

**Photonic Integrated Circuit**  
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**Lecture 28**  
**Directional coupler**

Hello everyone, welcome to another series of lecture on photonic integrated circuits. So, in this series we are going to look at integrated photonic passive devices. So, what do we mean by passive devices? So, we could broadly define passive devices as class of device that passively manipulate the property of light, let us say.

So, when we say passively, we are not going to apply any external energy into the system of any kind. So, the structure itself takes care of the manipulation. For example, if you take a lens; so, lens is a passive device, it is structured in such a way that either it focuses or it ((01:09)). So, this is the property of a passive device. On the other hand, we have active devices where actively we give and take energy.

So, light sources for example, LEDs, lasers are class of active devices where you are giving some external energy, that means, there is a current flow and this current flow results in photon generation. Similarly, light detectors, so, light detectors are again an active device, where you have to bias it, give some voltage to the system, so, that you can do photo detection.

And also there are electro optic devices, acoustic optic devices, where you are actively giving some external energy to manipulate the property of light. In case of passive devices, we are not going to do that. So, we rely on the structural property. So, we are going to fabricate a design is structured in such a way that the structure modifies the property. So, what all are the properties one could modify, let us say you want to split the power.

So, you want to reduce the power let us say of the propagating beam. So, you can do that. So, you want to filter out multiple wavelengths. So, I have all the wavelengths combined in the system, let us say, but I want only one wavelength out. So, which is called filtering. A simple wavelength filtering could be realised by simple gratings. For example, so, gratings are passive again.

So, the structure of the grating is making this wavelength selective functionality, we are not actively putting anything there. So, similarly polarisation. So, if I want to split the polarisation, the polarisation beam splitting now, so, we had we just discussed about beam splitting power splitting so now I want to take two polarisation and one polarisation should go in direction x the other should go direction y you can do that by using a passive device.

So, you can also realise other functionalities like combining and also interference. So, by using various passive elements that we just discussed, it could be combination of devices that one could use to make passive functionality. So, that is broadly how we look at passive devices. But in this lecture series, we are going to look at individual devices there are a couple of devices that are very key in building a photonic integrated circuit.

So, let us look at each and every component that we initially briefly discussed in the early part of our lecture. Probably you might remember that we talked about different geometries, coupling, splitting different passive devices or integrated photonic components we looked at. So, now we are going to look at each and every component with key components. We will not have time to discuss about everything but there are key components with which you can realise, different functionalities. So, we are going to discuss that. So, let us have a detailed look at some of these components.

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Optical Couplers

- polarisation conversion
- mode conversion
- power splitting/combine
- wavelength multiplexing/de-multiplexing

Directional Coupler

X-section

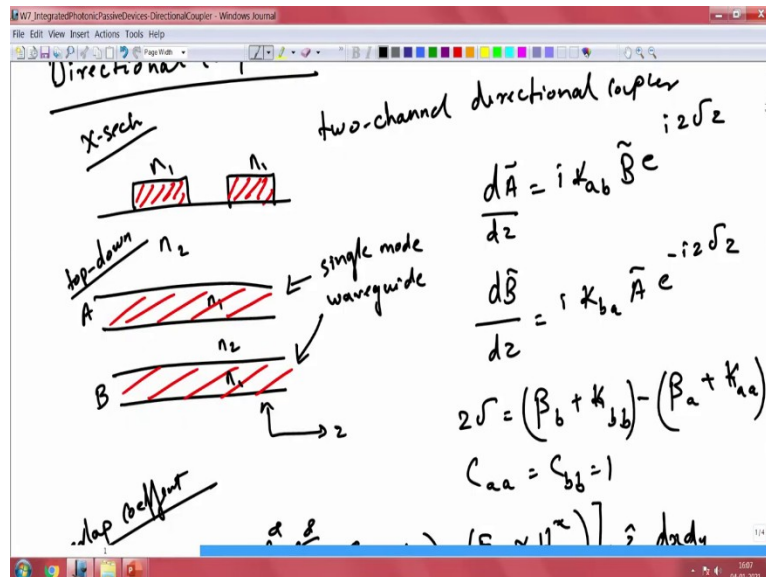
Two-channel directional coupler

k mode

$$\frac{d\vec{A}}{dz} = i\kappa_{ab} \vec{B} e^{i2\delta z}$$

So first, let us start from where we left in the last lecture, so, we had a lot of discussion about coupling like mode coupling and so on. So, optical couplers is a general word you could use to cover a lot of different optical functions. So, you could have polarisation conversion, you could have mode conversion, you could have power splitting, power combining operation and you can also have wavelength a selective operation; wavelength multiplexing, wavelength demultiplexing. So, these are all the optical coupling could do.

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$$\frac{d\tilde{A}}{dz} = iK_{ab}\tilde{B}e^{i2\delta z}$$

$$\frac{d\tilde{B}}{dz} = iK_{ba}\tilde{A}e^{-i2\delta z}$$

$$2\delta = (\beta_b + K_{bb}) - (\beta_a + K_{aa})$$

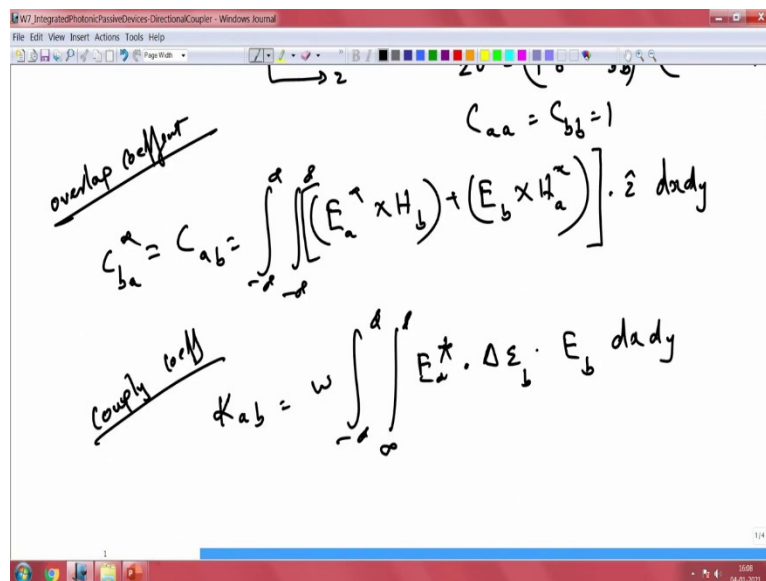
$$C_{aa} = C_{bb} = 1$$

So, let us look at a very simple structure here what we call? Directional coupler. So, what you see here on the left side is how the coupler looks like in the cross section. So, it has two rails here two waveguides made out of material index  $n_1$  surrounded by material index  $n_2$ . So, this could be two channel directional coupler you could have many, but then the coupling also is going to be complex here.

Let us take a very simple case of two waveguides here and this is top down how they look. So, there are two wave guides A and B the waves there in A and B as a function of, of length how are they going to evolve as they propagate is given by this equation this coupled equations

which is something we already know of. So,  $\kappa_{ab}$  and  $\kappa_{ba}$  and then you have the phase mismatch component  $\delta$  and what is that  $\delta$  is nothing but  $\beta_b$  plus  $\kappa_{bb}$  minus  $\beta_a$  plus  $\kappa_{aa}$ . So, this is this is something that we already know from our earlier discussions.

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$$C_{ba}^* = C_{ab} = \iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} (E_a \times H_b^*) + (E_b^* \times H_a) \cdot \hat{z} dx dy$$

$$K_{ab} = \omega \iint_{-\infty}^{\infty} \iint_{-\infty}^{\infty} E_a^* \cdot \Delta \epsilon_b \cdot E_b dx dy$$

So, now, we want to understand how one could use this understanding in order to couple light between these two, to transfer energy. The other thing that we saw was on the overlap integral so, that is something that we also very clear about. So, that there is again something that we know, so, one can write this overlap integral as we know from the two different fields the field in A and field in B that is overlap coefficients and let us look at coupling coefficient.

So, the coupling coefficient kappa ab is nothing but  $E_a$  dot delta epsilon of b and  $E_b$  dx dy. So, this is our coupling coefficient. So, the other one was overlap integral, this is the coupling coefficient. There is nothing new here we have seen this earlier as well when we discussed about mode coupling. So, let us look at a few scenarios here.

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Handwritten notes on a computer screen showing waveguide diagrams and equations for overlap and coupling coefficients.

Diagram 1: Two parallel waveguides, A and B, with refractive indices  $n_2$  and  $n_1$  respectively, separated by a distance  $d$ . The z-axis is horizontal.

Equations:

$$\frac{d\beta}{dz} =$$

$$2\beta = (\beta_b + K_{bb}) - (\beta_a + K_{aa})$$

$$C_{aa} = C_{bb} = 1$$

Overlap coefficient:

$$C_{ba}^* = C_{ab} = \iint_{-\infty}^{\infty} \left[ (E_a^* \times H_b) + (E_b \times H_a^*) \right] \cdot \hat{z} \, dx dy$$

Coupling coefficient:

$$K_{ab} = \omega \iint_{-\infty}^{\infty} E_a^* \cdot \Delta \epsilon_b \cdot E_b \, dx dy$$

Handwritten notes on a computer screen showing a cross-section of a directional coupler and the phase matching condition.

Diagram 2: Cross-section of a directional coupler with two waveguides of width  $w$  and refractive index  $n_1$ , surrounded by a medium with refractive index  $n_2$ .

Equations:

$$K_{ab} = \omega \iint_{-\infty}^{\infty} E_a^* \cdot \Delta \epsilon_b \cdot E_b \, dx dy$$

Phase matching condition:

$$\beta_a + K_{aa} = \beta_b + K_{bb}$$

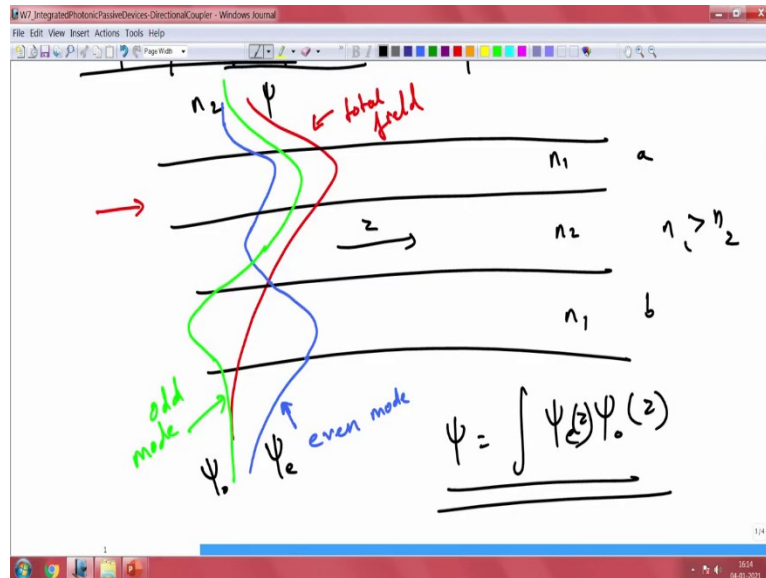
$$\beta_b + K_{bb} = \beta_a + K_{aa}$$

So, again recall here so, we are just recalling from what we have seen earlier, let us look at symmetric directional coupler. So, there is a directional coupler where we have two waveguides. This is cross section of equal width. So, I have  $w$  here and then I have  $w$  here and  $n_1$  and  $n_1$  and surrounded by refractive index  $n_2$ . So, if you want the phase matching to be done that means the delta here.

So, this is the phase matching condition here. So, the delta should be equal to 0. So, when you have phase matching. So, then you will have, so this is the condition for phase matching. So, once you have the phase matching, it will automatically come into play. So, let us now

look at how the energy transfer is going to happen when you have, two wave guides like this. So, let us know simply draw these wave guides and then see how we could understand this.

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$$\varphi = \int_z \psi_e(z) \psi_o(z)$$

So, I have two waveguides here, waveguide a, waveguide b. And now, we all know these two are two different waveguide systems. So, that means I am going to transfer energy from a to b, but I have a solution in a. So, I am putting energy in this. So that means my solution the effective I will have effective solution like this, so this is how my solution or mode is going to look like.

But then, it is not perfect to say that, this is the only solution that we have, because this is a coupled waveguide system, they are closely spaced and because of the closeness of this instead of having individual solutions, they will have combined solutions, what we call the super modes.

So, now, you will have even and odd modes now. So, we know this refractive index is n1, this is n2 and this is n1 and we know n1 is greater than n2, if that is the case you should have oscillating solutions in n1 and decay solution in x2, sorry n2. So, if that is the case, so, this is our, our total field, let us say this is our total field, but now you can decompose this total field as a function of individual fields.

So, you will have a solution that will look like what we call the super mode solution, so, I will pick one colour here. So, we can have a solution like this. So, this is one solution, this is what we call even mode or even solution and then you will have odd solutions, so, odd solution would look something so, this is our odd mode. So, now, this total field is a combination of the odd and even that you have.

In this particular case we only have 2. So, this total field is nothing but the overlap integral. So, if I have to mark them if this is  $\chi$ , so, you have  $\chi E$  and then let us say, so, this the total field that you have  $\chi$  is nothing but your overlap between  $\chi E$  and  $\chi$  naught. So, the odd and even overlap is what gives you the total field here. So, similarly, you could have a field here in B.

So, as it propagates through the system, so, to this particular waveguide system. So, you could have evolution of this. So, you after passing through a certain length, you are going to have beating between these two. So, because they are phased match, when these mods are phase match, then they are going to beat with each other, there is a this overlap field. So, the overlap field is a function of the propagation length as well.

So, they are going to be with each other. So, you will end up with highly different modes here. For example, after passing through a certain distance  $l$  you might have, the total solution would your total field might look something like this. So, this could be your total field now. So, in this case, you will have the odd mode will be similar. So, your odd mode could be the same here, what we saw and even mode again will be inverted in this case.

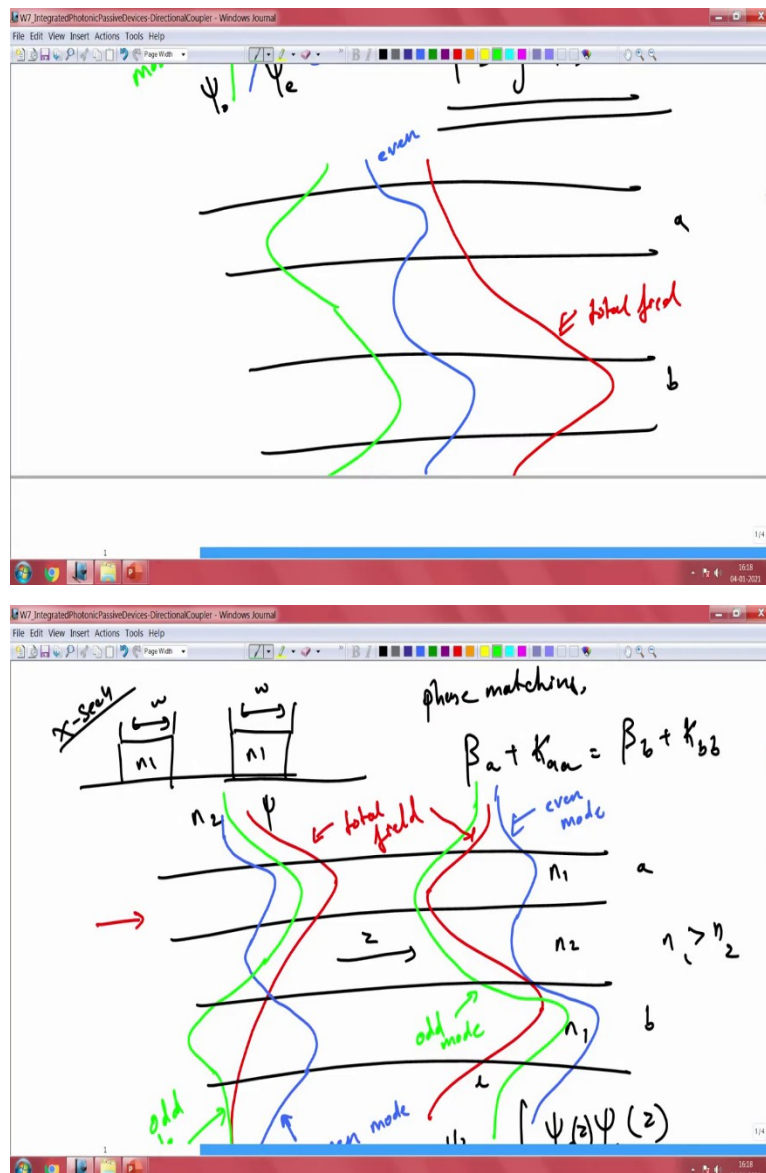
So, even mode will be, so this is our even mode. So, the odd more and you can see here the implication of having the phase difference between the odd and even mode and that puts the power, the major power. Maybe I should show it explicitly here this will be smaller here, larger here. So, the other one was correct. So, let me. So, this is our total field now, and this is our even mode and this is our odd mode and then we have the total field.

So, this is again the total field. So, you can see here the total field is now on the lower waveguide, so, or rather distributed between the lower and the upper waveguide. So, there is a equal power splitting between a and b now. So, that is because of the change in our odd mode. So, you look at the odd mode it has completely shifted, so, while your even mode remains the same.



So, the even mode remains the same your odd mode has completely flipped, you see, so, there is a 180 degree phase change that we see in the odd mode and because of that phase change, now, the total field is now distributed between the two waveguides. So, the red curve that you see here, so, initially it was only in a but now, we have a and also b. So, now, there is a power splitting that has happened between these two and if you keep flowing through the length of this directional coupler, you will come have a complete power transfer. So, that is what we are going to visualise now.

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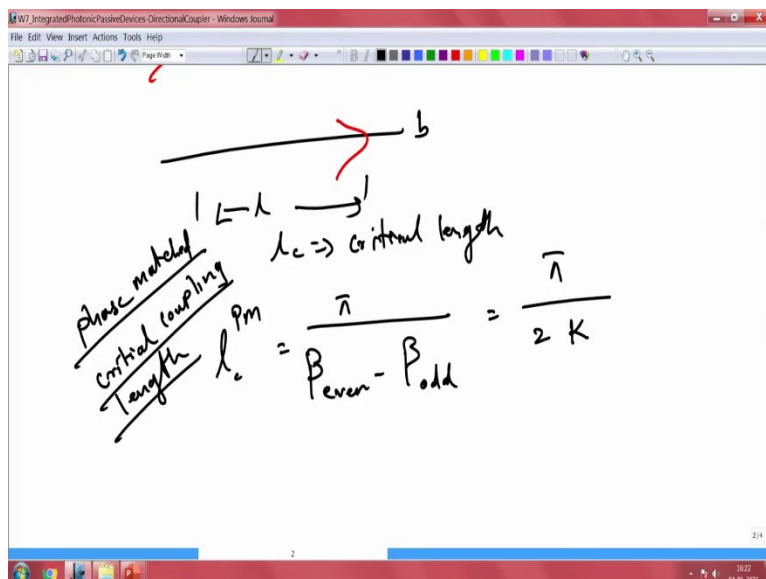
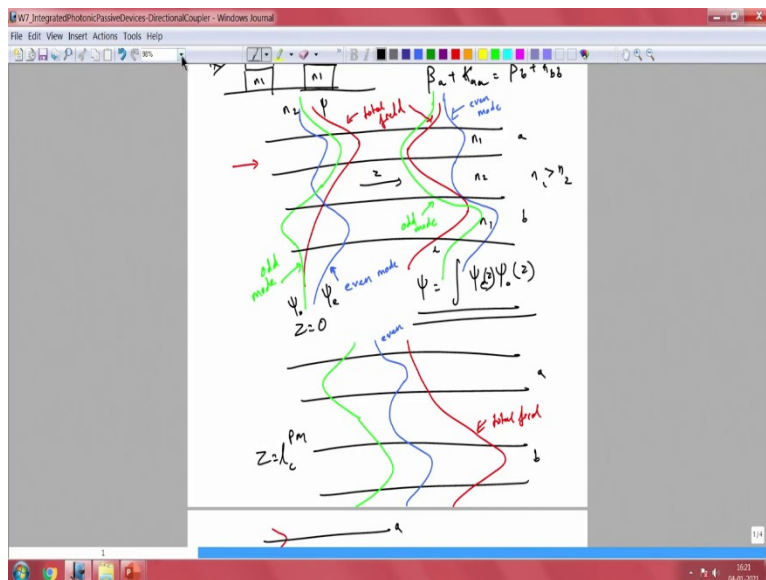
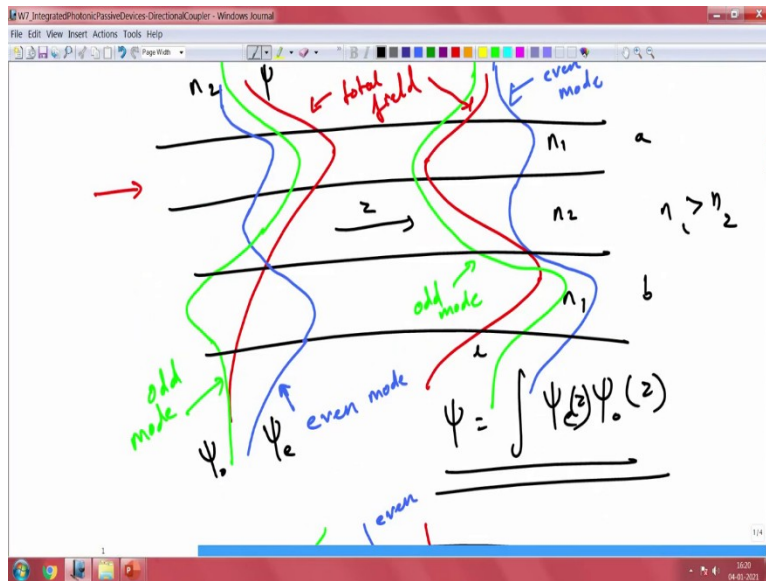
So, if you take the two waveguides here again a and b. So, when there is a complete shift here and say as it propagates through the system, you could have a complete power transfer to b.

So, this is our total field. And you will have similar kind of even and odd. So, the odd is going to be, even is going to have something like this.

So, this is even mode and you will have your odd mode something like this. So, now, the coupling would be slightly different here, but the length the travel will be also different and we are going to talk about the length shortly. So, these are all the things that you could expect as the wave propagates through the system.

But this all depends on, the propagation constant of these modes. And that is given by our phase matching. So, let us look at we have seen this in our early discussion as well. Let us recap that. So, when you when we talk about phase matching, we normally designated to  $\Delta$  and when the  $\Delta$  is 0, we have this, but what is the length that is required in order to transfer the energy from one to the other. The critical length probably you remember, so, that critical length?

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$$\eta_{PM} = \sin^2(kl)$$

$$l_c^{PM} = \frac{\pi}{\beta_{even} - \beta_{odd}} = \frac{\pi}{2k}$$

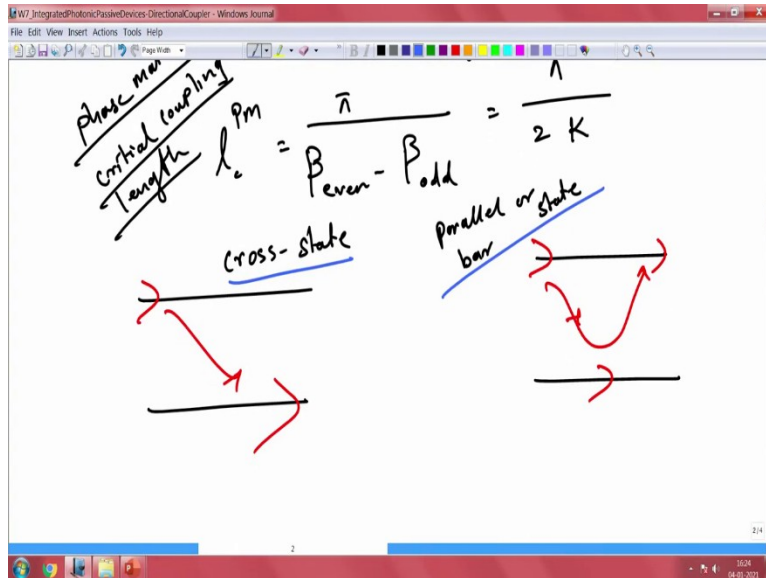
So, that is a critical length. So, I will draw a stick diagram here. So, you have a wave here and I want the wave to transfer from a to b and this length that is required is called critical length. This is the critical length. So, what should be this critical length? And as we saw, this critical length depends on these two modes that we have. So one is the odd mode and the other one is the even mode.

So, it depends on the odd mode and the even mode. So now, we can write it the phase match length. This is the critical phase match PMS, phase match. Critical coupling length is given as pi divided by beta even minus beta odd. So, why pi? That is the phase shift that you saw here. So, you want to have this a complete phase shift that is required in order to make sure between these two let me reduce it a bit so that we can see both of this together.

So, when you look at these two, this is position 0, let us say this is z equals to 0 here and here is z equals to the phase matched coupling here. So, what does it take for the wave to go from one waveguide to the other waveguide? It should have achieved the complete phase shift? They should be complete pi phase shift. So, that is what we want and that is given by this particular relation.

So, the coupling length is pi divided by beta even and beta odd, which is not difficult to find. So, we can easily find it out or in other words, it will be pi two kappa here. So, this is our phase matching condition and one can find this and solve this for any given structure. So, now, let us look at a special case here, where these two waveguides are identical. So, that is what we looked at.

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So, I have I have two simple waveguides. So, there are two things that can happen; one is I have a wave here and let me shorten this this to make. So, there are two case that you could do I start with wave the wave here and the wave goes to the other and then it comes to the next wave going from A to B and this is called cross coupling or cross state. So, this is cross state. The next one is where you have the wave in one waveguide it can go here, but then it will come back.

So, it does this so, it goes back to the same waveguide. So, in this case, we call this as a bar state or a parallel state, parallel or bar state. So, they are useful for various aspects. So, this is what you could do by just changing the length of it. So, let us look at how this coupling length can be calculated. So, we just saw it, but how what is the coupling efficiency we can get out of this. So, the coupling efficiency is given from our earlier definition of coupling efficiency of from the coupled mode theory.

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coupling efficiency  $\eta = \sin^2 |\kappa| l$

$l_c^{PM} = \frac{\pi}{2|\kappa|}$

Length to achieve desired coupling eff  $l = \frac{1}{\kappa} \left( n\pi \pm \sin^{-1} \sqrt{\eta_{PM}} \right)$

$l = 2 \left[ n \pm \frac{1}{\pi} \sin^{-1} \sqrt{\eta_{PM}} \right] l_c^{PM}$

$$\eta = \sin^2(|\kappa|l)$$

$$l_c^{PM} = \frac{\pi}{2|\kappa|}$$

$$l = \frac{1}{\kappa} (n\pi \pm \sin^{-1} \sqrt{\eta_{PM}})$$

$$l = 2 \left( n \pm \frac{1}{\pi} \sin^{-1} \sqrt{\eta_{PM}} \right) l_c^{PM}$$

So, your coupling efficiency  $\eta$  is given as  $\sin^2 \kappa l$ . So, this is our coupling efficiency. So, phase match length is now, so,  $l_c^{PM}$  is given by  $\pi / 2\kappa$ . So, now we can do rearranging these two. Let us say there is a perfect coupling happening. So, that means for  $\eta_{PM}$ . So, this implies a perfect phase matching will happen  $\sin^2 \kappa l_c^{PM}$ .

So, now we can write your coupling length here, so you can also call this one but we want to know this particular length, at which you will get the perfect phase match. So, for that, let us apply this  $\eta$ . So, what should be that  $\eta$  or what should be that  $l$ . So, let us go back and write this  $l$  can be given as  $1/\kappa$  this is  $n\pi \pm \sin^{-1} \sqrt{\eta_{PM}}$ .

So, this is what I want which is nothing but  $2(n \pm 1/\pi \sin^{-1} \sqrt{\eta_{PM}}) l_c^{PM}$ . So, this is our desired length of the coupler. So, this is desired length to achieve desired coupling efficiency of  $\eta$ . So, now, for perfect coupling you will have this  $\eta$  as 1 so, there is a complete power transfer. But then you may want to have this as 0.5, 50:50

power coupling in those cases, you may want to find what is the length and you can easily find it.

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$$l_c = \frac{\pi}{\beta_{\text{even}} - \beta_{\text{odd}}} = 2K$$

Critical Coupling Length  $l_c$

$\eta = \sin^2 |K| l$

$l^{pm} = \frac{\pi}{2|K|}$

Cross-state Parallel or state bar

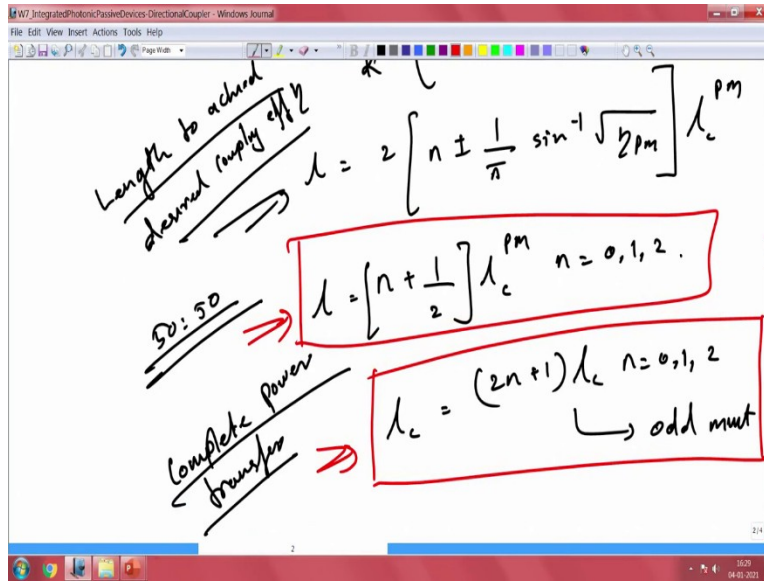
$$l = \frac{1}{K} \left( n\pi \pm \sin^{-1} \sqrt{\eta_{pm}} \right)$$

Length to achieve desired coupling efficiency

$$l = 2 \left[ n \pm \frac{1}{\pi} \sin^{-1} \sqrt{2\eta_{pm}} \right] l_c^{pm}$$

50:50  $\Rightarrow l = \left[ n + \frac{1}{2} \right] l_c^{pm} \quad n = 0, 1, 2$

All the power  $\Rightarrow l_c = (2n+1) l_c \quad n = 0, 1, 2$   
 $\hookrightarrow$  odd mult



$$l = \left( n + \frac{1}{2} \right) l_c^{PM}; n = 0, 1, 2$$

$$l_c = (2n + 1) l_c^{PM}; n = 0, 1, 2$$

So, let us say I want 50:50 power splitting. So, 50:50 power splitting the length would become  $n + \frac{1}{2}$  or let us let us start with, yeah, you can you can also do this in  $1 + \frac{1}{2}$  into  $l_c^{PM}$ , so, this is our 50:50 coupling where  $n$  is an integer. So, now, if you want to have complete power transfer, so complete and complete power transfer. So, your  $l_c$  so, that is the critical coupling will be equal to  $2n + 1$   $l_c$ .

So,  $n$  is equal to 0, 1, 2. So, this is nothing but odd multiples, all odd multiples is going to give you a complete power transfer. So, when you launch optical power in one waveguide you may want to completely transfer it to the other waveguide that is one possibility. So, that is the cross state and bar status going back and there will be an another possibility here. So, where you could have a light in both the waveguides.

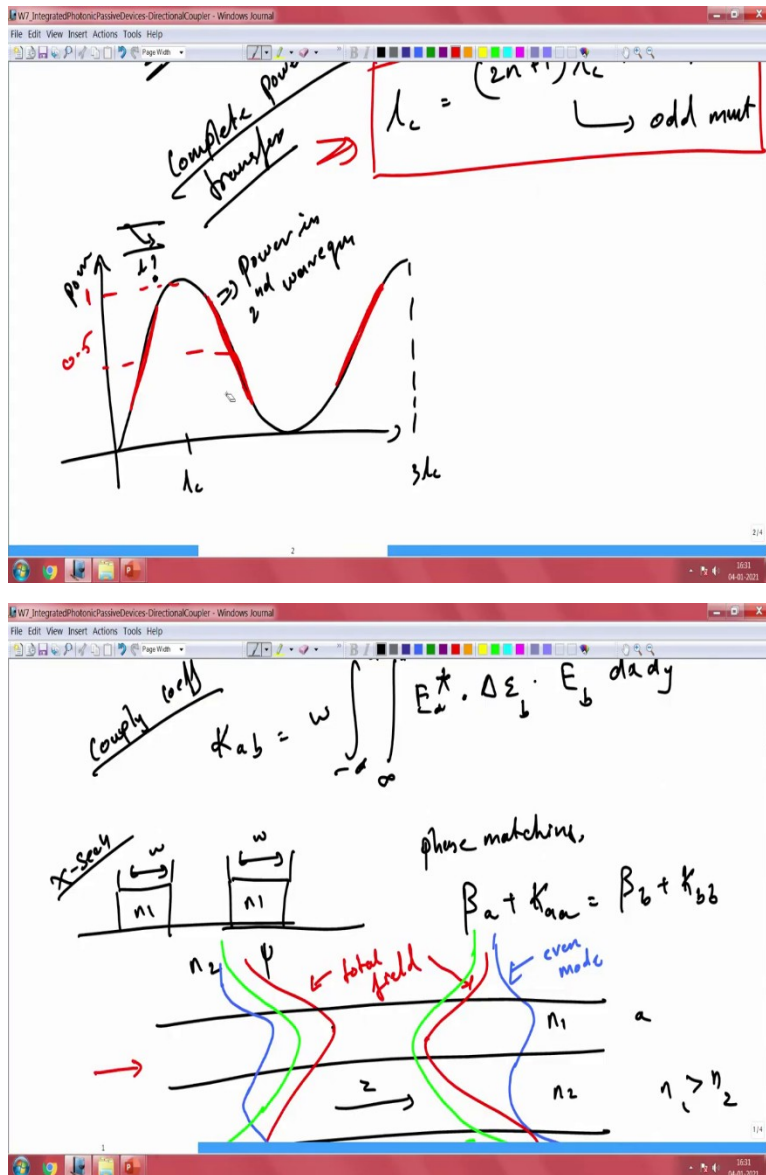
So, in between these two, you could have it in both. So, this state is a 50:50 condition. So, here is a complete transfer. So, what is the length required for this? And what is the length required for this could be found by simply using this. So, these are all two important equations. So, this is for 50:50 coupling and this is for complete power transfer. So, these two relations keep in mind.

So, now, the question to ask is what can we do beyond this. So, I can, this is basically building a beam splitter of arbitrary splitting ratio, I can do 50:50, I can do complete power



transfer and I can do even in between so that is where you are critical length comes in here. So, you can choose whatever is decide coupling and then find out what is the length required for that.

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So, this is anyway going to oscillate between the two states. So, you have going to go power will do something like this, a complete power transfer will require  $\lambda_c$ . So, it is an odd multiples is what we are looking at. So, this will be  $\lambda_c$  and here this will be  $3\lambda_c$  and so on. So, this is the power in second waveguide. So, you have two waveguides. So, what is the power coupling here?

And what is the length required in order to achieve power coupling. So, this region we can exploit for other power. So, this is 1 of course and here is our 0.5 when you can get anything arbitrary that you want, based on this curve it is reasonably linear in this region. So, you can exploit some length here. So, that you will get the power ratios that you are looking for when you are building a power splitter or power distribution network.

So, you can make this active if you want we will we will come to that shortly, but whatever we are discussing is all passively done. So, the structure itself takes care of it. So, what else can we do with this, so we are just talking about power and also we just spoke about only the single mode. So, these are our two identical waveguides. So, that means, the waves in these two solutions are fundamental normal solutions and relatively easy to handle. So, what happens if you want to do polarisation splitting?

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Length to achieve desired coupling eff

$$l = 2 \left[ n \pm \frac{1}{\pi} \sin^{-1} \sqrt{2pm} \right] \lambda_c^{PM}$$

50:50

$$\lambda = \left[ n + \frac{1}{2} \right] \lambda_c^{PM} \quad n = 0, 1, 2$$

Complete power transfer

$$\lambda_c = (2n + 1) \lambda_c \quad n = 0, 1, 2$$

never in eqm

Polarisation Splitting

TE+TM

TE

TM

$l$

$$l = \frac{n\pi}{K_E} = \frac{(2n \pm 1)\pi}{2K_M}$$

$$l = \frac{n\pi}{K_E} = \frac{(2n \pm 1)\pi}{2K_M}$$

So, that is again something you can do. So, let us look at, how one can look at polarisation splitting. So, what is polarisation splitting? I have two waveguides now, the same thing. So, in one of the waveguides I have TE plus TM. So, my job now is to split this. So, I want TE to come out this way and then I want TM to come out this way and this waveguide. So, that means, I have this polarisation, this.

So, this is how it is, but I want TE to come out this way and I wanted TM to, I wanted TM to come out this way. So, I need to design this length appropriately. So, we know this length is crucial. What is this length that is required to split this? So, in one case I want the TE there, the other case TM here.

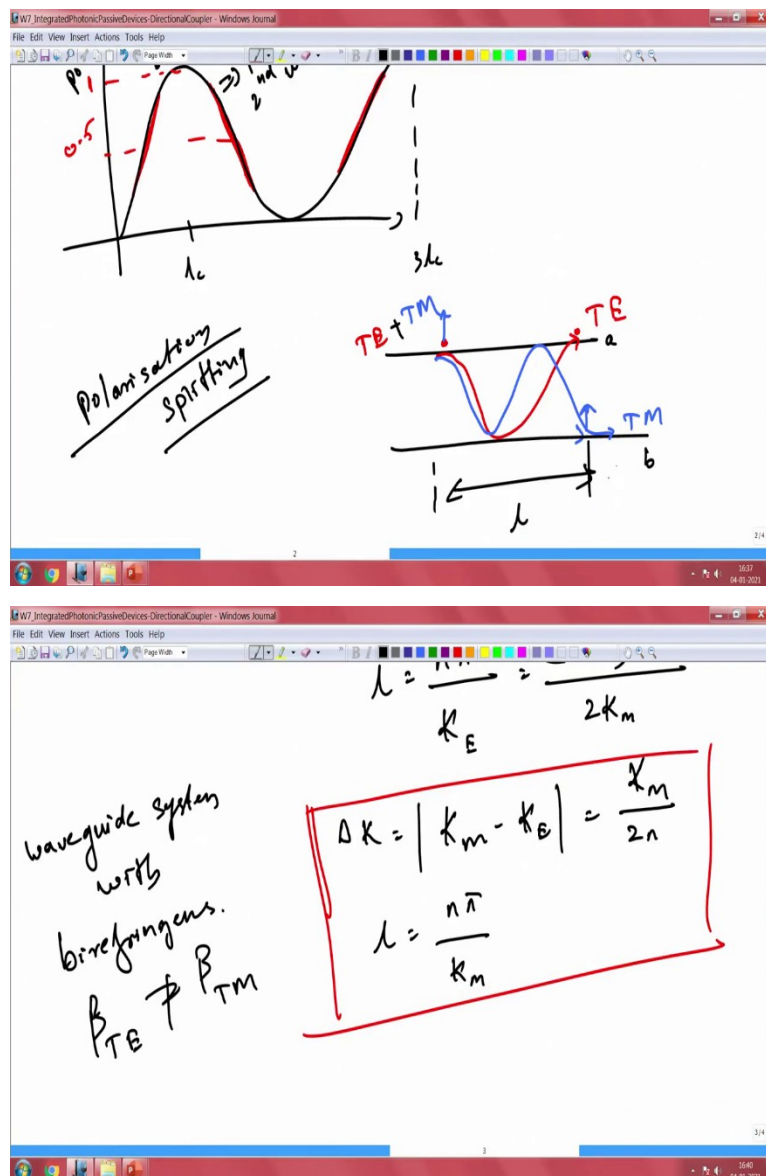
So, it could be done in different ways. So, we could do, you take the TE it can come down and then it can go back this is one way of doing and for TM could do like this. So, now, we can split this to polarisation. So, now, it is rather easy to draw it like this, but why TE has longer beat length compared to TM? That is because of the effective index or propagation constant of this.

So, now, we can write that equation by looking at the required phase matching condition. So, here we looked at the complete power transfer that is required from one waveguide to the other waveguide. So, we can use that in order to understand how to couple these two waveguides. So, the coupling length is going to be fixed, but what is the phase matching condition.

So, there if you want to have this happen, so, this is how it should be. So,  $\kappa_e$  electric, which should be equal to  $2n$  plus or minus  $1$  by  $2\kappa_m$ . So, that is electric and magnetic. So, this is the relation that you have to satisfy in order to get the 2 polarizations splitted. So, the coupling coefficient  $\kappa_e$  and  $\kappa_m$  are for TE and TM waveguides even though these waveguides are symmetric, you will have two different coupling because these are all two different waveguide polarizations.

So, two different solutions altogether and that is why you need to make sure that this is there. And this is possible. This is possible if there is a difference between the coupling coefficient that is equal to the following.

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$$\Delta K = |K_M - K_E| = \frac{K_M}{2n}$$

$$l = \frac{n\pi}{K_M}$$

So, if you have delta kappa is equal to kappa m minus kappa E in this case which is equal to kappa E by 2n. So, this is just 2n is nothing but your integral number there. So, you could from this you could easily calculate that what is the required length in order to split this. So, for polarisation splitting to happen where TE comes out in the cross and TM to be in the bar we need to satisfy this particular condition.

So, it has to be some integral of this form and your length will be equal to  $n\pi$  over  $k$ .

So, this is  $k$ . So, this is what we should use if you want to do our polarisation splitting. So, if you want to do polarisation splitting, this is what you should make sure. The reason why the length  $n\pi$  over  $k$  because you want  $m$  to be on the other side.

So, you instead of having it on the a, you want to have it in b. So, what is the condition for it to be in b it is nothing but odd multiples of this. So, it is all coming back to how you should have this. So, this is just basically what dictates your coupling here. So, that is how we explored this directional couplers for coupling or splitting power, combining power and also splitting and combining different polarizations as well.

So, this polarisation becomes even more interesting when you talk about particularly in some of the circuits where you want to do different sort of polarisation dependent processes, but most of the time you will you want to make sure that you only handle one single polarisation in the system. So, this kind of polarisation splitters allows you to take this particular polarisation splitting quite easily there is something important that we should also keep in mind here when we talk about the difference in the coupling.

So, this difference in the coupling, you want to make sure that this is taken care this only works with certain  $\Delta k$ . So, you should have some difference. But if you do not have this difference, you will not be able to split it at all. So, if you remember these are two different waves propagating through the system. So, unless there is a propagation constant difference between these two propagating modes TE and TM, you will not be able to do this polarisation splitting.

So, what is required in order to establish this you need to have birefringence for this, you need to have waveguide system with birefringence. So, that means your  $\beta_{TE}$  is not equal to  $\beta_{TM}$ . So, there should be some difference in this case and only when you have this difference, you will be able to establish this, you could actually use exploit this from the medium itself.

So, we know that ordinary and extraordinary axis or we could build a waveguide system where you could achieve this difference in the propagation constants. So, with this we have understood directional couplers. So, we have taken a very simple two waveguide system and couple light from one waveguide to the other waveguide. And you could do it by using critical length.

And what is that length depend on we looked at the super modes. So, the super modes are interfering. Super modes are nothing but when you do the solution with two waveguides as a single system, if you remember in a couple mode with theory, we studied how you can decompose it. If it is a 2 mode system could be decomposed into 1 mode, 1 waveguide system, and then you remove it and put the next mode, next waveguide and find the solution.

So now these two solutions should match each other, or you have to find the integral between these two waveguide. So, in this case, that is exactly what we are putting, when you put these two waveguides then the solutions are going to be different and that that solution could be odd and even. So, in this particular example, we only took single mode. So, there are only two modes allowed in this system, between these two.

We had the odd mode distributed between these two even more, because of the profile itself. And when you take these two modes, they are going to interfere as they progress through because there is a phase change as they progress. So, based on that phase change and the beating, we get this particular length, the critical length that we have, it is nothing but beat length, we call it  $\pi / (\beta_b - \beta_a)$ . So, that is all you want.

So, you can put it under modulus, modulus  $\beta_a - \beta_b$  is what is your beat length. So, based on that, what is the length required to completely transfer the power and the next thing we saw was how do you exploit this for polarisation splitting. So, one essential thing to remember is when you are doing polarisation splitting the waveguide system that you have should be birefringence and that means the  $\beta$  of TE and  $\beta$  of TM should be different.

So, they if they are same then you will have, you will not be able to split it because as they propagate, they are going to accumulate same phase or the beat length is going to be the same you do not want that. So, if you want to do that, you should have birefringent system and that could be inbuilt by design itself. So, you could design the waveguide system where the TE and TM waveguides are sorry TE and TM mode propagation constants are different.

So, unless you have it you will not be able to split or combine these two polarisation to put it into one of the waveguides. So, with that, we understand this directional coupler. We will revisit this directional coupler when we talk about a ring resonators, so in ring resonators, one of the components or one of the elements is directional coupler. So, with that understanding we close this discussion on directional coupler. See you in the next section with another passive device.