Photonic Integrated Circuit
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Lecture No. 26
Co and Counter Propagating Mode Coupling

Hello everyone, let us look at mode coupling in a completely practical sense now. So, so far we looked at what are all the basic that one should understand in order to couple energy from one mode to another mode, which we will be exploiting throughout our photonic circuit design, so this is one of the essential foundation should say in guided wave optics in particular photonic circuits.

So, let us look at two scenarios, this is something that I briefly mentioned in one of the earlier lectures, but I will again give you a recap, so when you talk about mode coupling, so that the understanding is there are two, but you can also argue that there could be only one mode coupling to itself, it is also possible, where you have a perturbation in the system. So, that is what we call self-coupling, but in this case, let us take two modes, mode a and mode b.

So, these two modes are solutions in the waveguide, it could be a single waveguide or this could be two different waveguide and they are propagating parallel to each other. So, when they are traveling parallel to each other there are two ways that you could look at this, one they are Co propagating, could be in single waveguide they are Co propagating or in two different waveguides they are Co propagating, that is scenario 1.

The scenario 2 is they are still parallel, but they are moving in opposite direction. So, one is going from right to left, the other one is left to right. So, when this is happens then we call that as counter propagating waves, but then these two are normal solutions, with the question here is, what is the coupling condition in order to transfer energy from mode a to mode b when they are together in same direction or in the opposite direction.

So, the condition here is there they are there is some perturbation that one could bring in, in order to couple this, so there is a kappa is non-zero here, so there is a kappa associated with this coupling, so we will see what is the nature of this kappa, so let us look at this and understand how one can couple these two waveguides. So, there are two scenarios as I mentioned, but let us look at a very simple waveguide system.

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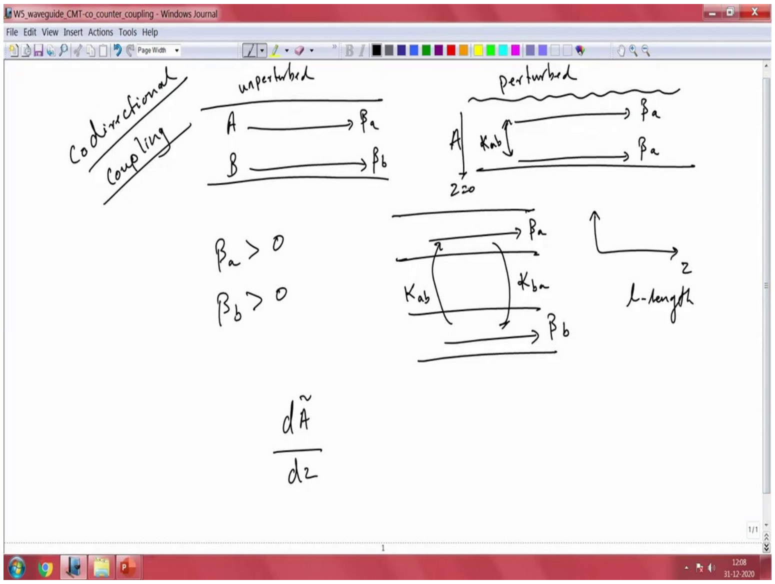

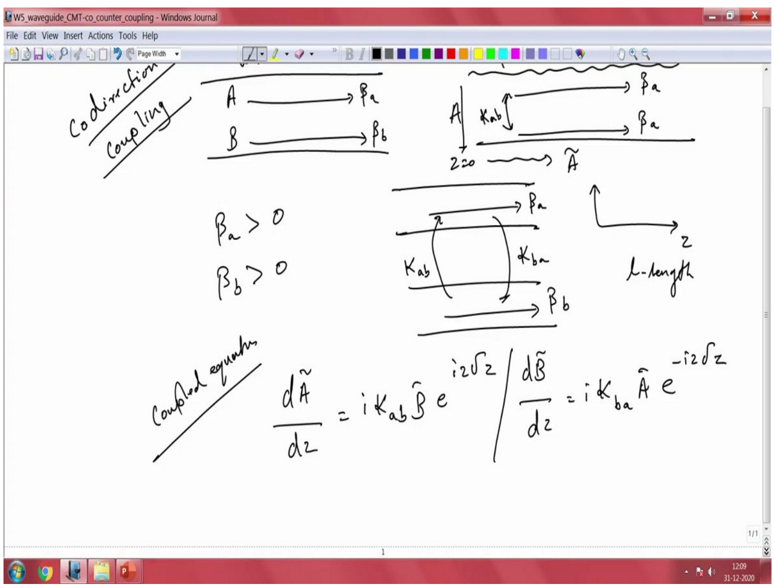

$$\pm \frac{d\tilde{A}}{dz} = iK_{ab}\tilde{B}e^{i2\delta z}$$

$$\pm \frac{d\tilde{B}}{dz} = iK_{ba}\tilde{A}e^{-i2\delta z}$$

So, if I take a very simple waveguide you would have beta a and then beta b in such a system you will not have any coupling at all, so these two are Eigen solutions let us say a and b, if they are Eigen solution you would not be able to couple at all. So, these are all normal solution, we saw that in the earlier discussion, you need to have some perturbation, so there should be some disturbance in the system.

And this disturbance could be brought in by let us say a simple roughness, so here we have a very smooth surface, but let us say the surface is rough and in that case you create a coupling between these two, so there should be a coupling between these two waveguides. So, this is unperturbed and this is perturbed and what we are looking at is co-directional coupling like they are moving in same direction, so that is what it means. So, this is scenario 1 where we have only one waveguide.

So, how about two waveguides, so now this is already a perturbed system we do not have to add additional perturbation here, so beta a is propagating there and this is beta b, so what we are looking at is whether we will be able to couple from here this is kappa ab and kappa ba. And similarly you have kappa ab here. So, what is the requirement in order to make this coupling happen?

So, in case of a single wave guide we need to have the perturbation like this the roughness let us say in order to enable this coupling, but in a two waveguide system that perturbation from, comes from the proximity itself. So, now let us say the waves are moving in forward direction and direction here meaning you know is that and they are interacting over some length l, so l is the length of interaction.

So, here both betas are positive sign, because they are moving the forward direction and if this is the case then our coupled equation that we saw earlier as a function of distance, so A tilde here is nothing but how A, so you can start with A here and as it propagates through the system, so let me take this as an example, you start as A when z equal to 0, but as it propagates through the system because of your perturbation it will be you know it will become A til.

So, that is the A inside or the magnitude or the wave inside this perturb system, so A til over dz is given by Ii kappa ab again B til e to the power i2 delta z. And similarly for B dB delta over dz is nothing but i kappa ba A tilde e to the power minus i, so this is our coupled equation. So, A tilde is the profile of A as it moves along z direction, so we can solve this using a very simple initial value problem.

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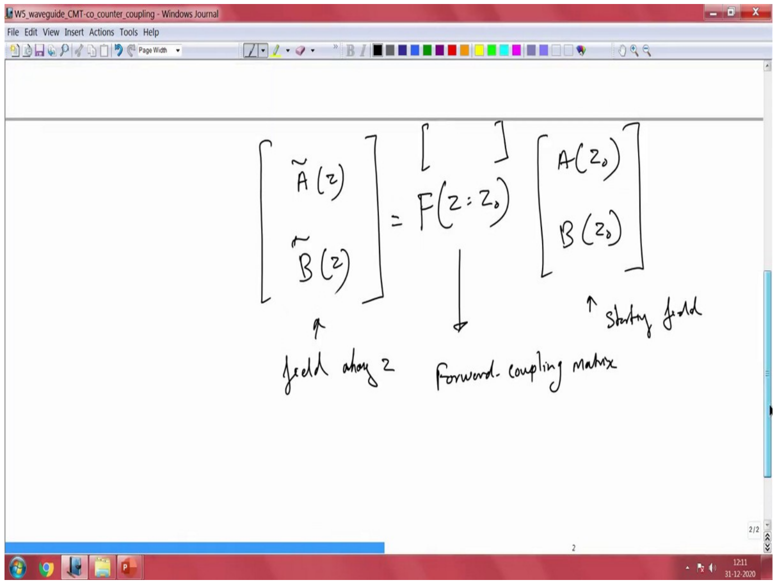

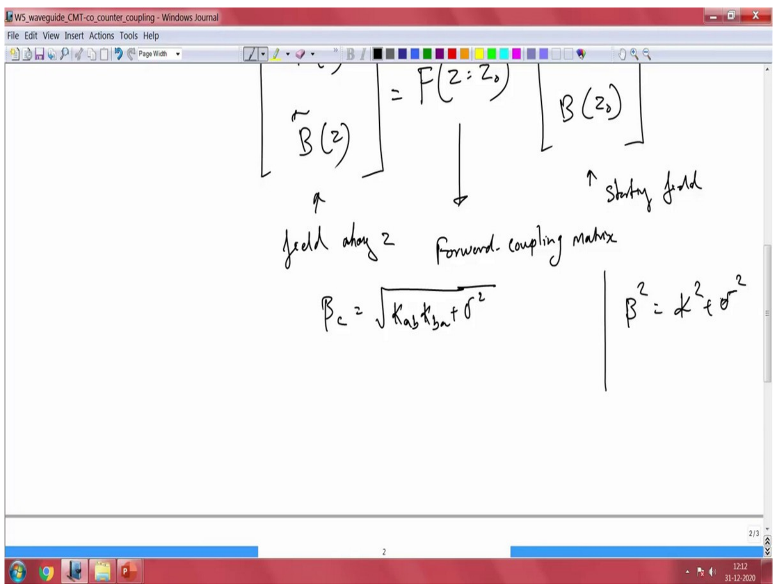

$$\begin{bmatrix} \tilde{A}(z) \\ \tilde{B}(z) \end{bmatrix} = F(z = z_0) \begin{bmatrix} A(z_0) \\ B(z_0) \end{bmatrix}$$

$$\beta_c = \sqrt{K_{ab}K_{ba} + \delta^2}$$

$$\beta^2 = K^2 + \delta^2$$

So, we will look at how one could do this, it is a very simple we can do it as a matrix, what is the field A and B along z when you are starting point is given by this? So, this is your starting field and this is field as you move along. So, this is starting field and then this is field along z. So, they it is coupled by what we call a forward coupling matrix, in forward coupling matrix is given by something of this kind. So, it is a matrix and this is what we call forward matrix

So, this is a reasonably a complex matrix might that takes into account the coupling between a b and ba in a more extensive way, but one thing to notice may now there is no need for us to derive this forward coupling matrix, but for the moment, let us say that this matrix contains all the coupling parameters, kappa a as a function of kappa ab and kappa ba.

So, we want to define a coupling parameter here what is called beta c which is given by root of kappa ab kappa ba plus delta square. So, this is nothing but kappa square plus delta square is what we have seen this in a earlier version it is basically beta square equals kappa square plus delta square is just a conservation equation, so the kappa here is ab and ba, so that just we need this for further discussion.

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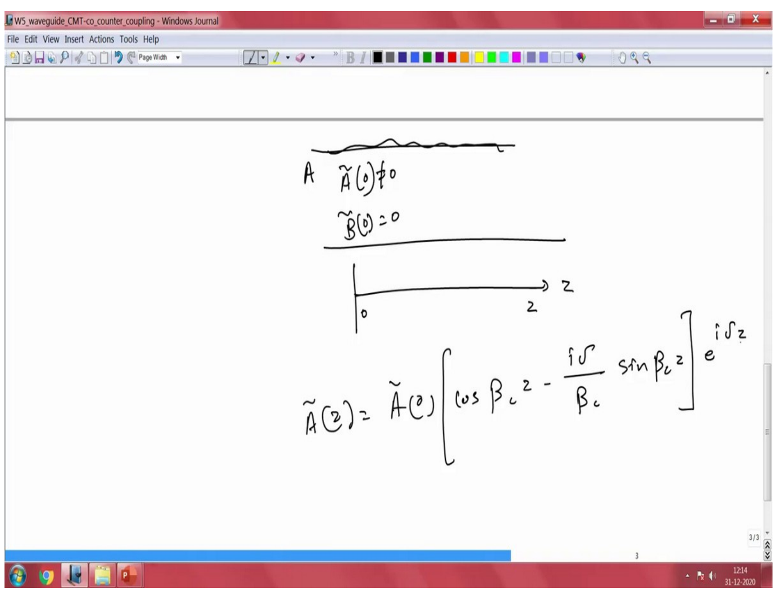

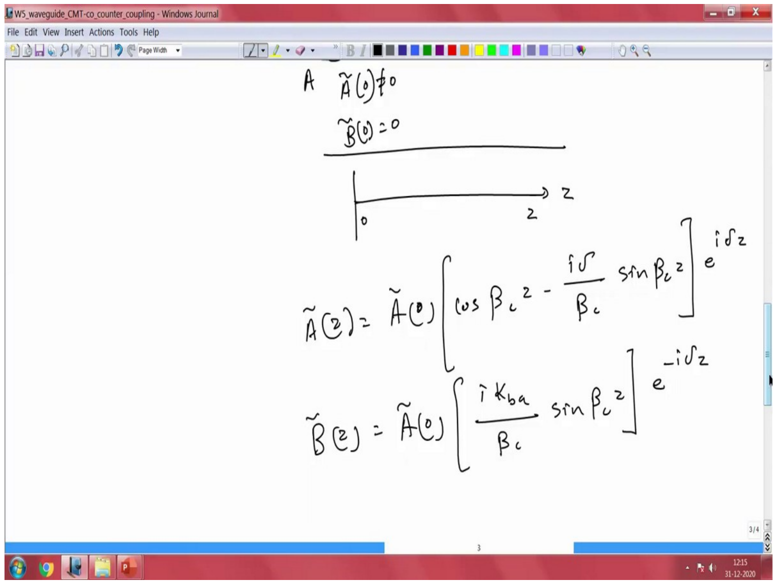

$$\tilde{A}(z) = \tilde{A}(0)[cos(\beta_c z) - \left(\frac{i\delta}{\beta_c}\right) sin(\beta_c z)]e^{i\delta z}$$

$$\tilde{B}(z) = \tilde{B}(0)\left(\frac{iK_{ba}}{\beta_c}\right) sin(\beta_c z)]e^{-i\delta z}$$

So, now let us take a simple case of a waveguide which has some perturbation here, it is a perturbed waveguide and we are going to launch only A, so we are going to launch A and this is z and along the direction this way and this is 0 and this is some z here. So, we are launching only A here at is the z equal to 0. So, that means you are filled inside the perturbed system A tilde 0 is not equal to 0.

So, that means there is a field that you have here, you are launching A, but you are not launching B, so it is the B when you start with is not present there, so that means there is no power in B at all, but it is a possible solution, so there is no power in B, but it is a possible solution inside the waveguide, but we are only launching A here. So, by applying this condition then we can write A along the length as a function of the starting field and then we need to have the coupling field cos beta c times z minus i delta over beta c time sin beta c to z whole e to the power i delta z.

So, we saw this one earlier let me adjust expanding this. So, this is this is how the field is going to evolve as a function of c. So, you have the starting field inside the perturbed system and your coupling is going to tell you how much energy that A of z is going to have or how the field A of z is going to evolve. Similarly, we can do this for B of z, so you start with A field and i kappa ba divided by Bc sin beta cz e to the power minus i delta z. So, this is how B is going to look like, as it propagates through the system here.

So, now we should understand what is the power in these two modes, how the power is going to vary as a function of z here. So what we have just gotten here is how the profile is going to vary, but what is the actual power inside.

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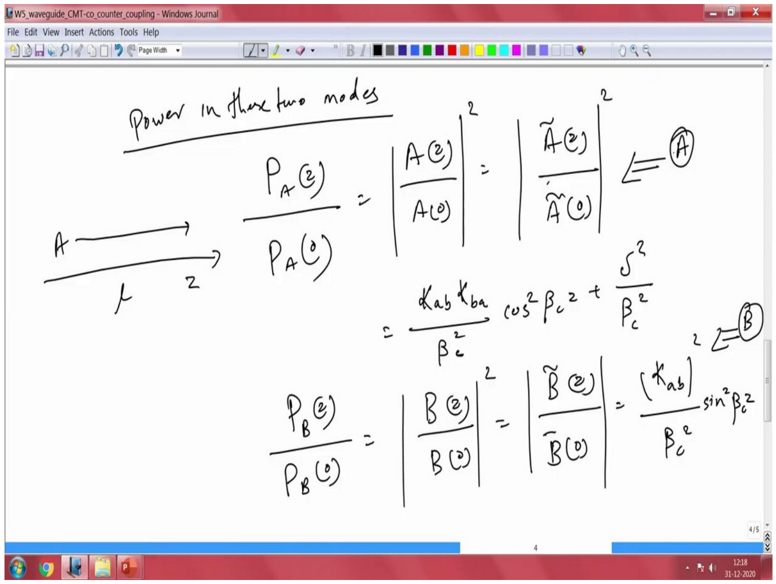

$$\frac{P_A(z)}{P_A(0)} = |\frac{A(z)}{A(0)}|^2 = |\frac{\tilde{A}(z)}{\tilde{A}(0)}|^2$$

$$\frac{P_A(z)}{P_A(0)} = \left(\frac{K_{ab}K_{ba}}{{\beta_c}^2}\right) cos^2(\beta_c z) + (\frac{\delta^2}{{\beta_c}^2})$$

$$\frac{P_B(z)}{P_B(0)} = |\frac{B(z)}{B(0)}|^2 = |\frac{\tilde{B}(z)}{\tilde{B}(0)}|^2$$

$$\frac{P_B(z)}{P_B(0)} = \left(\frac{|K_{ab}|^2}{{\beta_c}^2}\right) sin^2(\beta_c z)$$

So, this is just a field, so the electric field z k, so we need to know the power in these two modes, so let us look at how the power is going to change, so power of A at z over power A when you started, so that is can be given by this, in other terms, in a perturbed system this is your power. So, we can apply the equation that we had here and that would result in a coupled equation, kappa ab kappa ba divided by beta c square times Cos squared beta c z plus del square beta c square.

So, this is how your power is going to evolve and let us look at for B, so PB z over PB, so this is again rather simple you start from this mod square it is nothing but kappa ab mod square divided by beta c square into sin square beta c z. I am not sure whether you are able to see this, let me, sin square beta cz. So, this is how the power is going to look like. So, how the power evolves as a function of length?

So, as a function of length here, when the A wave is moving through this, this is how the power is going to look like, so this these factors are going to affect your coupling, so let us look at the coupling efficiency how efficient the coupling between the A and B are going to be. So, what we just saw is how the power in A and the power in B are going to look like, but now we need to understand how the coupling is going to look like. So, now we are looking at the actual coupling efficiency.

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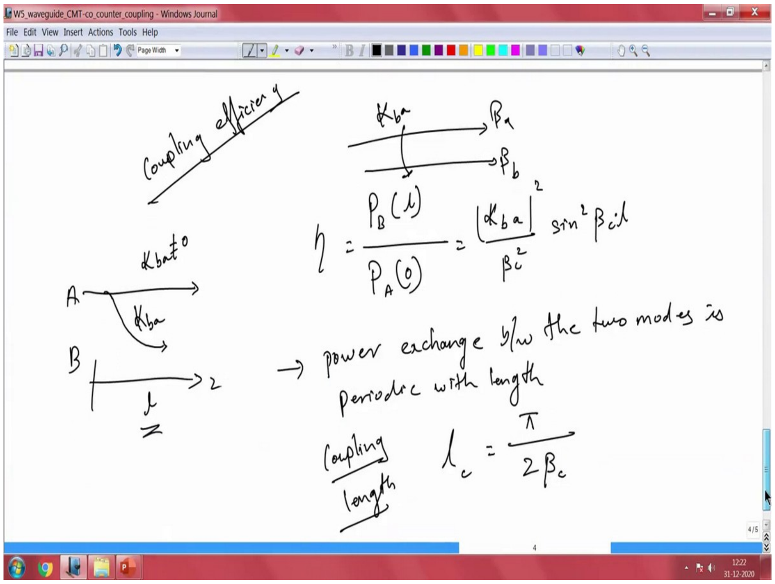

$$\eta = \frac{P_B(z)}{P_B(0)} = \left(\frac{|K_{ab}|^2}{\beta_c{}^2}\right) sin^2(\beta_c z)$$

$$l_c = \frac{\pi}{2\beta_c}$$

So, coupling the so far this is actually not the coupling efficiency, you should be careful here, we are looking at only the individual mode. So, in this case, we are only looking at A, how the A mode is changing. In the next relation, we are only looking at the B mode, so these are two modes how the power is going to change only these two we are only looking at, but now we need to understand how much power is transferred from A to B, as a function of length, how efficiently we can do that.

So, for a given length, this they are interacting with each other, so beta A beta B they are interacting with each other and they are coupled with Ba Bb constant. So, this is only one way let us say this way, so we are looking at how much we can couple from A to B. So, that coupling efficiency eta is given by the power that we have in B at a certain length A over power of A when we started.

So, we started with let us say A and B, B was not there at all. So, you started only with A, but as A propagates since kappa ba is not non-equal, it is a finite quantity here as the wave propagates along the length l, this is just length l, your power is going to coupled into B as well now. So, that means the power that I had in A is going to reduce and power in B will increase, so you are splitting the power now and this splitting is mediated by this coupling constant.

And as you can see, this is a function of length. So, what is the power of B at a certain length A to the power of A when you started? So, how much power is being coupled is given by this relationship and we can give this by using this relation that we just saw early or rather I will just put l, l is the interaction length you saw here, so this is basically how much of power we had. So, all we have done is added a length to this.

So, this is the length that is required, so now the power exchanges as you can see here this is a basically a sin function you see, so the power varies as a function of sin square here, and that means it is periodic in nature, so the power exchange between the two modes is periodic with length and that coupling length lc is given by pi over 2 beta c. So, what is beta c? So, beta c is nothing but the coupling effects so kappa ab kappa ba plus delta square over under root.

So, this is what you are Bc is, so it gives you an idea about how much coupling one would get as you propagate through a length lc. So, if you want to have a complete power transfer, this is to maximum coupling, if you want to have a complete power transfer that means you know from A to B, there is 100 percentage power is transferred from A to B, so there is no power left in a A, all the power is dumped into mode B. If you want to do that, then you are delta here, so delta is nothing but our phase difference, so it should be equal to 0.

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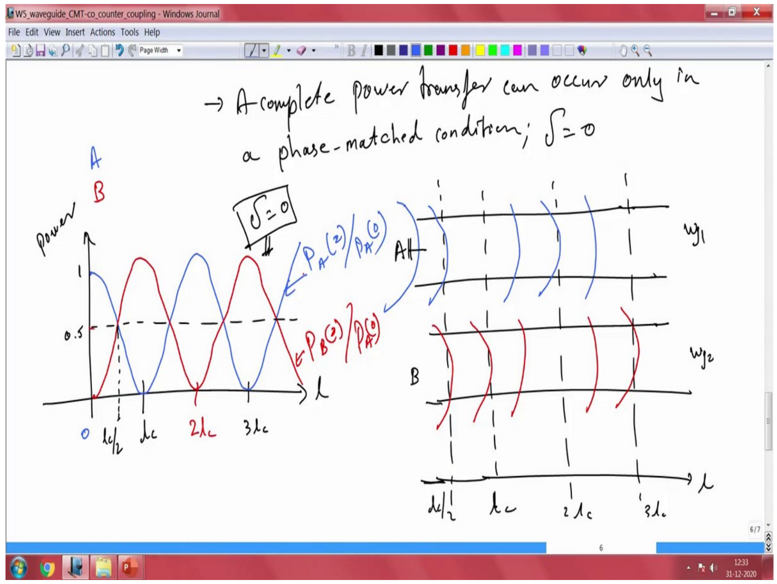

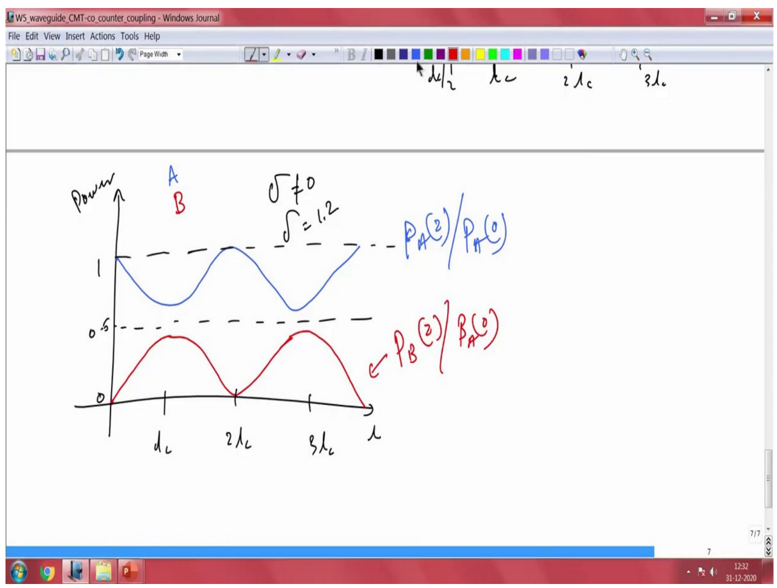

So, a complete power transfer can occur only in a phase match condition. So, what is that condition? That condition is delta should be equal to 0, so when you have 0, then you have perfect phase matching and you will get all the power coupling from one waveguide to the other, one mode to the other mode, let us look at how that will happen. So, you can numerically do that based on the equation we have, but let me show you here qualitatively how you this would happen.

So, let us say this is a function of length and this is for del equals to 0, so this is the power exchange let us say this is the power. So, let me take two colours here, so blue colour is for A and red is for B, so here blue gives you the A coupling, so that means used when the length is 0 so you go from 0 to a certain length when there is a length is 0 then all the power is in A, so all the power is in A, but then they start oscillating it starts oscillating and at a certain length lc we saw here, at this particular length lc you have maximum power coupling, that means at delta equals to 0. So, this is the condition we are looking at here.

So, this is at lc and then the next high will happen at 3 lc and 5 lc and so on, so odd multiples of lc you will get power low. So, now how is the power going to look like at B? So, when you start with B is 0, so there is no power in B, but then as it propagates you will see that this is for B, so B gets maximum power when it is lc and it gets lowest power at 2 lc so if you just to visualize I take a waveguide or even in this case I can take two waveguides both arguments are right, so I only launch A here this is waveguide 1 and waveguide 2, so waveguide 1 and waveguide 2.

So, I have A but I want to couple to B, so when I start with I only launch A, so I only launch A here, so there is no B there, but then this is as a function of length, so when the length is lc all the power is now in B, so that means light is now completely of let us say let me draw a line here this is lc and then you have 2 lc and that is a 3 lc. So, I launched it in the upper waveguide, now when it is lc, then all the power will be now in B. So, and then when it goes to 2 lc the power comes back to A and then as it propagates it is going to be like this.

So, it will switch from one waveguide to the other, so in between you will have other powers, so here you have 50 percentage of the power 0.5 here and here is 1 a complete power transfer happens here when it is 50. And if you are going to if you want to split the power between two different waveguides here, if it is a power splitter, you want to split between these two you should choose 0.5. lc or lc by 2, if you take lc by 2, so that is what here if you take lc by 2 you can split the power between these two waveguides now. So, as the wave propagates you will have points where the light will be sitting on both the waveguides.

So, that position is called the splitting position where you have 50-50 power between these two and when you reach lc or 3 lc, then the whole power is transferred. In this case, if it is 2 lc it will go back to your original waveguide. So, this complete power transfer 100 percentage transfer you can see here from going from 1 to 0 will happen when delta is equal to 0, so delta equal to 0 means you should be able to get that. But in practical scenario, you may not be able to get this perfect phase matching. So, perfect phase matching may not be possible.

In that case you will have a scenario where your delta is not equal to 0, in this case it is not equal to 0, so let us say delta is something like 1.2, let us say and again this is power and this is as a function of length, let us take the same scenario here, we have lc, 2 lc and let us say 3 lc, the same thing that we saw here as well. So, now how the power is going to be distributed, again A is blue now and B is your red curve here.

So, when I start Ba is 1, so A is 1 that is a maximum power we start with, but then as it propagates through instead of going to 0 they will oscillate between these dates, let me draw it again, so it is not going to go to a complete power transfer, that means it should be 0, if I draw B it will be very clear for you, so this is your B and B is going to do this, you can see here, so this is our 0.5 and this is our 0. So, the maximum is here 1, so when you have delta, which is non-zero, then it is impossible to do complete power transfer.

So, you will only do partial power transfer, in some of the cases this is still useful, but the only thing is you would not be having the complete axis of power that you can transfer from one waveguide to the other wave guide. So, when the phase matching is not proper, so what you see here is nothing but d let me draw this in blue, so PA z over PA naught and here PB z over PA naught, so this is what we have seen here.

The same thing is through here as well. So, how the power is to look like, so PA z over PA0 PB z over PA0, so that is the red and this is the blue one. So, the power transfer now happens as a periodic function as you move along the length of the device and we use this concept to design one of the device just called a directional coupler, we will look at it briefly when we talk about passive waveguide structure, but that the power distribution works with this code directional coupling mechanism. So, co-directional is all fine we understand it reasonably well now.

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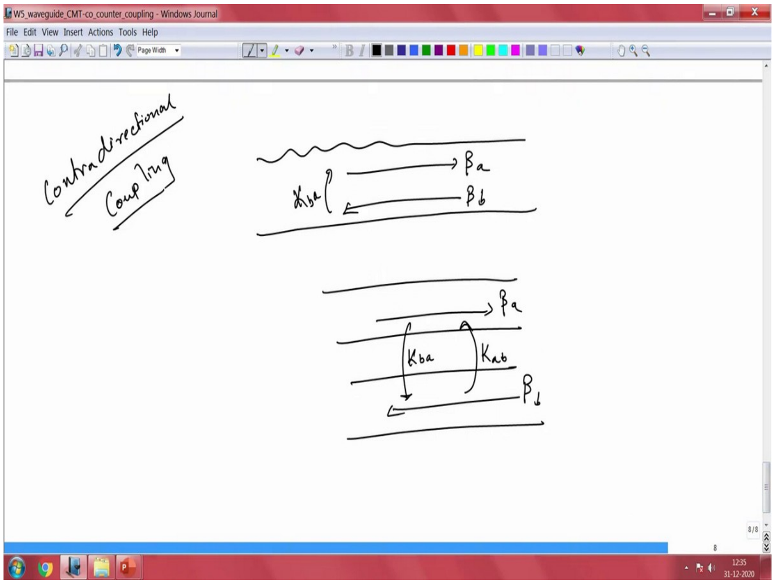

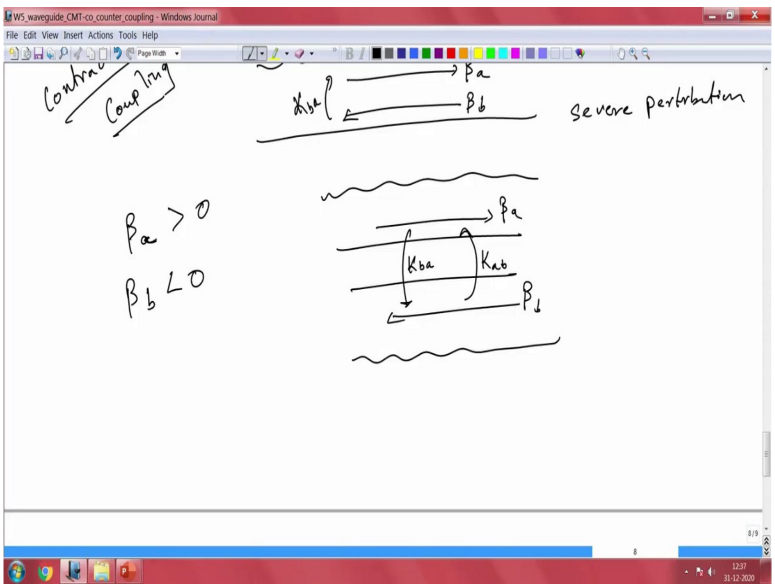

So, let us look at counter directional propagation, counter directional coupling, so here the scenario is different, let me take you back to this cartoon where we saw the waves are moving forward in a single waveguide and also in the stew waveguide system. So, in both the cases, we have waves that are parallel but they are moving in same direction, but here in counter directional or counter propagation we are going to have it in opposite direction.

So, let us look at a perturbed waveguide that we have, so beta a is going to do forward, but now beta b is backward, so we are looking at kappa ba here. Similarly, if you take a two waveguide system here we are having beta a and here we are having beta b, so now the question is, how much is your kappa ba and how much is your kappa ab? So, you have all these perturbation that we have in this system in order to do this coupling.

So, because the coupling has to happen, we said there should be a perturbation, but in case of counter-propagating coupling, counter-propagating wave coupling the perturbation has to be severe, so even in this case when you talk about simple waveguide two waveguide system you should have reasonable perturbation both here and also here. So, one of the requirement is severe perturbation, you can also say it is a significant perturbation that you need in order to make this happen.

We will see why we want this perturbation to be very strong, in the in the earlier case in the co propagating case we did not say the perturbation has to be severe we just said there should be perturbation, but in this case there should be very severe perturbation in a single waveguide system and also in the double waveguide system. So, now we can go back in and write our couple equation here. But one thing that we should again keep in mind is the sin of beta, so your beta a will be positive and your beta b will be negative, so it is propagating in the opposite direction and that is why we put this convention of positive and negative. So, now the coupled equation, let us look at the coupled equation.

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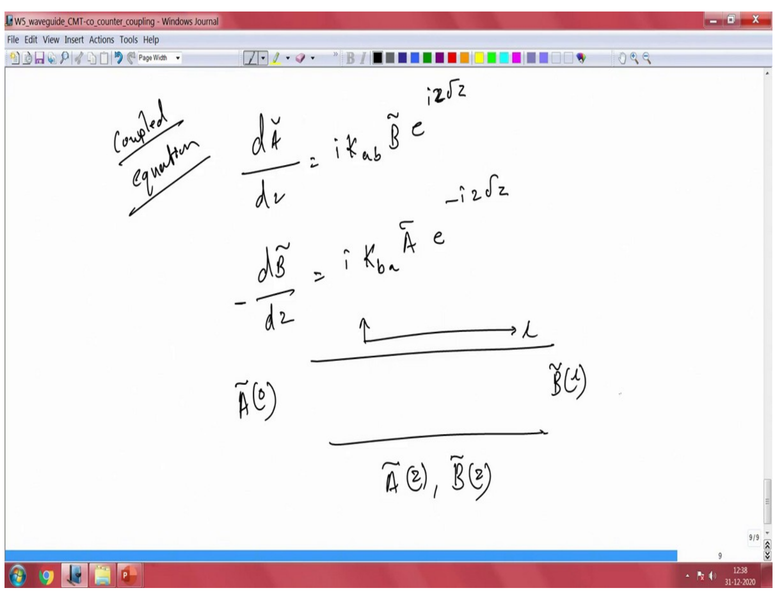

$$\frac{d\tilde{A}}{dz} = iK_{ab}\tilde{B}e^{i2\delta z}$$

$$-\frac{d\tilde{B}}{dz} = iK_{ba}\tilde{A}e^{-i2\delta z}$$

So, the coupled equation here would look like this, it is the same thing what we saw earlier, i kappa ab B e to the power i 2 del z and then d dz, so since it is moving in opposite direction, we need to put a negative sign here, kappa ba A e to the power minus i 2 del z. So, the equation for the counter direction coupling are again could be solved using boundary value problem with the following values that you start with A tilde at 0, you start with this waveguide at one end and you have B tilde l at the other end.

And we can find value of A tilde z and B tilde somewhere, in any location between the two ends. So, this is the length that we had, so in this case we are launching A naught and then look and also on the B from bl, so they are they could be propagating from both the ends. Let us look at our coupling matrix now.

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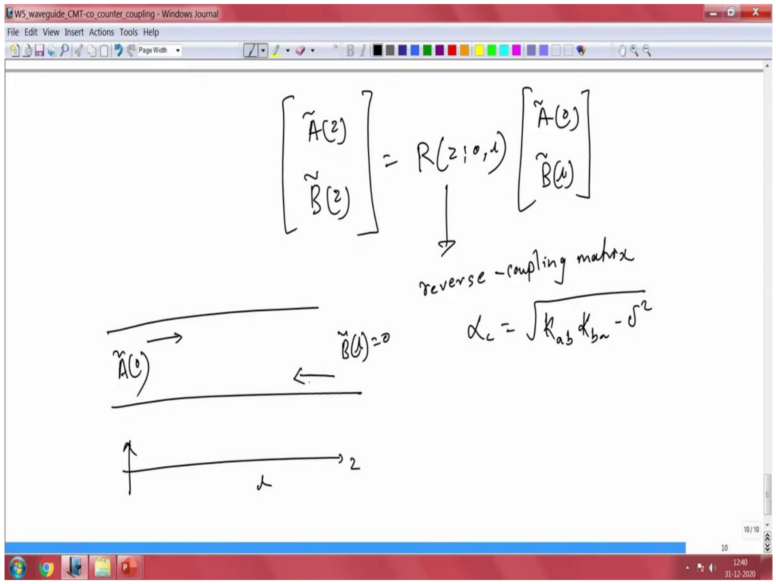

$$\begin{bmatrix} \tilde{A}(z) \\ \tilde{B}(z) \end{bmatrix} = R(z;0,l) \begin{bmatrix} A(z_0) \\ B(z_0) \end{bmatrix}$$

$$\alpha_c = \sqrt{K_{ab}K_{ba} - \delta^2}$$

So, the general coupling that we saw it is rather same, but the only thing is we talked about forward coupling, so A0 here and B of l here, so instead of forward coupling we have reverse coupling, so from 0 and l, so this is this is called our reverse coupling matrix, here again we had this beta c, so in this case we call it as alpha c, so alpha coupling is nothing but root of kappa ab kappa ba minus del square, so this is the conservation equation that we already know.

So, now let us consider the scenario of launching one of the modes and then look at how it will coupling at the other end or how it is going to couple to the other mode in this case. So, now again we will look at a single waveguide system, so there is along and it is a certain length l, so I am launching the mode from one end which is propagating in this direction and now I am not launching anything here from this end it is 0 but then the solution exists from this side. So, now the question is in the middle, how are they going to (())(39:13) so this is not equal to 0 and this is equal to 0. So, now let us write the field equation for A and B in some arbitrary position.

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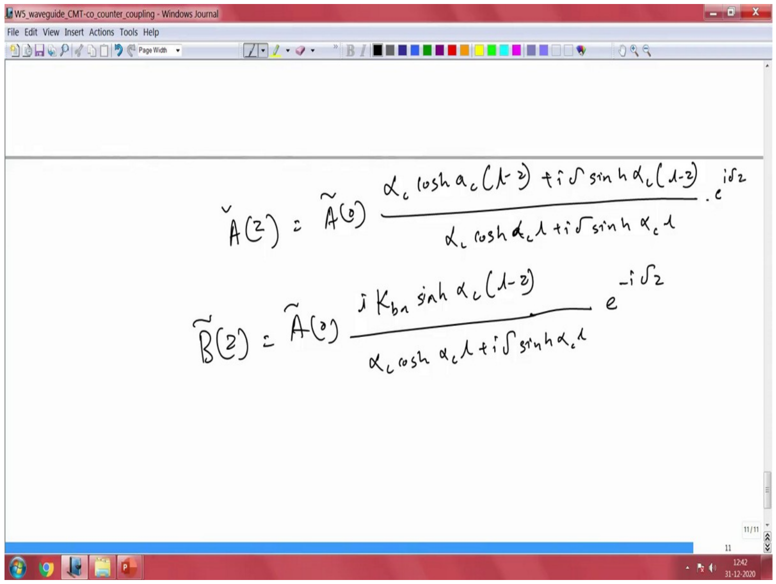

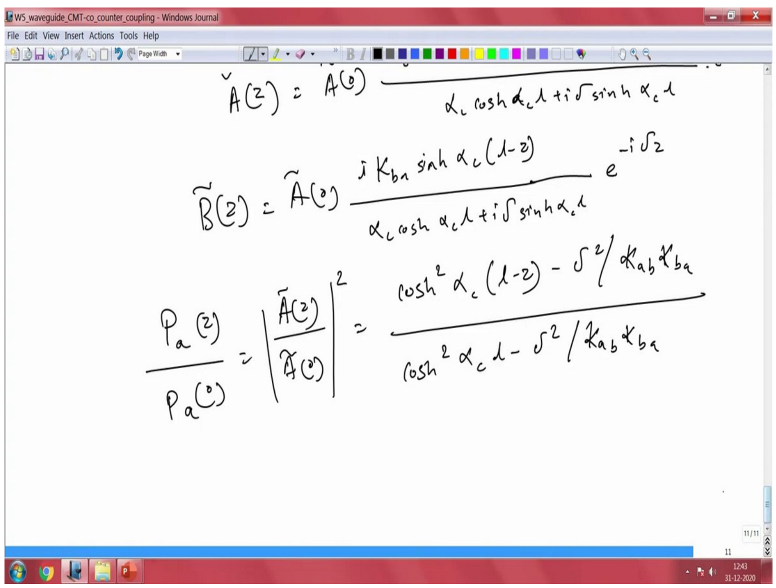

$$\tilde{A}(z) = \tilde{A}(0)\left(\frac{\alpha_c cosh\alpha_c(l-z) + i\delta sinh\alpha_c(l-z)}{\alpha_c cosh\alpha_c l + i\delta sinh\alpha_c l}\right)e^{i\delta z}$$

$$\tilde{B}(z) = \tilde{B}(0)\left(\frac{iK_{ba} sinh\alpha_c(l-z)}{\alpha_c cosh\alpha_c l + i\delta sinh\alpha_c l}\right)e^{-i\delta z}$$

$$\frac{P_A(z)}{P_A(0)} = |\frac{A(z)}{A(0)}|^2 = |\frac{\tilde{A}(z)}{\tilde{A}(0)}|^2 = \frac{cosh^2\alpha_c(l-z) - \frac{\delta^2}{K_{ab}K_{ba}}}{cosh^2\alpha_c(l) - \frac{\delta^2}{K_{ab}K_{ba}}}$$

So, A of z will be equal to A tilde 0, so here it is going to be little complex, but just stay with me l times z that is i delta sin h alpha c l minus z the whole thing divided by alpha c cos h alpha c l plus i delta sin h alpha c l times e to the power of i delta z. And the other thing is Bz can be written as A naught i times kappa ba sin h hyperbolic sin alpha c l minus z whole divided by alpha c hyperbolic cos alpha c l plus i delta sin h hyperbolic sin e to the power minus i delta c.

So, this is how the field is going to look like, as I progresses through the waveguide here. So, now we know that field but we have to understand how the power is going to be distributed, we will follow the same convention, the only thing here is between co-propagating and counter-propagating is the for the counter-propagating the relations are a little bit complex, so that we saw in the field. So, let us look at how the power coupling is going to happen.

So, power a along with z as a function when you started is nothing but A which is nothing but hyperbolic cos square alpha cl minus delta square by kappa ab kappa ba the whole divided by hyperbolic cos alpha cl minus del square by kappa ab kappa ba. So, this is how your field is going to look for A.

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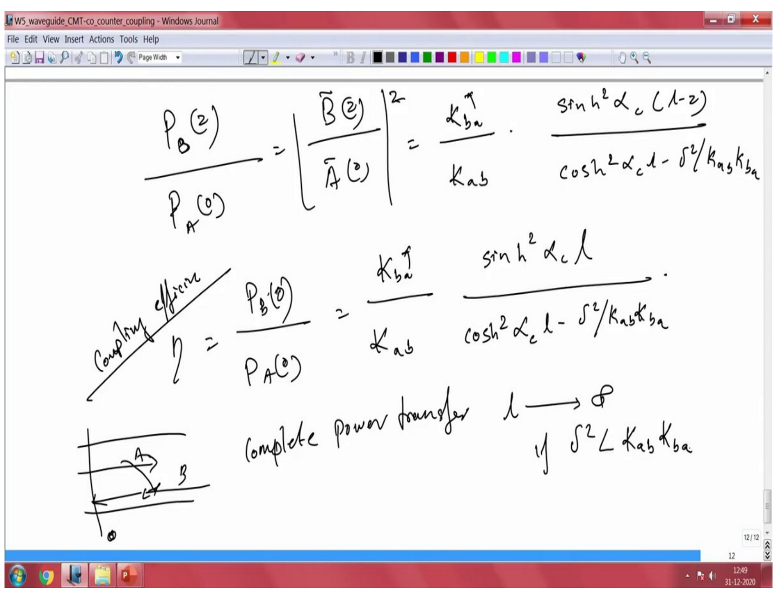

$$\frac{P_B(z)}{P_A(0)} = \left|\frac{\tilde{B}(z)}{\tilde{A}(0)}\right|^2 = (K_{ba}{}^{*}/K_{ab})\,\frac{sinh^2\alpha_c(l-z)}{cosh^2\alpha_c(l) - \dfrac{\delta^2}{K_{ab}K_{ba}}}$$

$$\eta = \frac{P_B(z)}{P_A(0)} = (K_{ba}{}^{*}/K_{ab})\frac{sinh^2\alpha_c(l-z)}{cosh^2\alpha_c(l) - \frac{\delta^2}{K_{ab}K_{ba}}}$$

And for B in just given by kappa ba by kappa ba Sin h square alpha c l minus c whole divide by del square K ab K ba. So, this is what we have, when we do our coupling to one mode to the other mode. So, this is how the power coupling happen. So, you can see here it is not straightforward, like we bought what we had for the co-propagating case, it is hyperbolic in nature, but then the efficiency will also be of that nature.

So, this is coupling efficiency. So, your coupling efficiency is going to be PB or PA which is nothing but kappa ba star kappa ab sin hyperbolic square times l, but then you can change this instead of having this you could place this as l, so we can just do this l. So, this is actual coupling efficiency in a counter-propagating wave, because B is propagating backwards with no input at is e z equal to l.

So, when e z in this case, so at set in length l, so there is no input, it is going from moving from A naught to Bz, so that is that is what we are trying to find out, so how much power will be coupled to that mode. So, what is the condition for complete power transfer? That means the power from A is dumped into b, so what is that condition?

So, that condition depends on the length as we saw earlier, so one thing that you may also want to notice here is the power coupling itself so the efficiency calculation itself look at this carefully, so PB naught over PA naught you are looking at the entry point, so you are looking at where z equals 0, so when you look at the other coupling efficiency that is the co-propagating efficiency let me show you that, so there we looked at PB at l compared to PA 0, so that means when you the starting point of your waveguide, so how much power is transferred to B at a certain length l?

But in this case we are looking at how much power is transferred from A to B at the initial position itself, because they are moving in opposite direction, so you are looking at how much power will be coupled from A to B when it reaches the initial position, so that is a coupling efficiency difference here. So, just keep that in mind, so condition for complete power sense were that is what we want, so when can we completely transfer the power?

So, complete power transfer will happen when l tends to infinity that is the only way, if del squire is less than kappa ab kappa ba, so that is the condition we should use for, let me write it here, if del square is less than kappa ab kappa ba, so this is the condition with which you can have complete power transfer, so complete power transferred could happen when you have this one. So, similar to go propagation let us look at how the coupling is going to happen as a function of length. So, we talked about length l where you would have complete power transfer.

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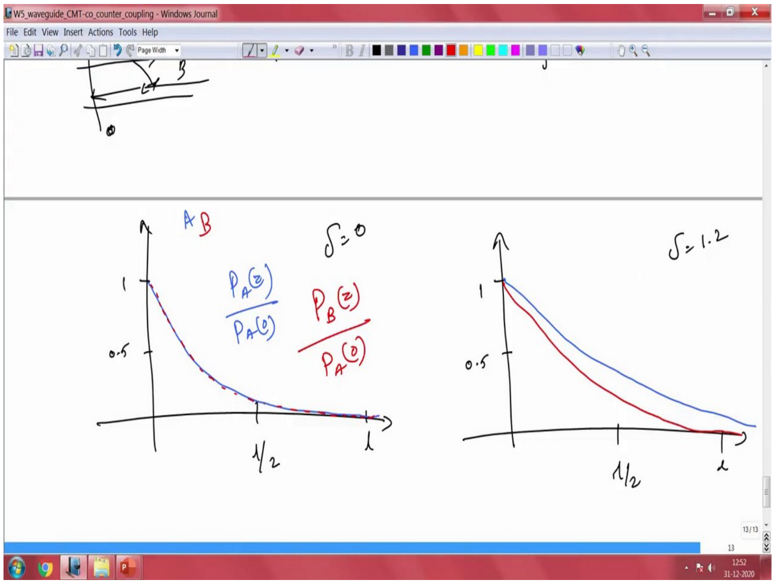

So, let us look at power coupling, qualitatively here as a function of l, so this is l and this is l by 2 let us say and this is when delta is 0 and you start from 1 and this is 0.5, so I will pick up again A and B, so the power exchange between the counter directional coupling modes would look in a like a decay functions it is a hyperbolic here, so you start from 1 and what you will see is just the k down and the go like this and for B also you will have the same thing.

But now when you have a condition where your delta in this case also we will take something like 1.2 or that we saw it is not 0 that is what we want to do. In this case again the power will not change much, so it is going to be exactly the same with very small difference, where your A we will see something like this while your B we will see a much dipper, so this is all you will get. So, what is A? A is nothing your blue is nothing but da PA z over PA0 and your blue red curve is PBz over PA0, so, this is how the power coupling is going to be.

So, you can you can clearly see that the implication of having the counter propagating wave here, so you will get maximum coupling only at infinity, but why are we actually looking at this? I mean this is as complex as it gets when it comes to couple mode theory the counter propagating wave coupling is rather difficult to transfer energy which leads severe perturbation and so on, but if you look at the problem, it is it is very practical we see it all the time, we see this all the time, where do we see this?

This is nothing but reflection, one could perceive this as a reflection of the A naught, so you have a forward propagating wave that is beta a and then beta b let us say it is nothing but your reflection of what you see here. So, this will be an interesting way to look at it, it is nothing but reflection, so if that is the case, can we write the reflection coefficient of this?

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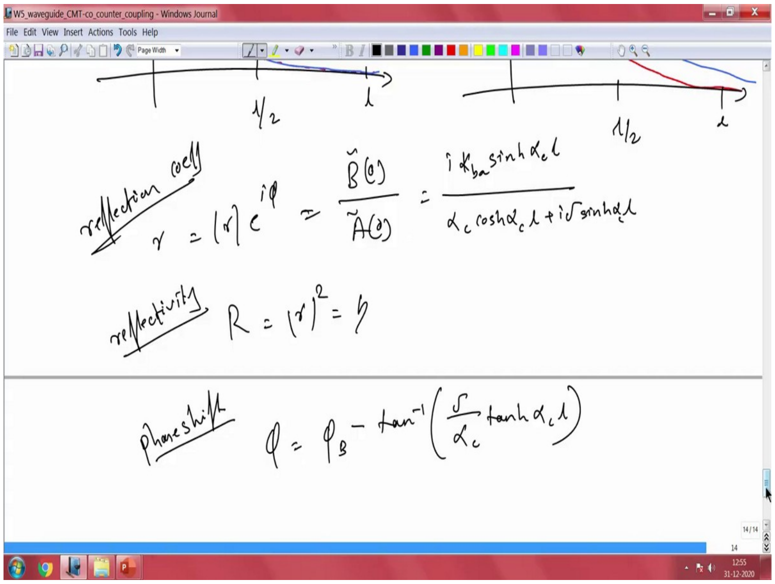

$$r = |r|e^{i\varphi} = \frac{\tilde{B}(0)}{\tilde{A}(0)} = \frac{iK_{ba}sinh\alpha_c l}{\alpha_c cosh\alpha_c l + i\delta sinh\alpha_c l}$$

$$R = |r|^2 = \eta$$

$$\varphi = \varphi_b - tan^{-1}(\frac{\delta}{\alpha_c}tanh\alpha_c l)$$

So, the reflection you can see this as a reflection, let us say your reflection coefficient r, so this is reflection coefficient that nothing but r e to the power of i phi, some phase difference is what you see. And this is nothing but B naught with respect to A naught, so this is what reflection is all about. How much power you get at a for power that you put in B in whatever power you put in A. And that is nothing but i kappa ba which we saw sin h hyperbolic alpha c l divided by alpha c cos h alpha cl plus i delta sin h alpha at alpha c times l.

So, this gives you the reflectivity, that is nothing but reflectivity so the reflectivity reflection coefficient and reflectivity is nothing but square or another case eta, so this is the reflectivity what you get, so let us say what is the phase shift that you may get and that phase shift is given by whatever from the reflection side and then the propagation tan inverse of delta alpha c and both alpha c times l.

So, this is how you know you can find the reflectivity if the phase shift that you may have when you are considering this counter-propagating coupling, so it is not that you know they do not exist, they do exist these counter-propagating coupling is similar to the reflection problem that we have and we can indeed a couple backward propagating waves by using very strong perturbation as we as we discussed here.

So, we want to have a very severe perturbation by doing that one should be able to couple it and people have shown this this is not just in theory people have fabricated these kind of devices in order to show that how one could couple to counter direct propagating modes. A good example here is a very simple gratings a black grating let us say, so you which we will see later on, while using this concept how light could be coupled back and forth in this using this counter propagation coupling.

And we use this concept for making lasers, so distributed back reflectors is a concept that we use to make lasers on chip lasers and they rely on this sort of coupling. So, it may look complex but it is very practical, so we use this in order to exploit again properties in the medium. So, let us look at summarize this with important phase matching conditions, I think for the moment we will stop with this understanding of Co and counter-propagating waves.

So, far we looked at these two waves how they could couple and also derived the nature of power coupling as they propagate through the waveguide. And one can use put this into use in practical devices which we will see in the passive device section. So, make sure that you revise these topics, it needs some time, you would not be able to just digest from just looking through this video relook, re-look at it, stop whenever you need, work it out so that you completely absorb this this concept of coupling, which is very very important when you are designing really complex devices with that thank you very much for listening.