Photonic Integrated Circuit

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Lecture No. 03

Electromagnetic Theory Review

Hello all, let us look at how we can explain light using electromagnetic theory. So, historically speaking electromagnetic theory was a breakthrough in physics and understanding of light in general because the theory really brought in two different fields together. So, one is electricity and the other one is magnetism. So, this was done by Sir James Clerk Maxwell.

So, in about 1864, so he proposed this unifying theory is that let us take electric energy and magnetic energy and these things are together, they are coupled and when they are coupled you can explain the whole spectrum of waves at that point of time primarily looking at electricity as separately and magnetism and then we had light which was coming from a completely different direction.

So, this unifying theory created a breakthrough and why this theory is one of the breakthroughs is because it was able to explain everything that was already out there. There is one of the important requirement for a unifying theory. So, whatever we understood about light it was able to explain but further, it was able to explain things like polarization very clearly. So, polarization is an important property of light and it made even better understanding of some of the phenomena's.

So, that is the reason why Maxwell proposal was really appreciated and still we use this theory's and we constantly evolve in understanding, how material interacts with light and how one can manipulate the properties of light using this electromagnetic understanding of light. So, let us look at what is that magic equation and why we are so excited about using this electromagnetic theory or field theory to understand light.

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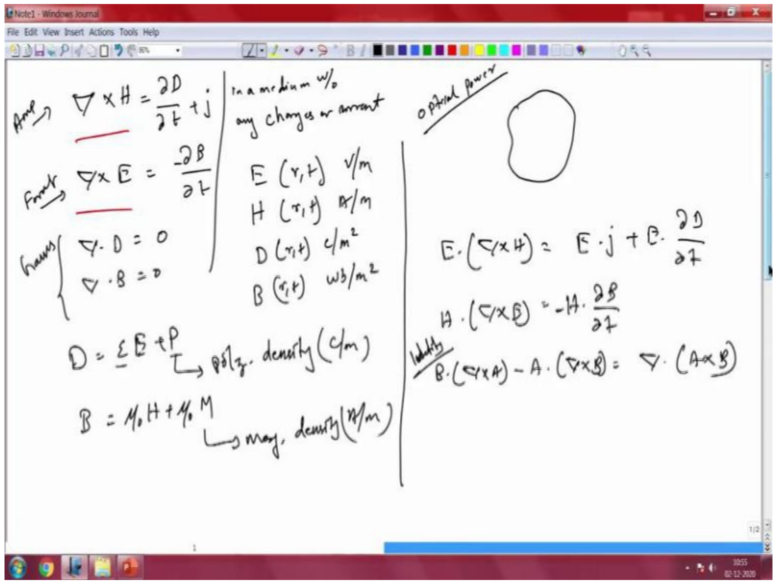

Let us, look at that. So, the electromagnetic theory actually couples as I mentioned electric field and magnetic field and they were coupled equation that was proposed and those equations were the following. So, $\nabla XH=\frac{\partial D}{\partial t}+j$, $\nabla XE=\frac{-\partial B}{\partial t}j$ and then you had $\nabla . D=0$, $\nabla . B=0$. So, this is how the wave occur let us say symmetric the partial differential equations in a medium without any charge or current let us say this is a dielectric medium.

So, if it is a dielectric medium, this is what you see. So, E is the electric field which is represented by this, this is nothing which is represented in unit wise volt per meter and H again as a function of position and time amp per meter and D is the displacement current which is nothing but coulomb per meter square, B is similarly represented by the weber per meter square. So, this is this how supposed this whole coupled equation. This was a lot new at that point of time. Because this is basically the Ampere’s law and this is the Faraday this is Ampere this is the Faraday and this is Gauss basically.

So, he coupled of this and said this is the beauty of the proposal. So, when it was thought that the Ampere law and Faraday laws only was applicable to electricity and magnetism, the whole beauty of Maxwell's is to bringing this together in order to represent the energy or in this case waves. So, D is represented by $D=\varepsilon E+P$. So, where ε is the permittivity, E is the electric field and P is the polarization density. So, this is nothing but polarization, density coulomb per meter and a similarly you can represent B as $B=\mu_o H+\mu_o M$. So, M is nothing but magnetisation density which is amp per meter.

So, μ_o and ε_o were probably you this is all free space permittivity and permeability that you might have already studied in your basic courses. So, we do not have to go into details of that. So, this is how you can represent an electromagnetic wave propagation it is nicely done. But then you want to know how much energy this wave is going to carry. So, than is what you want to do.

Let us, say I take a simple volume here and I want to know how much of energy that this particular volume is going to handle or carry whichever way you want to call it. So, the electromagnetic energy now is carried by both magnetic field and the electric field. So, both are carrying this energy. Let us, see how one can find the energy or power or energy density that these waves could take forward.

So, the way that one could understand is by looking at the space that the do you want let us, say optical power show how you can use optical power. Let us, take these there is Maxwell equation that we already know. So, these is two guys and then we are going to do some manipulation here.

So, let us let us do it here itself. So, let us multiply $E.(\nabla XH)=E.(\frac{\partial D}{\partial t}+j)$ and then $H.(\nabla XE)=H.(\frac{-\partial B}{\partial t})$. So, if we do that then it becomes $E.(\nabla XH)=E.j+E.\frac{\partial D}{\partial t}$. Similarly, we get $H.(\nabla XE)=-H.\frac{\partial B}{\partial t}$ it should not forget that negative sign here.

So, now we can use the identity. So, basically what we are trying to do here is optical power. So, how much power that you are the wave will carry so that is an interesting thing to do. So, by using the identity, so basically the identity is $B.(\nabla XA)-A.(\nabla XB)=\nabla.(A XB)$. So, this is the identity.

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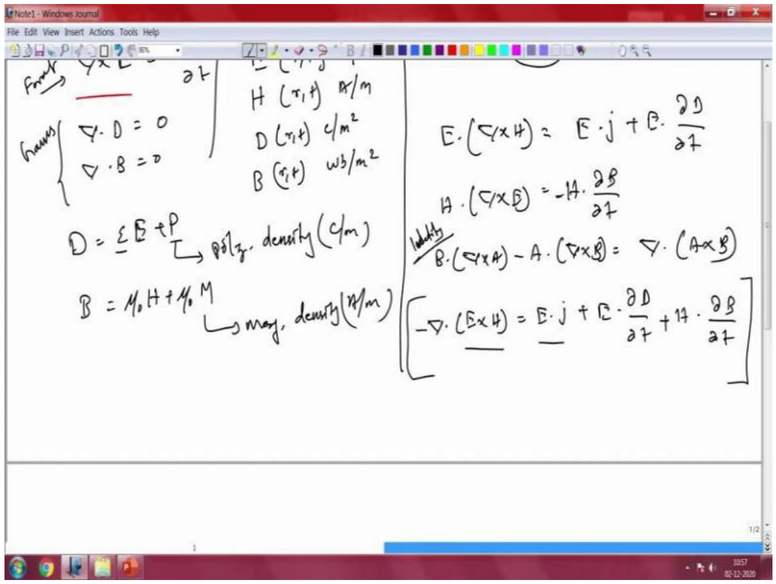

So, if we use this identity you can apply it to the above equation and if you do that, this is what you would get $-\nabla.(E\,XH)=E.j+E.\frac{\partial D}{\partial t}+H.\frac{\partial B}{\partial t}$. So, this is what you would get. So, what do we see here? So, if you look at this equation we will quickly notice that this is something that you must have encountered earlier on in your electromagnetic courses or in your introductory course in photonics that this is nothing but you are pointing vector.

So, E cross H is nothing but the direction of your energy flow and $E.j$, so E is the electric field and j is the current that you have and what is this, this $E.j$ is nothing but your powers basically this is voltage times similar to VI. So, this is similar to that. So, this is actually the energy that actually the power that we use in our conventional electric circuits.

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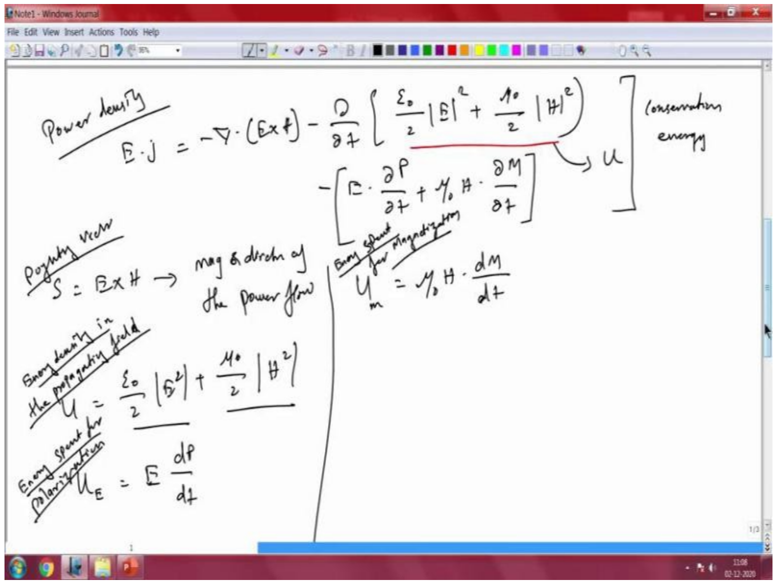

So, let us move on and then see how we can we can rearrange this. So, $E . j$ is basically your the power density so that you are. So, your power density is basically $E . j$ and the way that we could rearrange this is not sure whether we are using small j or capital J it does not really matter. So, now if you rearrange it, so it will become $E . j = -\nabla . (E\,XH) - \frac{\partial}{\partial t}$¿ .

So, all we have done here is bring in the expanded form of D and B. So, you want to use our earlier definition of the B and D here and you can expand it this way and now this tells us how the energy should be. So, what should be the energy or how what should be the power density of this. So, this is basically power density or energy density if you like. So, now the let us closely look at these factors, what they actually represent and what is the significance of this. So, let us do it one by one.

So, this basically is the pointing vector that we are already know. So, S is the pointing vector and that is nothing but E cross H. So, this is it represents the instantaneous magnitude and direction of power flow. So, this is telling you the magnitude and direction of the power flow. So, we do E electric field and magnetic field are orthogonal to each other. So, if we represent like this, this is x and y and z let us say you are electric field and magnetic field are this direction. Let us, say E and H and your energy flow should be perpendicular to this. So, the energy is going to move in this direction.

So, that means when you have oscillation when you have field oscillating in H direction and then when you have field oscillating in x direction here and y direction here, then the wave should move or the energy should be in the direction of z and this is how waves propagate. So, you have oscillation in x, y plane but then the energy it will be moving perpendicular to this, neither into this screen or out of the screen. So, this is what the pointing vector actually tells us.

The scalar quantity here this is the scalar quantity meaning this thing so whatever you have. Let us, look at that. So, what is that? So, this scalar quantity is a unit energy per volume. So, let us say this is u. So, this u is nothing but $\frac{\varepsilon_o}{2}¿E\vee¿^2+\frac{\mu_o}{2}¿H\vee¿^2¿¿$. So, what is this? This is basically energy density stored in the propagating field. So, this is energy density in the propagating field.

So, this is a very important quantity. So, it consists of two components as you can see here. So, this is the electric field the or the electric field component and magnetic field component at any instant of time. So, this is where the energy is actually stored. So, this is the stored energy of the propagating wave model and let us move on so that is the first factor is done and the second factor is done. Now, we know what this factored represent. Let us look at the third factor.

So, what is the third factor? So, third one is we call this as let us say u_E so which is $u_E=E\frac{dP}{dt}$. So, what is this? This is power density extended by the electromagnetic wave on the polarization. So, you are putting certain energy for polarization of this so energy spend for polarization, so you so you spend this energy for the polarization of the atoms inside the medium. So, this is energy spent inside the medium so there is a little correction here. So, the negative sign is this way. So, let us look at the third factor here or the final one the fourth factor here. So, that is u_M and u_M as you have guessed this is $u_M=\mu_o H\frac{dM}{dt}$.. So, what is this?

So, this is nothing but energy or the power density extended by the electromagnetic wave on the magnetization. So, this is energy spent for magnetization. So, if you look at the composition of this equation, so this is the power density that we saw. So, the magnitude and the direction of the power is represented here and then the energy stored component at a given point of time and then this is the third quantity here represent energy spent in the medium. So, the energy spent in the medium is very medium dependent. So, whether what is the magnitude of polarization and what is the magnitude of magnetization that one has in the material nonetheless this energy is spent.

So, overall when you look at this whole equation there the power density equation it is basically conservation of energy. So, conservation of energy as it propagates through a medium. So, we saw we took a small volume that we had here. But then when energy is when the wave is propagating through this there are a couple of things that can happen. So, how much energy is get taken through this material that is represented by these two the first two factors.

But then the third factor that you see here is going to be spent on the material itself. So, you can say that it is kind of loss or work done on the material. So, this is the way that you could find what is the power density of the wave or how much power a wave can carry as it propagates through a medium. So, once we understand this we can use these properties in order to discriminate various loss mechanisms, when the wave is traveling through a medium as well.

So, you can use this for characterization as well particularly, when it comes to material that are disposal and material that are non-dispersive. So, those all the things one can utilize to understand how much energy or optical power wave could carry. So, with us this part is covered. Let us, look at what is the effect of wave if it is a simple linear and non-dispersive medium. Let us, look at that rather quickly. So, you can take it from here.

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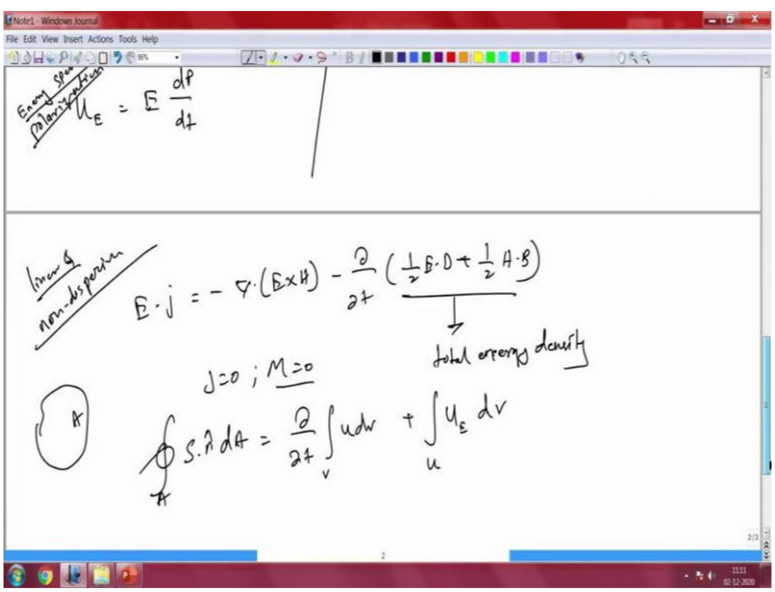

So, if it is a linear and non-dispersive. So, if it is a linear and non dispersive medium, you can write $E.j=\nabla.(E\,XH)-\frac{\partial D}{\partial t}(\frac{1}{2}E.D+\frac{1}{2}H.B)$ and basically this is the total energy that is being carried this is the total energy density. So, when you take an optical field where j is 0 and also let us say M is 0. So, it is a linear non dispersive medium if that is the case. Let us, say an A around a closed area A. So, one can use this to find out how much is the power density.

So, that means you are going to do let us say with around $\int S\lambda.dA=\frac{\partial}{\partial t}\int udv+\frac{\partial}{\partial t}\int u_E dv$ (()) (24:00) power that we had let us say $u_E dv$. Because the magnetic component is 0 so we are only looking at the electrical part alone here. So, using this we could quickly find out how much of energy we have in any (())(24:24) space were in any volume that you are interested in. So, it is just detection from the general equation that we saw here. So, this is more or less comprehensive general representation from here based on whether you have magnetization or not, what is the degree of polarization and so on you should be able to find out the energy in the system.

So, with that we understand how a wave could carry energy inside the system and when it is propagating through a medium it is the medium properties also influences the energy that you are going to carry. It is not that when you put the optical field into the medium, all the power is going to go through. So, the field is going to interact with the material and that is where you are polarization and magnetization property of the material comes in, most of the material the magnetization is 0, most of the energy that you spend is for polarization of the material that you have.

So, that is energies spend. So, you are going to spend some energy there and in what do you have left over is what you are going to transport through the medium. And this energy that you are spending on the polarization also depends on certain properties of the material itself. Because the polarization is not again a single constant number. So, it is again a series so probably when you think back you will write polarization as I expanded series, so you will have linear factor and then you have square and cube as a function of electric field.

So, it will be $\chi^{(1)}$, $\chi^{(o)}$ or $\chi^{(1)}$, $\chi^{(2)}$, $\chi^{(3)}$, so $\chi^{(2)}E^2$,$\chi^{(3)}E^3$ and all those. So, chi’s are nothing but your susceptibility that is actually creating your polarization. So, your when you spend energy to the polarization eventually you are going to lose that and it also depends on the electric fields strength and you have very low electric field you are you will only spend the linear loss there. So, you will not have sufficient E square and E cube magnitude and that will avoid this energy losses. There are a lot of other interesting effects that would come in when you have E square and E cube terms become significant.

So, with that brief understanding now we know how the energy propagates. In the next lecture we will see how to understand the solution to this wave equation and also a polarization. Let us, look at that in the next section.