Photonic Integrated Circuit
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Lecture No. 25
Two-Mode Coupling

Hello everyone, so let us continue our discussion on the mode coupling here. So, we would like to look at coupling between two modes in a waveguide, so this is very general interest because whenever you are making any waveguide based device you want to couple energy from one mode to the other mode it could be a the same waveguide system or waveguide next to it.

So, this is this is a very general application and even in a single waveguide you could have perturbation that is the waveguide, so when there is a perturbation in the waveguide the wave that is propagating through it, it goes through changes. So, when it goes through changes, it has to recover itself and then it has to make sure that it propagates through the perturbation.

So, that is something that is essential. In some of the cases you deliberately do this to transfer the energy and in some other cases you want to modify the propagation constant of this propagating mode, so the face could be modified by this participation is what we are looking at. So, let us look at two more coupling problem.

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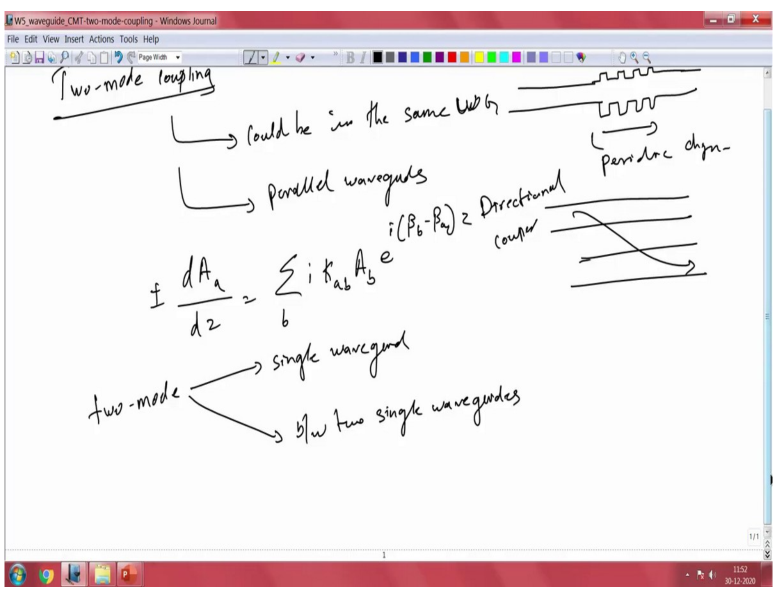

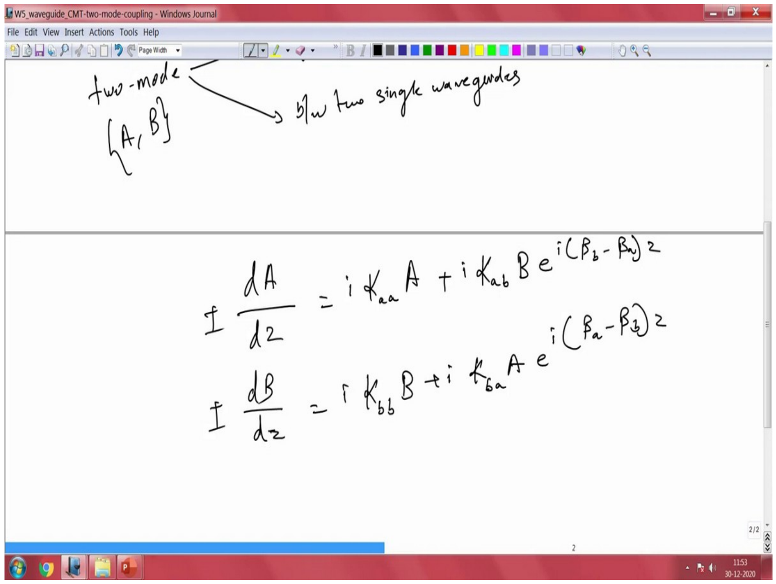

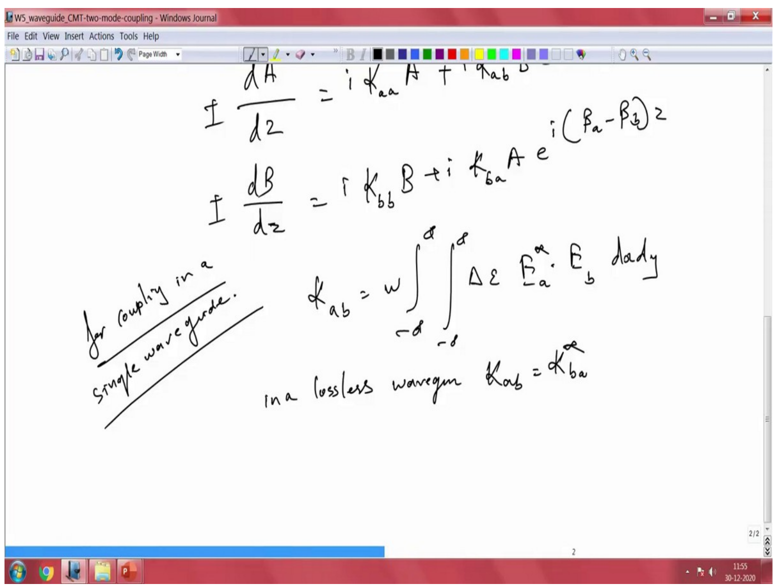

$$\pm\frac{dA_a}{dz} = \sum_b iK_{ab}A_b\, e^{i(\beta_b-\beta_a)z}$$

$$\pm\frac{dA}{dz} = iK_{aa}A + iK_{ab}Be^{i(\beta_b-\beta_a)z}$$

$$\pm\frac{dB}{dz} = iK_{bb}B + iK_{ba}Ae^{i(\beta_b-\beta_a)z}$$

$$K_{ab} = \omega \iint_{-\infty-\infty}^{\infty\infty} E_a^*.\Delta\in.E_b\, dxdy$$

So, let us take a simple case of two more coupling, so in the earlier case we had a generalized formulation, so let us look at the two mode coupling and most of the time these things could happen two modes in a same waveguide let us say so two modes could be in the same waveguide, this could be in the same waveguide for example, or we could have things like parallel waveguides.

So, in the same waveguide, we could have some periodic perturbation, for example, you could have a straight waveguide with some profile like this. So, this is periodic change, so one could look at the system and also consider this is a perturbation can we understand coupling in this. And within parallel waveguides it you can have two parallel waveguides where you want to take the light one to the other and this is called the directional coupler.

So, these are all the places where we should understand this two mode coupling for practical use. So, as we already saw that you know the solutions in these two modes a greatly affect the coupling, so we could form a generalized couple equation here again you know this is just a recap of what we saw in the earlier discussions. So, from how mode A is going to look like as it propagates through a system that has more B in it right and the coupling between mode B and mode A here.

So, the only difference in this in the coupling coefficient for multiple waveguide coupling is that the solutions are going to be orthogonal when they are isolated, but when you put together it may not be all. So, then we can write this two modes that are either in single waveguide or between two separate waveguides, let us say there are two modes that you take and we want to expand their fields and the field could be A and B let us say.

So, we are taking two modes and this could be either in single waveguide or they are between two single waveguide it is possible and let us say they are amplitudes are A and B and let us say how the amplitude of A and B are going to look like. So, the two modes are A and B let us say, so the amplitude of mode A as it progresses through the system plus and minus will depend on the first thing its self-coupling, it is coupling to itself.

So, it has a certain amplitude A and there is a self coupling that happens plus the cross coupling with mode B and now let us look at how mode B is going to look like, here again we have self-coupling with itself plus the cross coupling kappa ba A times e to the power i beta a minus beta b times c. So, this is the coupled equation of this two different modes that we are talking about.

And for the coupling in a single waveguide, your coupling can be if it is isotropic and so on your kappa ab is something that we already seen that is delta epsilon E times a dot E times dxty. If the waveguide is lost less then your kappa ab will be equal to kappa ba star. So, that is something that we know so this is for coupling in a single wave guide.

So, this is your coupling equation as it looks like, for in a lossless medium, you know loss less waveguide your kappa ab will be equal to kappa ba star. So, let us look at coupling between two waveguides, so this is where things get very interesting, so you are talking about two different waveguides.

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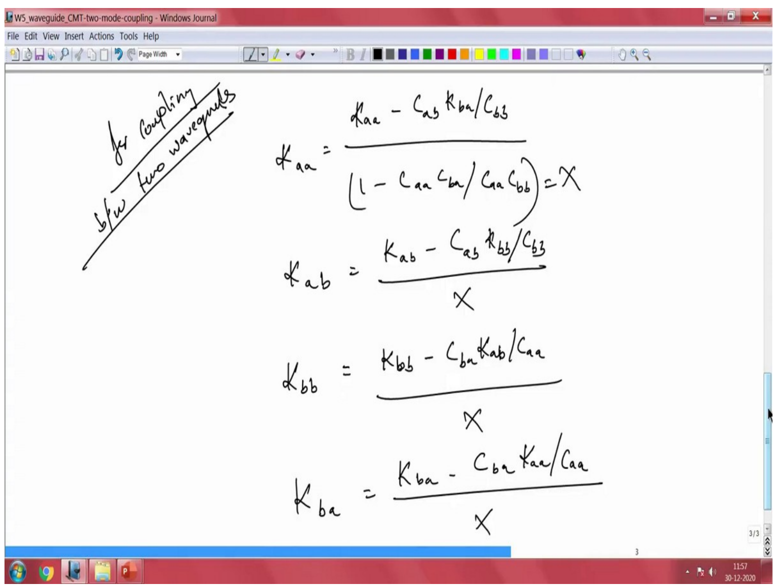
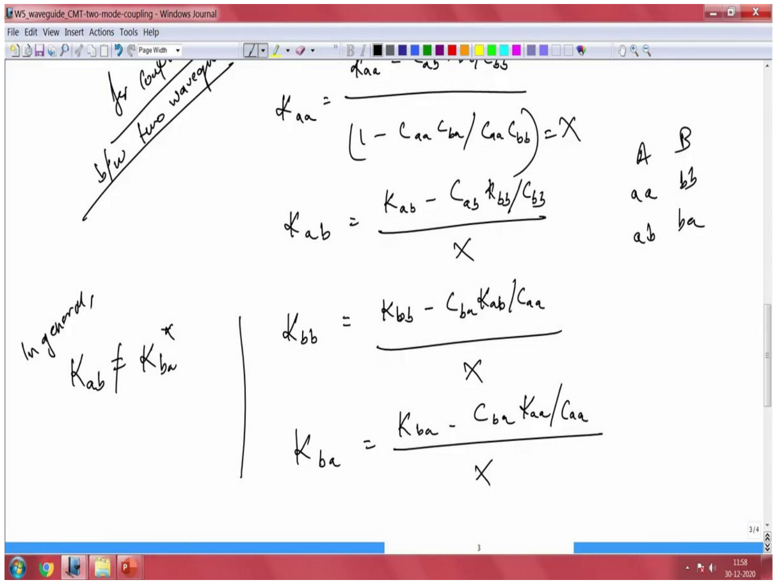


$$K_{ab} = K_{ba}^*$$

$$K_{aa} = \frac{K_{aa} - \frac{C_{ab}K_{ba}}{C_{bb}}}{1 - \frac{C_{aa}C_{ba}}{C_{aa}C_{bb}}}$$

$$1 - \frac{C_{aa}C_{ba}}{C_{aa}C_{bb}} = X$$



$$K_{ab}=\frac{K_{ab}-\frac{C_{ab}K_{bb}}{C_{bb}}}{X}$$

$$K_{bb}=\frac{K_{bb}-\frac{C_{ba}K_{ab}}{C_{aa}}}{X}$$

$$K_{ba}=\frac{K_{ba}-\frac{C_{ba}K_{aa}}{C_{aa}}}{X}$$

So, for coupling between two waveguides, so now that has something elaborate, so there we have self-coupling factor, which is given by an elaborate kappa aa minus Cab kappa ba by Cbb divided by 1 minus Caa Cba divided by Caa Cbb. So, this is this is one factor which I could call as X let us say, I can call this as X so that I do not have to repeat everything again and again, so kappa ab equal to kappa ab minus C ab kappa bb divided by Cbb divided by X and now we have kappa bb, so that is the coupling between that kappa bb minus C ba kappa ab Caa divided by X.

And finally, kappa ba which is nothing but kappa ba minus Cba kappa aa divided by Caa divided by X. So, you can see here, it is rather elaborate, so it has a lot of coupling coefficient and kappa here, so one thing that is clear from all of this is there is a cross coupling factor and there is a self coupling factor, so there is aa factor and there is ab factored, aa ab, bb and ba, so you can see here there is self-coupling between the a mode and that is self coupling between the b mode and then there is cross coupling between a and b and b and a.

Here something that we should also note down is in general this is a general case your kappa ab would not be equal to kappa ba, when you are talking about two different wave guides. So, this is again reiterating what we already looked at. So, the self-coupling in this terms, so everywhere we see kappa aa kappa bb.

So, since they are normal, the fact that these are all normal modes and the index profile in the perturbed waveguide is different from the original wave guide, that we have, we could remove this self coupling all together. So, we do not have to worry about this self-coupling at all. So, one can remove these self-coupling by expressing your normal modes as a function of this perturbed one.

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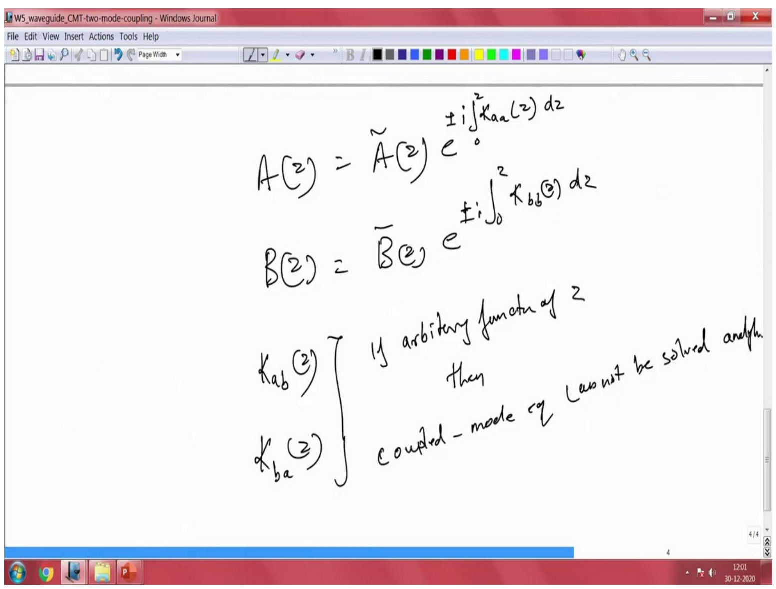
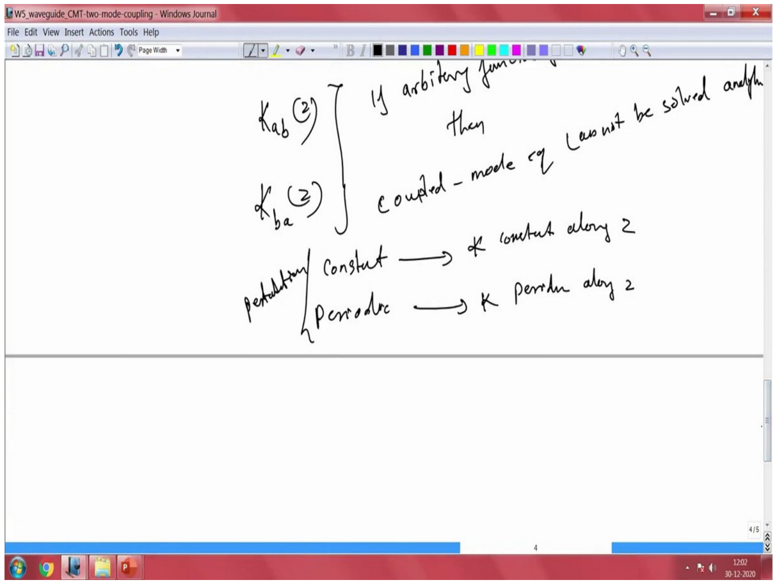


$$A(z) = \tilde{A}(z)e^{\pm i\int_0^z K_{aa}(z)dz}$$

$$B(z) = \tilde{B}(z)e^{\pm i\int_0^z K_{bb}(z)dz}$$



So, let us say the for the A mode we could look at some sort of perturbed mode here e to the power of i kappa aa as a function of distance dz, so 0 to z. So, this is how your field is going to look like as the mode is propagating the A mode is propagating through the medium along z distance. So, we have this self-coupling, so we initially had cross coupling in this case self-coupling and the same thing can be done for Bs well some form here e to the power you could put plus minus here depends on how they propagate kappa bb times z dz.

So, here the plus sign minus sign is just to denote whether the wave is moving the forward direction or backward direction your kappa aa and kappa bb are nothing but self-coupling and they self-coupling along the length of its propagation. So, this perturbation that we have, this kappa aa or kappa bb is going to uniformly effect and we do not have to worry about this. And that is the reason why one could remove this kappa aa and kappa bb factor out of our coupling equation altogether.

And how about kappa ab and kappa ba? So, let us look at those two. So, kappa ab as a function of z and then we have kappa ba as a function of z. So, these two are let us say arbitrary in nature, if it is arbitrary function of z if that is the case then coupled more equation cannot be used, cannot solve analytically, so it can be used but it cannot be solved analytically, we need to numerical solution.

So, coupled mode equation cannot be solved analytically. So, however for waveguide structures these things are not arbitrary, so you normally create some sort of predefined or periodic change in this perturbation. So, in those cases your delta epsilon or kappa will be periodic in nature so we do not have to worry about that once you have a periodic perturbation or you know perturbation that is dependent on z let us say.

If it is independent then you do not have even that problem, so if the perturbation is uniform there is no it is constant let us say, then the coupling constant is again constant, so let us say there are two scenarios so there could be a constant change or there could be periodic. In this case your coupling constant will be your kappa will be constant along z and in this case it will be kappa will be periodic along z, this is constant along C, so based on the perturbation. So, this is perturbation. And now we can come back to our coupled equation coupled mode equation.

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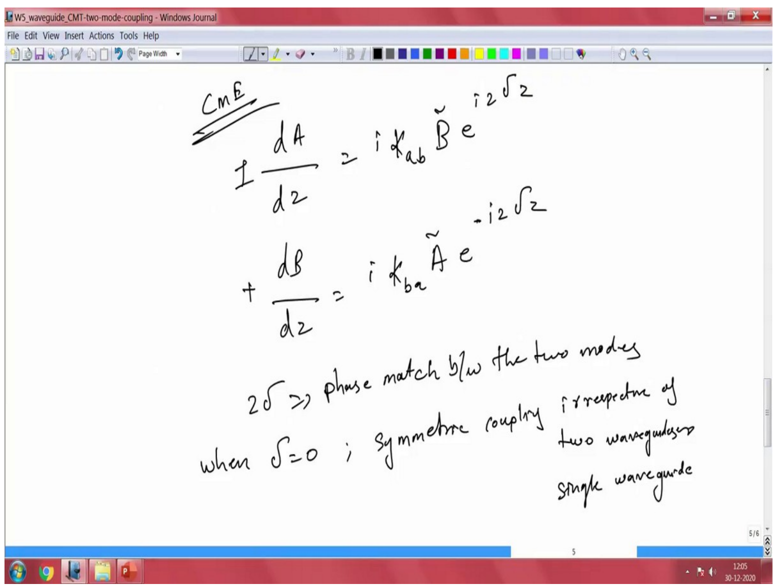

$$\pm\frac{d\tilde{A}}{dz} = iK_{ab}\tilde{B}e^{i2\delta z}$$

$$\pm\frac{d\tilde{B}}{dz} = iK_{ba}\tilde{A}e^{-i2\delta z}$$

So, let us look at our coupled mode equation after doing this understanding of self coupling and now the field that is propagating through either positive or negative depends only on the cross coupling B e to the power of i 2 delta C, so I will just mention what that delta is shortly, dB over dz, which is i kappa ba A times it some modified field here e to the power minus i 2 delta z.

So, now these could be analytically solved and the solution applies to many different two mode problems, you can take any two modes and then we can apply this and then one should be able to apply this one. So, what is this delta? So, 2 delta that we have here is 2 delta that we have this is nothing but the phase matching, phase match between the two modes and when delta is 0, so then you have symmetric coupling, when delta is 0 we get symmetric coupling.

Irrespective of whether these two modes belong to the same waveguide or two different waveguides that is very important, symmetric coupling irrespective of two waveguide or single waveguide, either two waveguides are a single waveguide. So, this should be understood, so the delta is the phase matching between the two and what is that phase matching? Shortly I am going to tell you that for the moment let us just assume that this is the phase matching and if you have a perfect phase matching then the delta is 0.

And that means you should be able to couple the light back and forth, either in a single waveguide system or two waveguide system. And the perturbation the two coupled equation now can have two different perturbations, one is a constant perturbation and one is a periodic perturbation, so let us look at the context perturbation and then look at the coupling.

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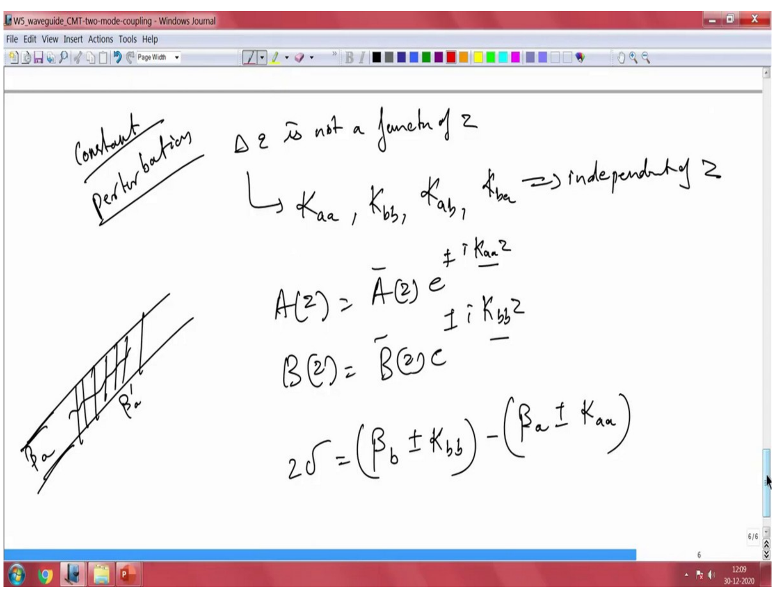

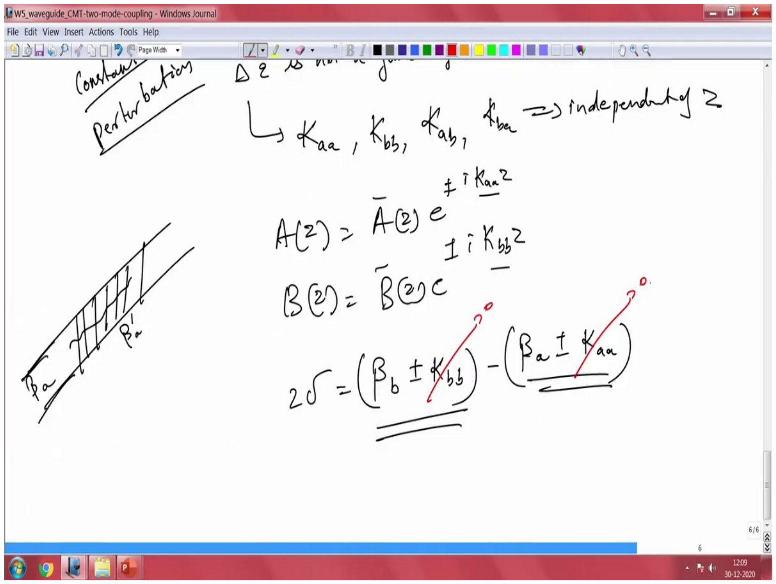

$$A(z) = \tilde{A}(z)e^{\pm iK_{aa}z}$$

$$B(z) = \tilde{B}(z)e^{\pm iK_{bb}z}$$

$$2\delta = (\beta_b \pm K_{bb}) - (\beta_a \pm K_{aa})$$

$$2\delta = \Delta\beta + qK$$

$$2\delta = \beta_b - \beta_a + qK$$

So, let us say this is constant perturbation, so that means your delta epsilon is not a function of z. So, you said if kappa is constant across is that for a constant coupling, so here the constant meaning there is no change at all. So, that means the implication here is kappa aa kappa bb kappa ab kappa ba all this are independent of z, so there is no dependent on z because it is constant. So, now our field along the propagation direction is nothing but the self-coupling like this and for B we have a modified field like this.

And now we need to look at the delta, so this is the coupling factor, so what is the phase matching here? Phase matching is nothing but beta b plus or minus kappa bb minus beta a plus minus kappa aa. So, if you remember the cell if you take out the self-coupling in this your coupling is be there your delta here is basically beta b minus beta a. So, the physical meaning of self-coupling coefficient is a change in the propagation constant of each normal modes here, so that is what here the self-coupling is all about. So, the what the self-coupling is going to do is to modify your propagation constant.

So, you as I mentioned let us say there is a perturbation here you start from beta a, you go through this perturbation and this perturbation is actually going to change the propagation constant to something beta a dash, so that change is brought about by this self-coupling. So, the propagation constant of the normal modes in the original waveguides are beta a and beta b, there is no change there. But then the change, the values are changed because of the perturbation in the waveguide, so in the original unperturbed system, you do not have any change, but in the perturbed system you will have this change.

So, now once you have this perturbation, then they propagate at this slightly modified propagation constant here. So, this is the propagation constant difference because of your the self-coupling that happens because of the perturbation, if you do not have any perturbation, without perturbation this goes to 0. So, there is no self-coupling there, in that case, your delta is beta a minus beta b, but in this case you are going to have this perturbation. So, that is what we need to understand for this.

So, the propagation constants are now going to be different and when you look at a periodic perturbation, this is for a constant perturbation, where kappa aa and kappa bb are constant as you move along. So, what happens when you have a periodic perturbation? So, let us look at a periodic perturbation now.

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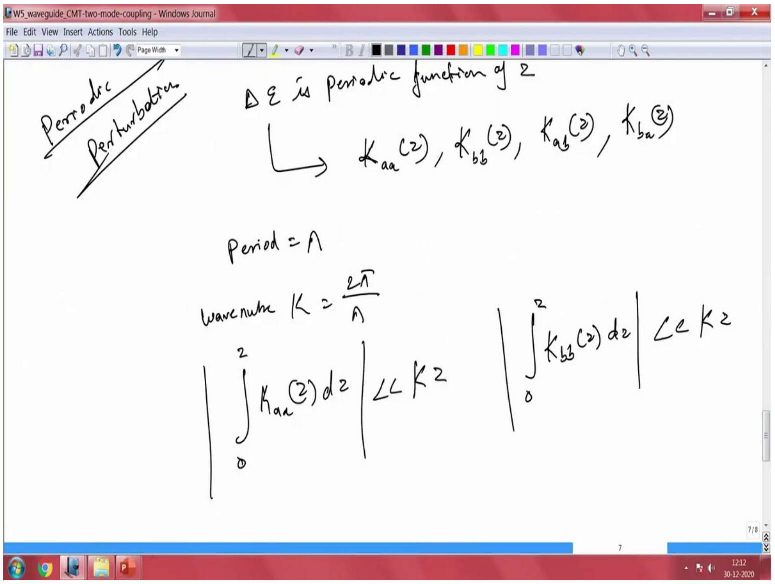

So, in a periodic perturbation, so here delta epsilon is periodic function of z, so if this is the case then kappa aa is also function of z, kappa bb will be also function of z, kappa ab is also function of z and kappa ba is also functional z. So, what is the periodicity that you have the periodicity would be period here is here it says a capital lambda and your wave number k will be 2 pi over lambda.

So, now the coupling coefficient kappa ab and kappa ba or periodic along z, with the period capital lambda and this can be expanded in a Fourier series with a constant kappa ab and kappa ba and phase factor q times k vector here. So, that is your wave number here. So, let us look at how your kappa aa and Kappa bb are periodic and once you have that then what is your wave number because of that.

So, the wave number is going to be modified because of this kappa you have which would be much much less than the change is less than capital K here z and the same thing 0 to that kappa bb that is at very less than K time's z. So, now the contribution of this phase mismatch term is given by this kappa aa and kappa bb is negligible compared to the phase change that the perturbation is going to bring in. So, the kappa aa and kappa bb or rather small the self-coupling is small, however your kappa ab and kappa bb or not and also your perturbation because of your periodicity is going to add up.

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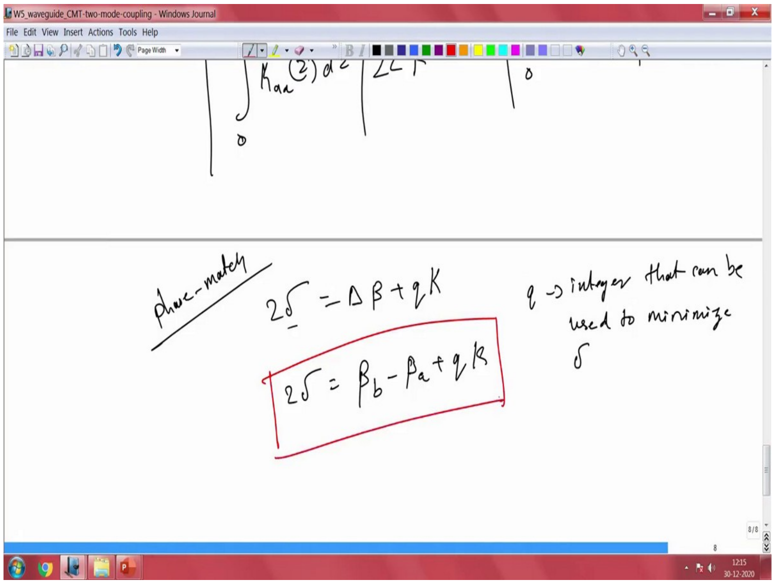

$$2\delta = (\beta_b \pm K_{bb}) - (\beta_a \pm K_{aa})$$

$$2\delta = \Delta\beta + qK$$

$$2\delta = \beta_b - \beta_a + qK$$

So, now let us look at our phase difference here phase matching. So, here phase matching to delta is nothing but delta beta plus q times K, so q is nothing but the order here, it is again in an integral number here, so once you have this, your delta beta is nothing but beta b minus beta a plus qk. So, this is your coupled equation in a phase matching condition when you have a periodic disturbance or periodic perturbation.

So, here q is integer that can be used to minimize delta, so all you are trying to do is you want to get this delta to 0 and one can look at this q, so that you should be able to make the phase difference to 0. So, this is about a coupling between two different modes either in same waveguide or different wave guide we have looked at the phase matching conditions and so on, where the direction of propagation does not really matter, in this case we are just looking at coupling between two modes in a constant perturbation and in a periodic perturbation.

So, in both these cases the phase matching conditions comes out to be slightly different, just to note it down let me mark it, so this is our phase matching condition when you have a periodic perturbation when you have a constant perturbation, this is what we have. So, with this we can we can summarize whatever we have looked at so far, we have looked at two waves coupling with between each other, constant perturbation here and also periodic perturbation here. So, later on we will see how to exploit this when the modes are Co or counter-propagating. Thank you very much.