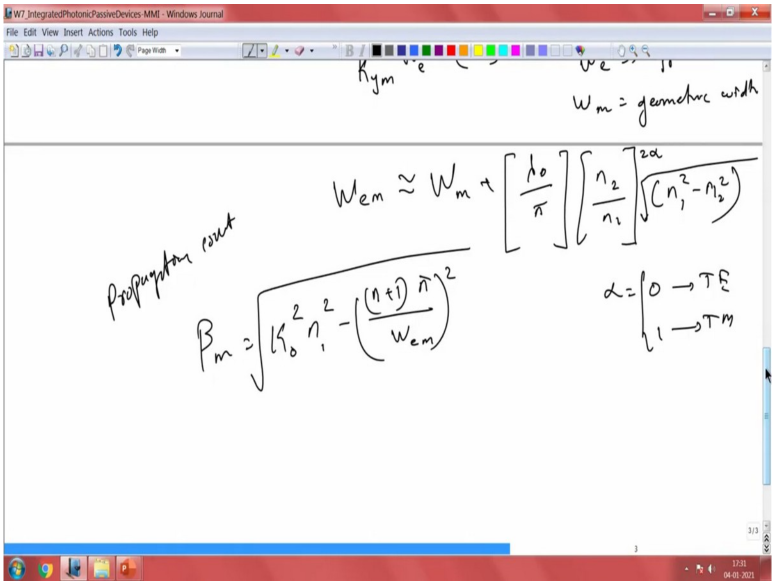

$$k_{ym}^{\ 2} + \beta_m^{\ 2} = k_0^{\ 2} n_1^{\ 2}$$

$$k_{ym} W_e = (n+1)\pi$$

$$\beta_m = \sqrt{k_0^{\ 2} n_1^{\ 2} - ((n+1)\pi / W_{em})^2}$$

So, you will have a very simple waveguide here, so this is the cross section of certain refractive index n. So, n n1 in this case is n1 and this is n2. And when you have a large multi-mode section as we saw in the earlier case, you will have multiple modes here, so you will start with first order mode; and then you have first order mode. And then you will have your second order mode, and then you will have third order mode and so on; so 0, 1, 2, 3 and so on. So, this is what you will have in a typical multi-mode waveguide section.

So, if this is your y section and this is your z; so what is your propagation constant or your dispersion equation along y? So that is what you want to know, so that is given by Ky some mode. So, mode number let say m square plus beta square m is equal to K naught square into n1 square. So, this is something that we all know, so it is K naught is nothing but 2 pi over lambda; so, we have these different modes. So that different modes that we have that condition for that is also something that we know of this; what is call the standing waves or the modes that are put into this particular section.

What we called W is the width, the effective width is n plus 1 times pi; so this is just the matching condition here. So, here W is nothing but the effective width, so W e is effective width; so there is actual width and there is effective width. The effective width takes into account the leakage as well; so that is why we we say this effective length and to differentiate it from the actual length. So, now the effective length since we know this is the relation for that; so now we can write the effective length for a certain mode. So, it can be any mode is approximately equal to the actual width.

So, Wm is geometric width of this plus lambda naught over pi, n2 by n1 to 2 into let say to the power 2; let say 2 alpha to n2 minus n1 square minus n2 square under root. So, here alpha is nothing but 0 if it is TE; 1 that is TM; so, now this is your effective width. Now, the propagation constant could be written slightly modified way; so the propagation constant could be written as K naught square n1 square minus n plus 1 pi, by Wem the whole square. So, this is our propagation constant. Now, you can do the Taylor series of this to find out the propagation constant.

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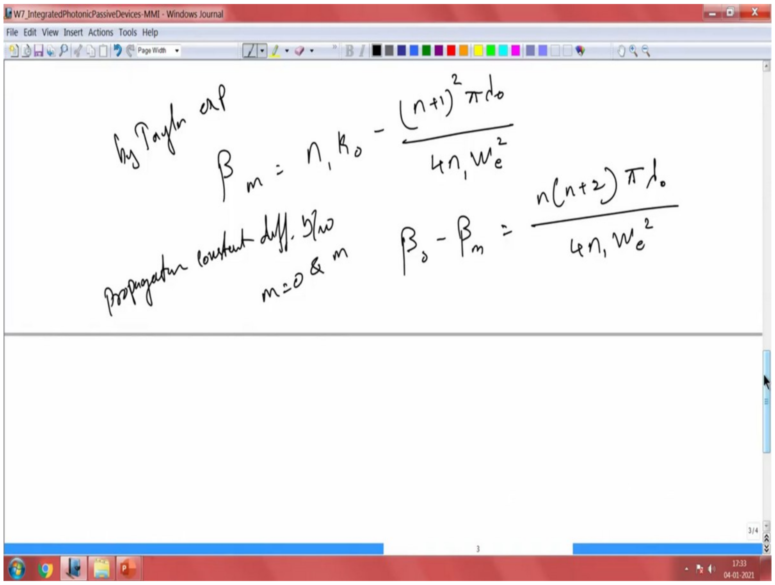
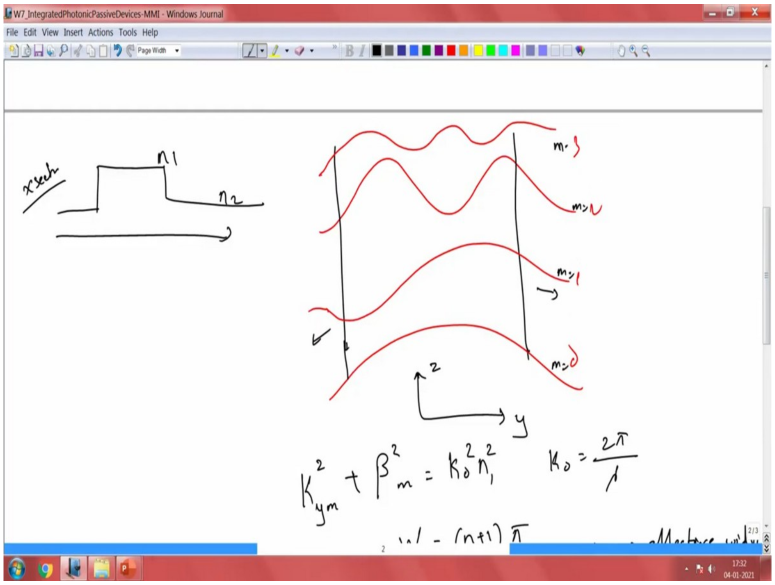


by Taylor exp

$$\beta_m = n_1 k_0 - \frac{(n+1)^2 \pi \lambda_0}{4 n_1 W_e^2}$$

propagation constant diff. b/w m=0 & m

$$\beta_0 - \beta_m = \frac{n(n+2)\pi\lambda_0}{4 n_1 W_e^2}$$



$$K_{ym}^2 + \beta_m^2 = k_0^2 n_1^2 \qquad K_0 = \frac{2\pi}{\lambda}$$

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$$\beta_0 - \beta_m = \frac{n(n+2)\pi\lambda_0}{4n_1 W_e^2}$$

So, by Taylor expansion we find that beta m is nothing but n1 K naught minus n plus 1 the whole square, pi lambda naught divided by 4n1 W e square. So, now this is for a particular mode, so this is for a particular mode index beta m. So, the m here is m 0, 1 and so on; it could have any of this. Now, the question to ask is what is the difference between the two modes that you have? Because we are always interested in the interference, interference between the modes; so that interference depends on the propagation constant now.

So, what is the propagation constant difference between different modes? Let us start from the zeroth order mode that, or the fundamental mode. So, propagation constant difference between the (propaga) the fundamental mode; propagation constant difference between let us say m equal to 0 and some mode m. So, what will be that difference? So that difference is given by beta naught minus beta m; which is nothing but n n plus 2 pi lambda, divided by 4 n1 W e square. So, this is our difference in the propagation constant. We know that the beat length here, so this is something that we saw when we are doing the directional coupler.

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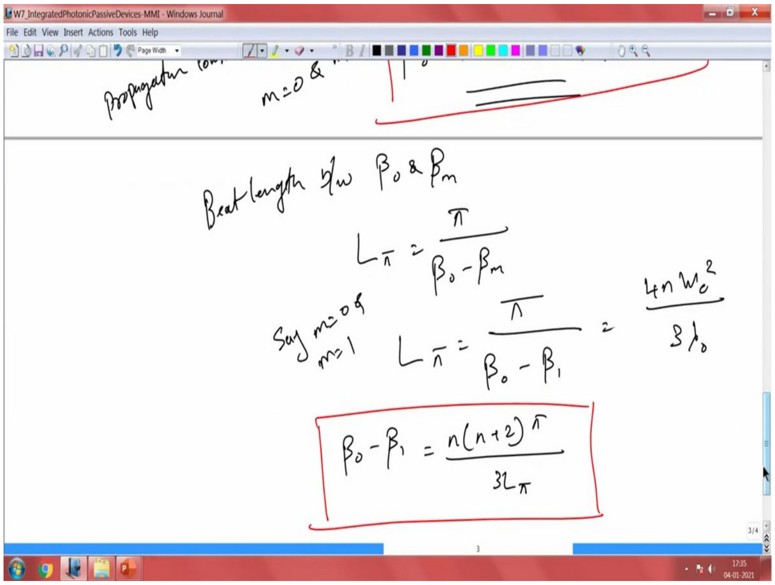

Propagation con

$m=0$ &

Beat length b/w $\beta_0$ & $\beta_m$

$$L_\pi = \frac{\pi}{\beta_0 - \beta_m}$$

Say $m=0$ & $m=1$

$$L_\pi = \frac{\pi}{\beta_0 - \beta_1} = \frac{4nW_e^2}{3\lambda_0}$$

$$\boxed{\beta_0 - \beta_1 = \frac{n(n+2)\pi}{3L_\pi}}$$

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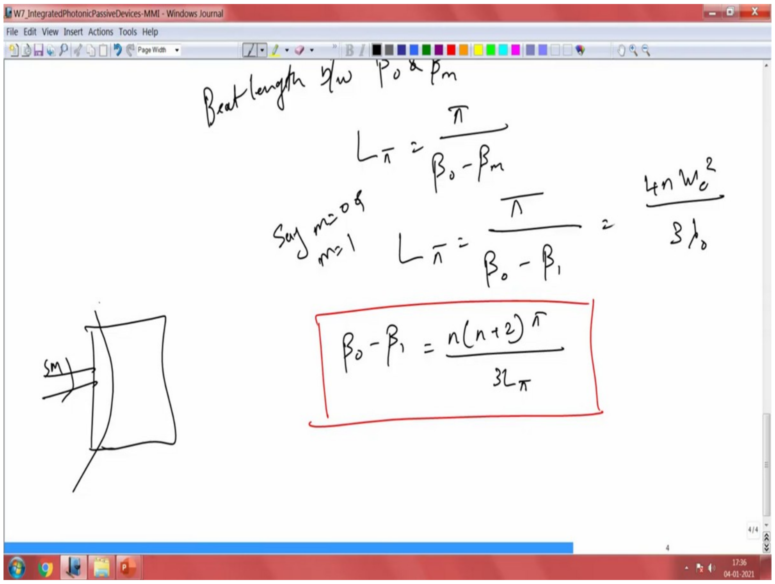

Beat length b/w $\beta_0$ & $\beta_m$

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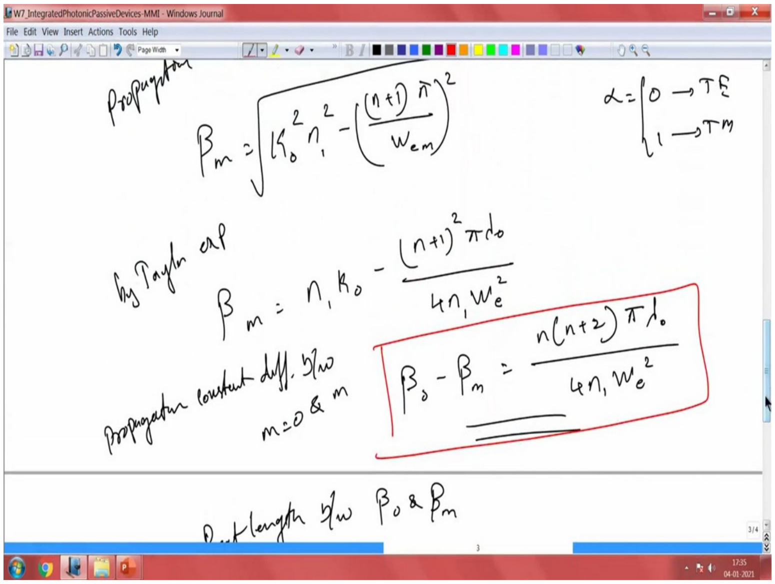

Propagation

$$\beta_m = \sqrt{k_0^2 n_1^2 - \left(\frac{(n+1)\pi}{W_{em}}\right)^2}$$

$$\alpha = \begin{cases} 0 \rightarrow TE \\ 1 \rightarrow TM \end{cases}$$

by Taylor exp

$$\beta_m = n_1 k_0 - \frac{(n+1)^2 \pi \lambda_0}{4 n_1 W_e^2}$$

Propagation constant diff. b/w m=0 & m

$$\boxed{\beta_0 - \beta_m = \frac{n(n+2)\pi\lambda_0}{4 n_1 W_e^2}}$$

Beat length b/w $\beta_0$ & $\beta_m$

So, the beat length between beta naught and beta m; so why we need this beta beat length? It is nothing but the interference between these two; so that will happen when this particular condition is met. So, if we say m equal to 1, so let say m equal to 0 and m equal to 1. So, what is this L pi is going to look at the; the zeroth order mode and the first mode. So that is nothing but beta naught minus beta 1, which is nothing but 4n W e squared, divided by 3 lambda naught. So, this is the propagation constant difference that we see, and your propagation constant difference could be written as taking it from our relation here.

We could write this as n into n plus 2 times pi, divided by 3L pi. So, this is the propagation constant, the difference in the propagation constant; or how far the two modes are sitting can be given by this. So, some of the important equations that you may want to note down is this; that we already saw between the two the first order mode, and this is the general equation that we saw. The rest of the things probably you remember. So, remember this we will revisit this equation as we progress through this understanding.

So, let us look at how one can look at the mode profile; now we are just looking at the beta. This is nothing but the coupling; the how the coupling is going to happen between different modes here; in this case 0 and 1 or 0 0 and any other mode. Why we are starting from beta naught, because when it comes in; let us look at very simple coupler here. So, when you have a single mode waveguide here, so the first mode that that has a clear overlap with the fundamental mode is our beta naught. So, that is why we always take to start with what is the difference from beta naught to the other modes. So, once you have this, then we should be able to understand how this particular mode is going to evolve. So, let us try to do that by looking at the coupling length; so now let us look at the field.

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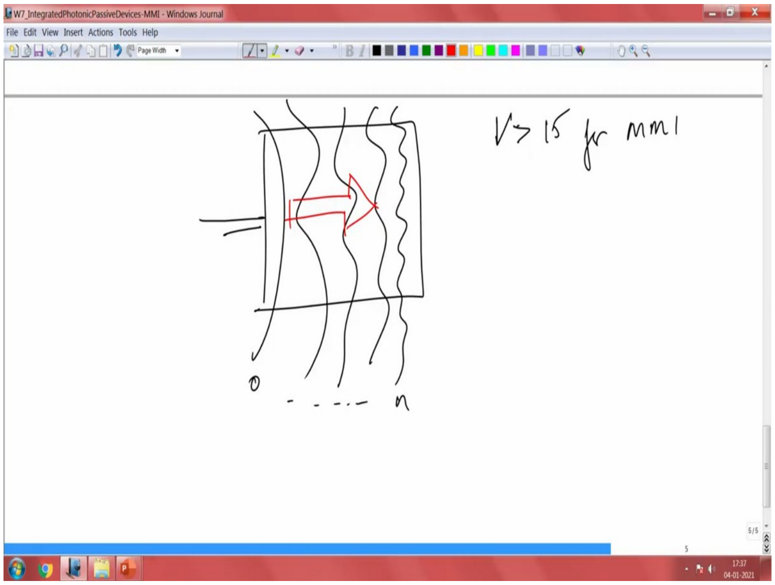

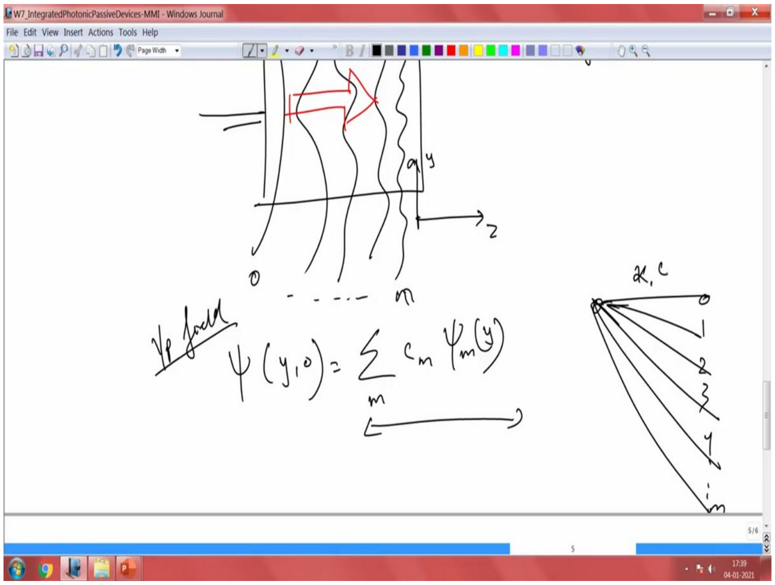

So, I take a piece of multi-mode configuration, I start with; so there are multiple, multiple modes are here starting from 0 to some n. So, normally you want this V number to be greater than 15, for multi-mode interference device; so that is something that you may want to do. Now, let us look at how these guided modes propagate through this particular section. So, you want to see how these modes are going to travel and how the mode field is going to look like? So, the mode field when you start from, so you can want to look at why, then this is the input field. So, this is z and this is in y direction.

So, when you have an input field, we could write that input field as a distribution of all the modes that you have in the system. So that this could be written as some summation or linear combination of all the modes that you have inside the system. So, this is the m number of modes that we have inside; so this is something we already know. Now, the field excitation is something that we should understand; so now this is the input field, and it can be it can excite any of this.

But, then how do we know that the coupling efficiency between mode 0 we have from outside, and then 1, 2, 3, 4, 5 till m. So, now you need to couple, you can be able to couple to any of this; so that should be based on the coupling constant that we know kappa. And also the overlap integral that have here; so, the field overlap.

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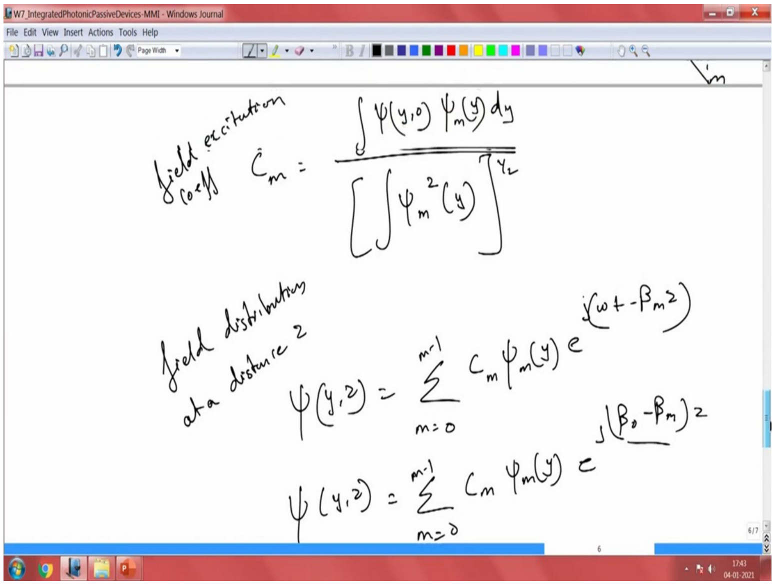


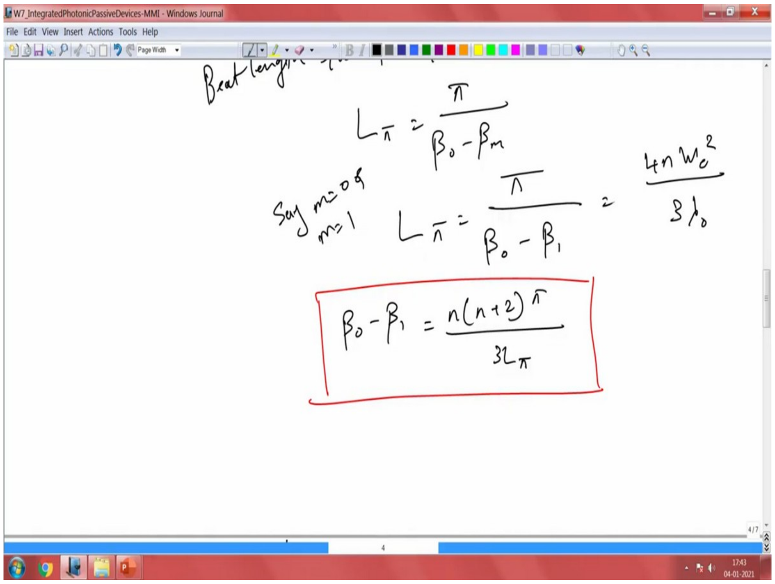
field excitation coeff

$$\hat{C}_m = \frac{\int \psi(y,0)\, \psi_m(y)\, dy}{\left[\int \psi_m^2(y)\right]^{1/2}}$$

field distribution at a distance z

$$\psi(y,z) = \sum_{m=0}^{m-1} C_m \psi_m(y)\, e^{j(\omega t - \beta_m z)}$$

$$\psi(y,z) = \sum_{m=0}^{m-1} C_m \psi_m(y)\, e^{j(\underline{\beta_0 - \beta_m})z}$$



Beat length

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Say m=0 & m=1

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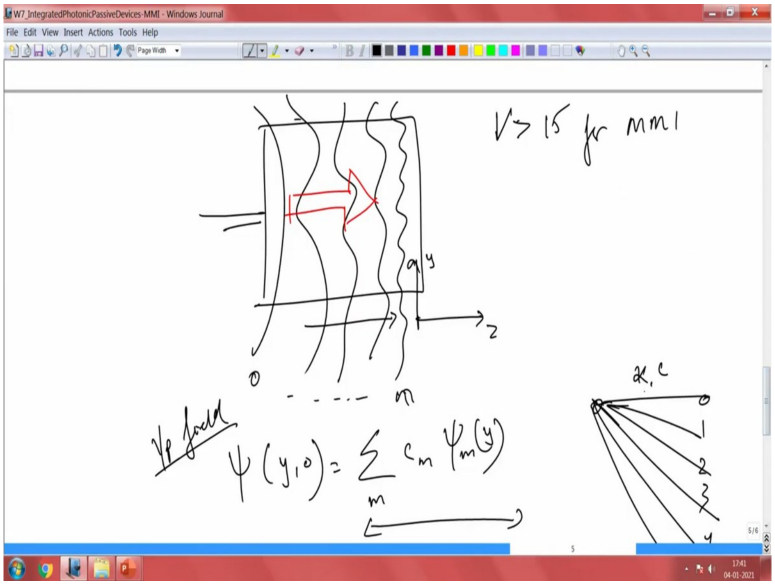
$$\boxed{\beta_0 - \beta_1 = \frac{n(n+2)\pi}{3L_{\pi}}}$$



i/p field

$$\psi(y,0) = \sum_m c_m \psi_m(y)$$

The field excitation coefficient or the field overlap can be given as field excitation coefficient Cm, is nothing but the overlap of your field that you have when you start. And a particular mode field at any position y to the total field that you have square. So, this is how one can write the overlap; so this is how we can find the overlap integral of a particular excitation mode; and the mode that that we have inside the system. So, this is all decomposition one can do. Now, look at the field profile at a certain distance z; so we are propagating through this particular medium along z.

So, let us say how the field is distributed as you progress through; because all these modes that you have, they are all going to interfere with each other. So, they are all propagating through the system; you can, you are exciting using a certain kappa here. So, all these modes are there and they are going to interfere with each other. And it will create a superposition of all these fields. So, how does that particular field distribution is going to look like. So, here field distribution at a distance z let us say; this is not going to be very different. But, then the move from what we just wrote here; but, the only thing is this is just the input field.

Now, I want to see how this field has evolved as it moves through the structure, the multi-mode structure. That means phi y of z, again it will be some linear combination of modes; some 0 to m minus 1 mode. You have your overlap of each mode and the mode profile e to the power j omega t minus beta m z. So, this is this is how your field is going to going to move based on the individual modes that are present in the system. Now, you know taking the phase of the fundamental mode as a common factor; we want to see how this is going to look like. Because the beta m is for individual modes; but we have we can normalize it to our fundamental excitation mode that is beta naught.

So, beta naught is our fundamental mode here; so based on that beta naught, how one can modify this. So, that is relatively useful, because now you can normalize all your propagation constant to your beta; so, this is how it is going to look like. So, now the field as it progress through, depends on phase difference that you have between the modes that we have; so, beta naught and beta m.

And now what we can do is this beta naught beta m, we know how this looks. So, let us go back and then see beta naught beta m are the general case here. So, we already know that beta naught minus beta m is of the form n n plus 2 into pi over 3L pi. So, this is our earlier understanding of what this propagation constant difference is. So, we are just going to plug in that.

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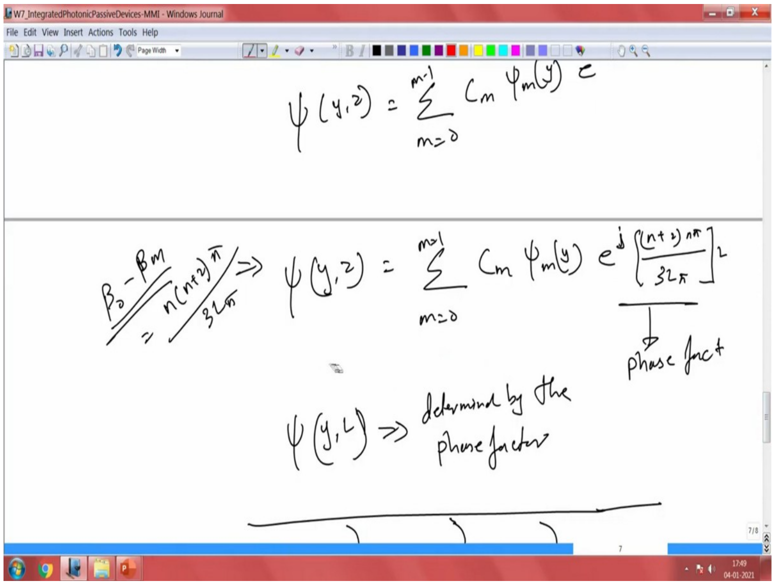
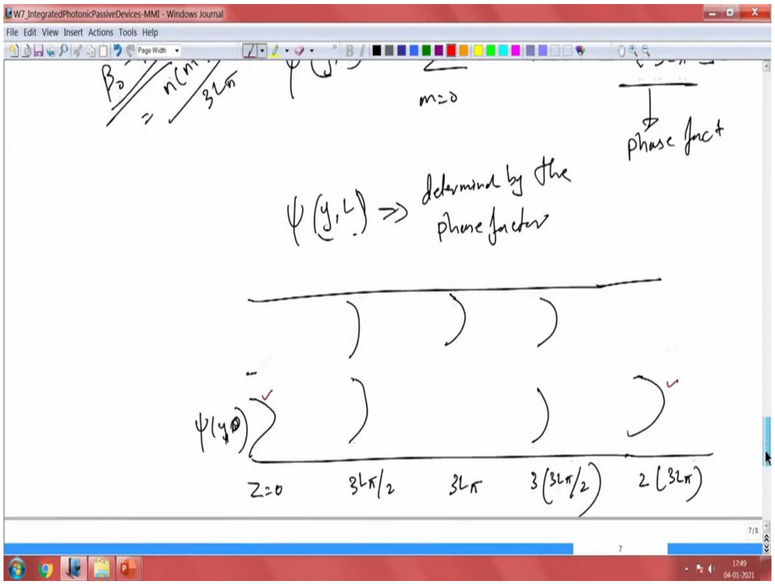


$$\psi(y,z) = \sum_{m=0}^{m-1} C_m \psi_m(y)\, e$$

$$\beta_0 - \beta_m = \frac{m(m+2)\pi}{3L_\pi} \Rightarrow \psi(y,z) = \sum_{m=0}^{m-1} C_m \psi_m(y)\, e^{j\left[\frac{(m+2)m\pi}{3L_\pi}\right]z}$$

phase fact

$\psi(y,L) \Rightarrow$ determined by the phase factor



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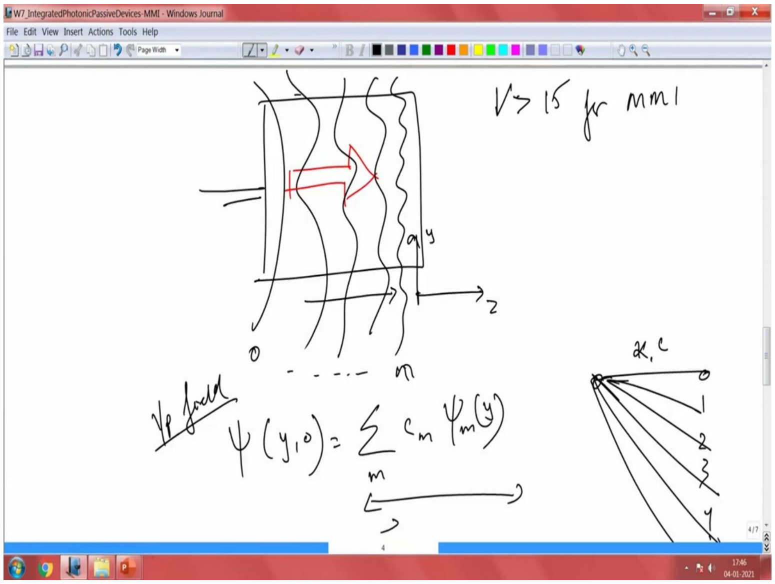

$\psi(y,0) = \sum_m c_m \psi_m(y)$

So, when we plug in beta naught minus beta m will be equal to n n plus 2 times pi divided by 3L pi. When we put that this would become sigma m is equal to 0 equal to 1 Cm phi m of y e to the power j; so now n plus 2 times n pi divided by 3L pi into L; so this is how it should be. So, now this phase factor, this is our phase factor; so how the mode is going to or how the field is going to look like is characterized by this phase factor now. So, what is the phase difference between these two? Is going to tell us how the phase is going to evolve or how the field is going to evolve.

So, one can write this as a function of now L; this is phi y comma L is basically determined by by the phase factor. Now, as it propagates through they are going to interfere; and this interference will result in what is called imaging. So, it will when they all interfere with each other, they are going to produce some superposition of these fields. And that field will create image as it propagates through our image of the field; that were, that you will see as it propagates through. So, we want to understand how this image is produced; so for that we need to look at the effect of length on let us say the image that is being formed.

So, the phase difference that you have between different modes are going to give you different type of images. So, let us look at the multi-mode section, so you have the multi-mode section. And in the multi-mode section this is 0 to start with, and at a particular position you are going to launch a particular mode. So, you are launching a mode here that is of phi y comma 0; so this is where you are launching the the field here. So, now it is propagating through the medium; so when it propagates through the medium for different length scales, there are interesting things that would happen.

So, at length of 3L pi over 2, you will see splitting of this particular power. So, you will see image splitting of this image or at least the field that you have here. And then as it goes through 3L pi, you will have image formed on the other side. So, you will see an image like this and when it goes through 3 times 3L pi over 2, then you will again see the splitting of powers. And when it goes to 2 times 3L pi, then it will come back to the original position here. So, as you can see here you will come to original position after making 6 pi L. So, the length has to be 6 pi L in order to get the image to the same location. The other times you can distribute it into multiple places or multiple field distributions.

So, this is how you can split the power that you have in in this particular multi-mode section. So, one can do detailed field expansion of what we have here; so I encourage you guys to jut have a look at this, you can split this into odd fields and even fields, in order to see how the interference is going to happen. So, that interference would actually result in such a mode splitting here.

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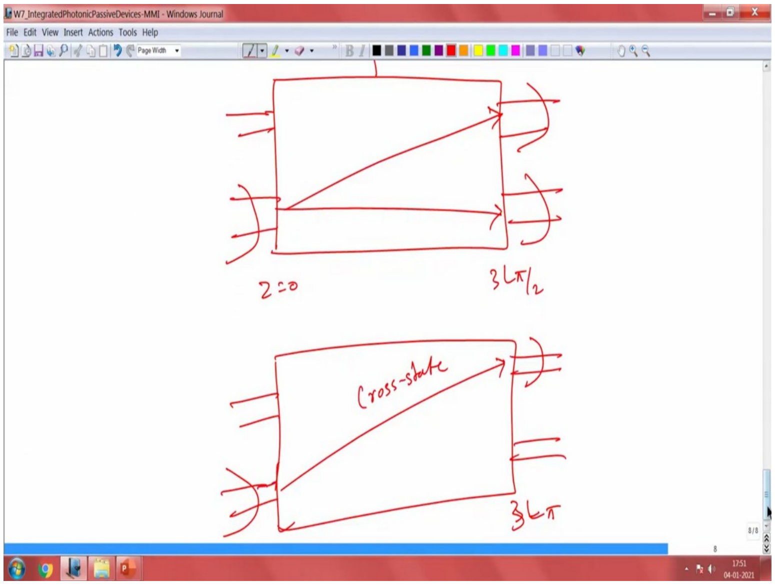

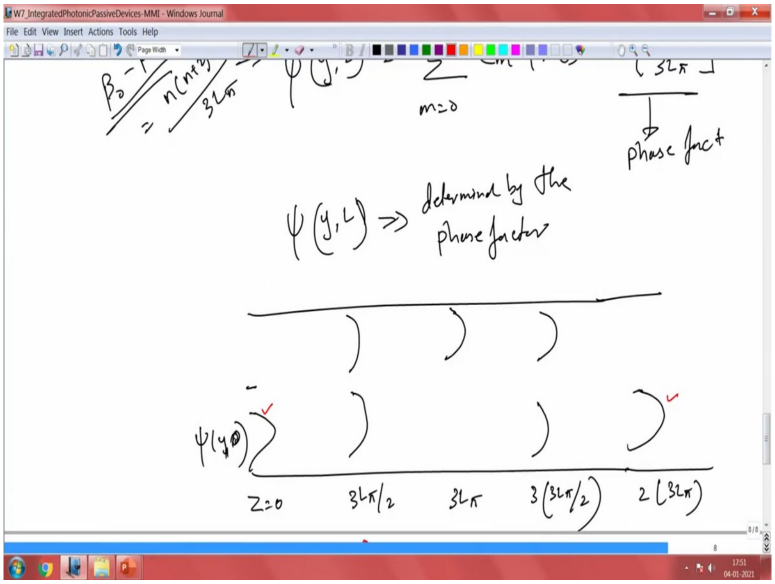

So, when you take a very simple 2 by 2 splitter; so let say I have a 2 by 2 splitter. So, when I launch my light here at z equal to 0, and if I design my lengths to be 3L pi over 2; then I will get equal distribution in this. So, that is corresponding to this particular length; and if I want to so that means you have equal splitting. And now if I take 3 pi 3L pi, so now the same structure I have; now I have 3L distance, so now I am launching it here. In this case, you will get it at the opposite end; so you are going to have cross. So, you will have coupled to this particular length.

So, you can choose your length appropriate length to transfer it to one particular waveguide; or to the other waveguide, or split between these two. So, all depends on just the length; you play with the length that you have here, in order to transfer power from one to the other. So, what is the use in in doing this? Because you do not have to design anything else; all you do is change the length. You have to find appropriate length is all is required if you want to do power splitting or power combining. You can do 3 db coupler or 50-50 coupler, anything of that kind. You can do cross so this all about cross and bar.

So, this is basically your cross, so this is cross-state; that you can create. But then if you go to 6 pi, so sorry 6L pi; then you it becomes a bar state. So, you can just transmit from one input to the other input on the same side; so this is image transfer. Basically, it works on the interference of this different modes that are propagating through this in order to produce this. But the only thing that you should also keep in mind is the splitting will also result in the power reduction as well. So, when you are propagating through this particular multi-mode splitting, your power will be equally divided here. So, if it is 2, it will be by 2; if it is 4 by 4. And as the n increases; your power is going to be split proportionally.

So, that is something that you should keep in mind when you are doing this kind of splitters. So, with this we have understood the multi-mode interference based power splitting or power combining. You can do this in passive mode, but you can also do this in active mode. When I say active mode, you can make this device electro optically active; so, you can actively control the propagation constant. So, you can actually switch between cross and bar, by applying an electric field; and the material should be able to react to this electric field, and change its refractive index.

So, some of the active materials or non-linear materials like lithium niobate or barium titanate should be able to do this. So, birefringent materials are known to create this sort of electro optic effect, where you can actively tune this. Though this device is passive in operation, when you apply an electric field and if the material is active, we should be able to exploit that to act as a switch. So, you can use this for switching as well; so making a passive device and then making it active in some other material is also possible. So, with that we have understood one more passive device with which we can do power handling. So, with that thank you very much for listening.