Photonic Integrated Circuit
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Lecture No. 24
Coupled Mode Theory 1

Hello, all. Let us continue our discussion on the mode coupling. So, so far we looked at how we can couple mode in a single waveguide. So, we took a waveguide and then first established that normality and then induced some perturbation and this perturbation is going to mediate our coupling.

So, with the, in the absence of this mediation, you will not have any coupling. So this is true for a single coupling waveguide. So, we will we will later on look at a little bit more into two mode coupling problems, but physically you may have more number of modes in a real circuitry. So, you will have multiple waveguides sitting next to each other and you may want to couple light from one waveguide to another waveguide.

So, now the problem becomes even more interesting. So, you have two different systems altogether, so there will be Eigen solutions on waveguide 1 and there will be Eigen solutions on waveguide 2, but they are Eigen solutions within the system. So, they are isolated. But now if you are going to bring it together, so you create a super system here so that means they are not isolated waveguides anymore. They act as a system with two waveguides in it. So, now when you when you look at these two systems, you will have Eigen solutions for this system, so what we call system modes.

So, in that case, the problem becomes interesting. How so how do you couple light from one to the other when you have a complete system to to describe the Eigen solutions that we have? So, most of the time when we are using such structures, we can solve the whole geometry or cross-section using Maxwell. So, that is something that we normally do, but then another way of doing it is by considering this isolated waveguide system, finding the solutions and then creating a perturbation when they come too close to each other.

So, when they come closer then you are essentially disturbing the dielectric space. You remember your delta P is equal to delta epsilon dot e. So, your delta epsilon, so that tensor matrix is going to be affected now because you are going to change the space where initially you only had a rectangle there but now there is another rectangle coming in. So, that means your delta x, y is now completely different. So, you are going, the perturbation is different now. So, because of this perturbation, you can consider coupling between these two.

So, one way of doing it is by not looking at this whole perturbation at all, handling is as a complete system, I am solving it using Maxwell. And the other way to do that is decompose this system into individual waveguides. If I have 4 waveguides, I can take the first wave guide and solve for the Eigne modes and then in the next I can only take the second wave guide and then do the solution for this. Then now I have to set of solutions.

There are two waveguides, and now I have a set of normal modes from waveguide 1 and I have set of normal modes from waveguide 2. So, now my job is to see the coupling between these two. And then I can I can come up with a strategy that how much mode one will couple to mode 1, mode 2, mode 3, and mode 4. There are kappa 1, kappa 2, kappa 3, let us say. And I can take mode 2 and then see how much of that mode is going to couple to mode 1, 2, 3 on the other side. So, there will be set of kappas. So, you have normal solutions on the left and normal solutions right, and then in the middle you have the coupling matrix. So, this is another way of doing it, analytically, one can try to come up with this.

So, in this lecture section we are going to look at how such system could be conceptualised and understood in a multi waveguide coupling system, let us look at that. So, we are going to look at multi waveguide mode coupling in this case.

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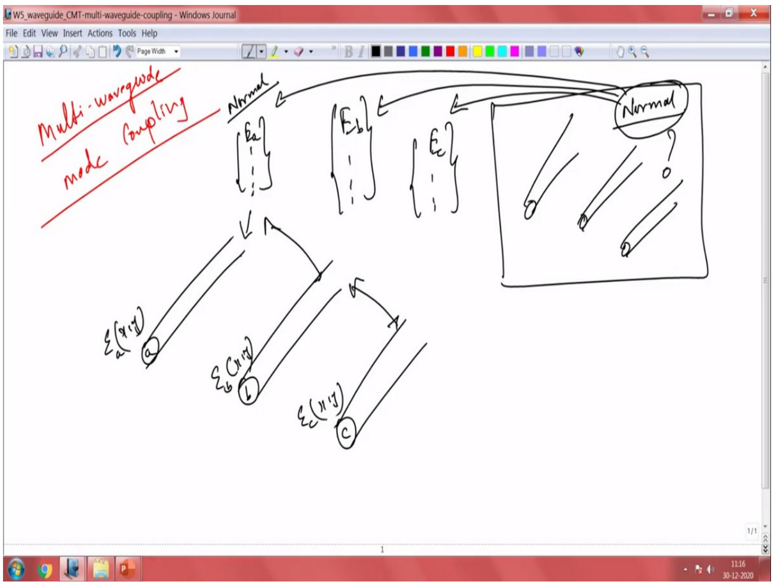

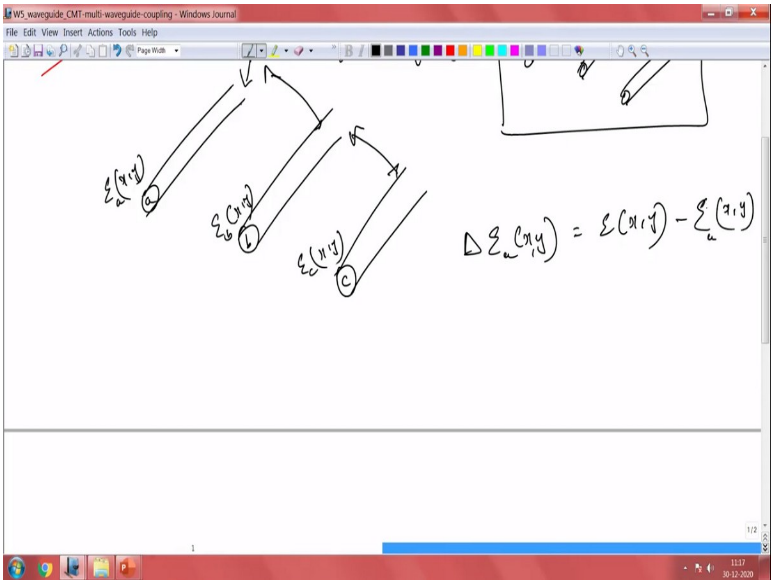

So, there are not just one mode, there are multiple modes and there are multiple waveguides. So, a simple system could be thought of as you have you have a waveguide like this. So, it has a certain refractive index profile epsilon x,y and then we have another waveguide, let us say goes by, which is epsilon b x,y. So, this is waveguide B and you could have another waveguide C, epsilon C x,y. So, how do we couple these, the energy between these waveguides?

So, there are multiple waveguide structures and there are individual single waveguides as well. So we we we just saw how one can understand coupling here, but now we are talking about how the energy could be transported between these waveguides. So, we already know the coupling formulation, so how one can formulate the coupling between two different modes, and now we, a multiple waveguide structures that we have, need to be solved. So, the way to do that is by dividing it into a combination of single waveguide system. So, that is what we actually did here.

So, epsilon x,y A x,y, epsilon B x,y and then epsilon C x,y, and so on. So, this is basically dividing the structures into their individual waveguides. So, now there are normal solutions for each of these. So, you have, let us say, normal solution here, the normal solution set for this one. So, Ea, and Eb set, and you have a solution set here and then you have a solution set here as well. So, this is all normal solution for each of these waveguide, but in the field in the entire structure that we have, so that means a combination of all of this.

So, so this is A, B, C so that is what we have done, but then if I remove all of this A, B, and C and then say this is this is my combined system, so how do I represent the normal modes here? So, individual modes are there. So for a system based approach, there are all these waveguides are now together, so how can I do that? So, in the case, I should be able to write this normal mode for the system as a linear combination of the individual modes that I have here. So, I can I can do that. So, this is something that we already discussed in the last lecture how you can do this linear expansion.

So, the field of this entire structure can be expanded in terms of the normal modes that we have for this, the waveguides that we have here. So let us look at simple perturbation that we have here. So, that perturbation that we have here is delta x,y so which is nothing but some delta x,y so that is the day, and some combination, let us say this is for example for a certain mode or certain configuration here, let us say A epsilon A x,y. So, this the perturbation that we have in the system.

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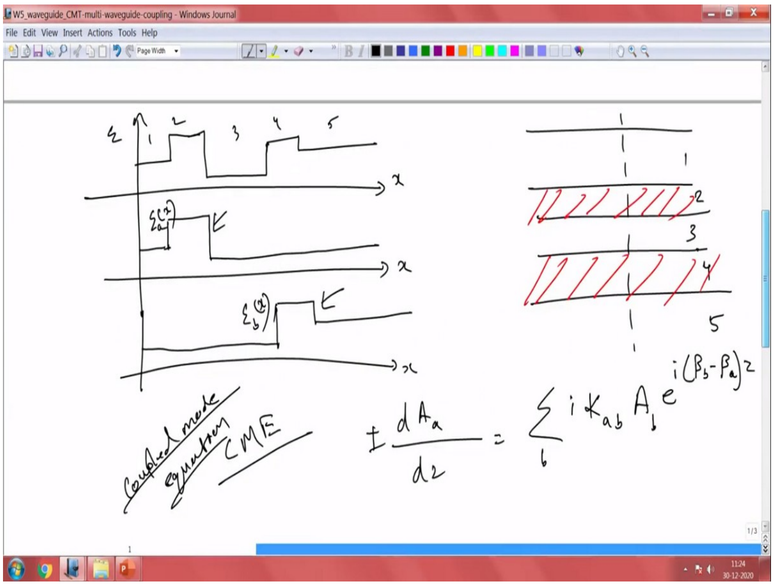

$$\pm \frac{dA_a}{dz} = \sum_b iK_{ab} A_b \, e^{i(\beta_b - \beta_a)z}$$

So, for all practical purpose, let us just look at a very simple waveguide geometry. So,, I have a waveguide with the epsilon having a function something like this, so now you can see as a function of X here, the epsilon looks like this. So there are two waveguides here.

So, from another way to look at this is I have waveguide 1 here, let us say if it is a slab section, so I have 1, 2, 3, 4, and 5. So, 1, 2, 2, 3, 4, and 5. So, this is a cross-section here so the the the guiding is happening or the refractive index of these two regions are larger while 1, 3, 5, the refractive index is lower, let us say, comparatively, and now the way to do this is by decomposing, so that is what we discussed earlier. So, we we can decompose this into two waveguide systems, two isolated waveguide systems. So, how can we do that?

We will take this particular waveguide and then do this. So, this is our waveguide 1, and the next structure will be here. So, we start with this. So this is our, let us say, epsilon a across X, and this is epsilon b across X. So, one can do this kind of combination and we know we can find the normal solutions for this waveguide and normal solutions for this waveguide, and we can do their linear combination there.

So, now the coupled mode equation for a multiple waveguide system, so this is the couple mode equation, let us say CME is coupled mode equation, could be written as a certain mode, let us say epsilon a, b, c will be equal to epsilon linear combination of system kappa a, b, a times b here. That is the amplitude, and this is our phase. So, this is beta a and beta b or the propagation constant of these two different modes that we have here. So, how much coupling we have here?

So, the plus and minus sign we already know that it is whether it is forward propagating or backward propagating. So, we have defined this and now the coupling constant here kappa, so here coupling constant is little bit complicated than what we saw earlier. So, in our last lecture this coupling constant was directly derived from the overlap of the field between the two modes that we talked about. So, here it is going to be a little different because we are talking about two different waveguide system.

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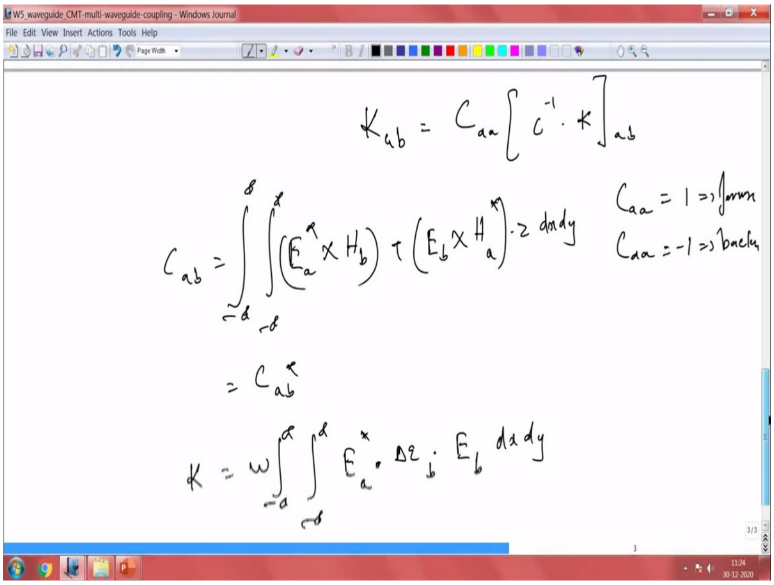

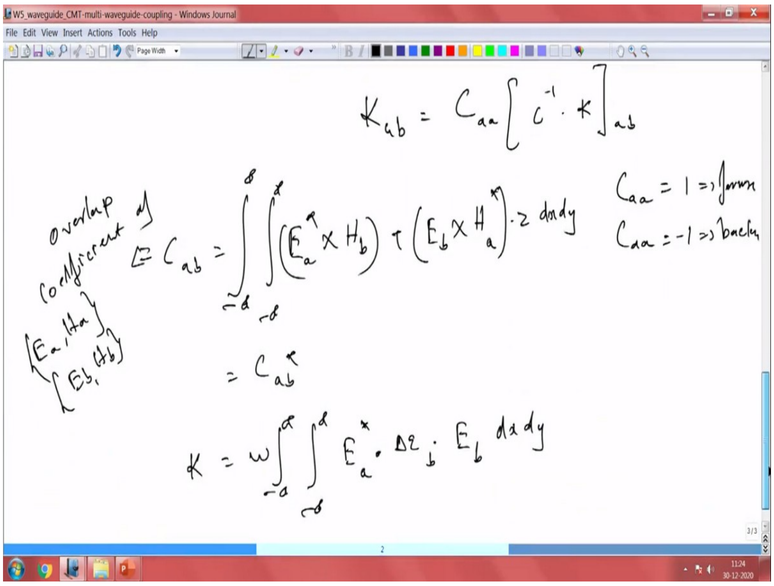

$$K_{ab} = C_{ab}[c^{-1}.K]_{ab}$$

$$C_{ab} = \iint_{-\infty-\infty}^{\infty\infty} (E_a x\, H_a^*) + (E_a^*\, x\, H_a).z\, dxdy$$

$$C_{ab} = C_{ab}^*$$

$$K = \omega \iint_{-\infty-\infty}^{\infty\infty} E_a^*.\, \Delta z_b.\, E_b\, dxdy$$

So, now the coupling here, this is kappa ab can be represented by something called C aa, C inverse kappa a. So, let us look at what these factors are actually. So, this C aa is nothing but forward coupling. So, this is when the when the wave is propagating forward, so C aa is just 1, and if it is minus 1 if it is back propagating, if it is propagating backwards. So, this C aa will be equal to 1 when it is forward, and C aa will be equal to minus 1, backward and let's see what is this matrix is all about? So, it is a C ab matrix is going to look like.

So, basically an overlap integral that we saw earlier across H of b plus E of b times H of dot Z dx dy. So, Z is the unit vector here and this is, which is also equal to C ab star. So, again the the the coupling coefficient kappa is something with is something that we already seen is Ea dot delta epsilon b dot E a, or E b in this case, dx dy. So, the coefficient of C ab, this just represents the overlap coefficient of these two different fields. So, the field overlap is captured by this C ab, the mode field of different individual waveguides, the a and b waveguides that we we we had here.

We are actually finding the overlap of this field. So this is nothing but overlap coefficient of Ea, a, and E b, Hb of this individual waveguide. So, as we discussed earlier, we are going to take individual fields and then see how we can couple it together. So, now we have coupling coefficient as a function of field overlap and and the perturbations that we have here. So, the C ab is going to be non-zero. It is it will not be 0, the reason for that is the modes that we are talking about are from two different waveguides. You, let us go back and look at this.

So, the epsilon a distribution and epsilon b distribution, you can look at it, you can clearly see that these are two different systems and two different waveguides and you will have normal solutions on this and that is what Ea is and the normal solution on b is Eb, and Hb, but they are normal solution on the isolated waveguides but then if you compare the orthogonality between these two, it is not going to be.

So, that is the reason why you are overlapped here. So, if they are not orthogonal then your C b will not be equal to 0. Your C ab will be 0 if these two modes are orthogonal or normal modes. If they are normal modes, you will have C a.b to be 0, but C ab will not be equal to 0. That is what you want. You want this coefficient not to be 0.

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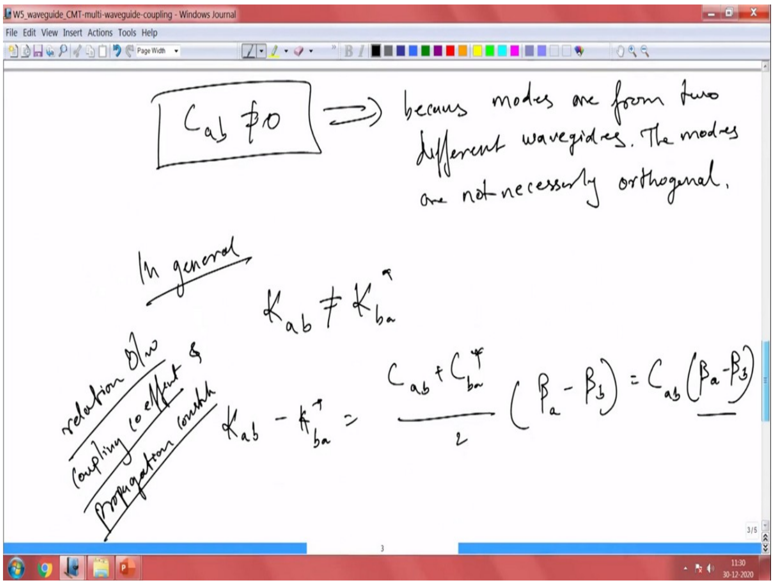

$$C_{ab} \neq 0$$

$$K_{ab} \neq K_{ba}^{*}$$

$$K_{ab} - K_{ba}^{*} = \frac{(C_{ab} - C_{ba}^{*})(\beta_a - \beta_b)}{2} = C_{ab}(\beta_a - \beta_b)$$

So, important thing to notice here is your C ab will not be equal to 0. This is an important to note. This is because the modes are from two different waveguides, the modes are not necessarily orthogonal, so this is an important statement that one should keep in mind that if you have two different waveguides, it is not necessary that those two solutions that you have, modes you have, should be orthogonal. It is possible that in some cases you will have identical waveguides. You may have two identical waveguides coming close to each other, there you you may have orthogonal solutions between these two. So, that is a special condition.

So, in general, in general, your kappa ab is not equal to kappa ba star, and your kappa ab is not equal. So, yes that that should be fine, so whatever we had, so where a, and b are from two different waveguides. So, coupling from a to b and coupling from b to a need not be the same. So, that is what it means. So, if I have two waveguides, so coupling this way and then coupling this way from a to b, so this need not be equal, so between the two different geometries.

So, if the waveguide is is is identical then it is true that you will have reciprocal coupling, but in this case you can avoid this. We could we could write the direct relation between the coupling coefficient and propagation constant. So, that is kappa ab minus kappa ba star, can be written as C ab plus C ba star raised to beta b minus beta a minus beta b, beta a minus beta b, which is equal to C ab, beta a minus beta b. So, this is relation between coupling coefficient and propagation constant.

This is a very important relation that you should you should keep in mind that beta a and beta b are the propagation constants of these two different modes that we are talking about, and C ab is the coupling between these two. If you want a coupling to happen, if you want this coupling to be happening then you you have to make sure that your betas are identical. So, you want to make sure that this term goes to 0. So, when this term goes to 0, then you can naturally see your kappa ab will be equal to kappa ba here. So, let us summarise something what we learnt here, some learning from here, I would say.

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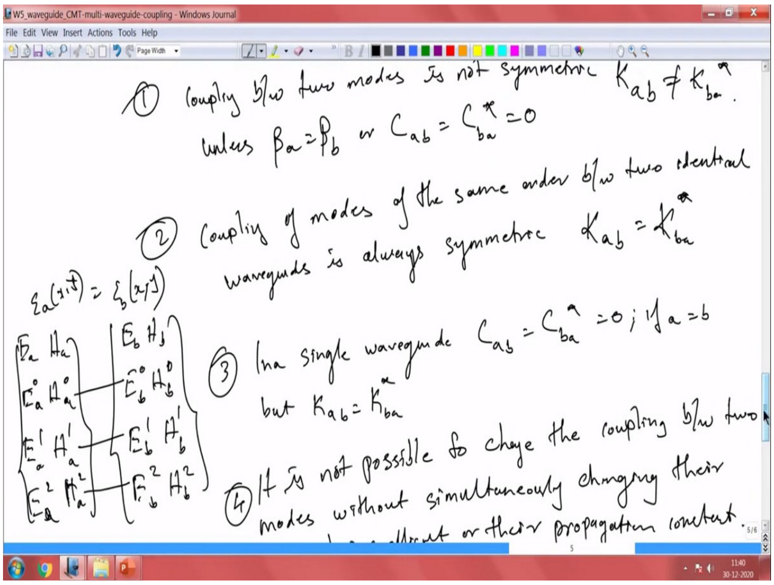

The first point from whatever we have learnt so far is that when coupling between two modes is not symmetric, so coupling between two modes is not symmetric, so this is a general statement, you can also say that the coupling between two modes can be symmetric provided the modes are, the two waveguides are identical. So, that is not a general case. That is a special case. So let us leave off the special case and look at the general case.

The general case here is you have two modes from two different system, so they will not have symmetric coupling that means kappa ab will not be equal to kappa ba star. So, this is very

very important. This is true unless unless your beta a is equal to beta b, that is what we looked at here. If beta a equals to beta b, this goes to 0 and kappa a will be equal to kappa b, a star. So, the reverse coupling will be identical or in other words your kappa ab or C ab or C ba star should be equal to 0.

So, this is because the normal modes in different individual waveguides are not orthogonal anymore. So, that C ab and C ba again if you if you want that C ab to be 0, you want this E a, and a, H a, Eb, Hb to be orthogonal between the two system. Again, we are coming back to the the same argument that unless your modes are orthogonal or your propagation constant are are are identical, you will not be having symmetric coupling. So, this is very very important point to remember. I am going to say it again, you can have you cannot have symmetric coupling unless your modes, the two modes that you are talking about are orthogonal in nature or your coupling propagation constant is identical.

So, those are all the two instances where you can have symmetric, other than that it is not going to be symmetric. So, the second point is is that, so let me, let us drive that point. So, coupling between coupling of modes of the same order between two identical, this is very important, two identical waveguides is always symmetric. So, that is kappa ab is equal to kappa ba star. So, when we are coupling between the same order between the two identical modes.

So, let us say, it is, that is you have two modes Ea and Ha and Eb and Hb. So this is one waveguide and then the next waveguide. So, this is a generalised form. So, we will have E naught, H naught. So, Ea, let us say, I will put it this way, naugh Hb naught, and then Ea first mode Ea, Eb first node Eb, anf then you will have E b second node, Ea second node, Ha second node, and Eb second node. So, this is the set you’re going to have between the the two waveguides.

So, now, the coupling of moles of same order, so same order here means 0 to 0, 1 to 1, 2 to 2. So, these are all two identical. So, epsilon a x comma y is equal to epsilon a epsilon b x comma y. So, this means they are identical. So, they are identical waveguides here. If they are identical, then coupling between the same order, E a naught to E b naught and the other way round is going to be symmetric. So, you can move between these two without any problem. So, this is very important. So, let us also remember this.

So, the third point, so the third point is so this is also true when you are having a single wavguide mode. When you are talking about a single waveguide, in a single waveguide you you could have C ab equal to C ba, which is equal to 0. So, this is this is this is possible. Let us say if a is not equal to b, because this is a this is in a single waveguide, that means there is a normal solution. You can have this but you will have kappa ab equal to kappa ba. So this is something that you can you can actually achieve in the same waveguide. We know that in a single waveguide you, it is possible to achieve a normal solution. So, then it becomes 0.

So, the C ab becomes 0, then you should be able to couple from one waveguide to the, sorry, one mode to the another mode that we have here and we should be able to do this coupling. We will later on see that this this is not universally true, there should be an overlap that is required. So, the field overlap that is required between C ab and C ba.

Just to give you an example, in this case, if you are going to take two modes for example a mode like this, and then let us say the first order mode. So, here the coupling, the kappa is going to be 0 because when you have overlap integral between these two modes, it will cancel out each other. So, you are going to put the overlap integral between these two, it will cancel out. So, the net will be 0. So, you will not have any field overlap.

However, if you have a waveguide mode that is the second order mode. So, this is mode 0 and this is mode 2, in this case if I do one overlap, the overlap integral between these two will not be equal to 0. That means the non-zero overlap is something that you can get. So, it is not just your your ortho normal solution that we are looking at but also the field overlap and the other thing is it is not possible to change the coupling between two modes without changing their overlap coefficient or propagation constant.

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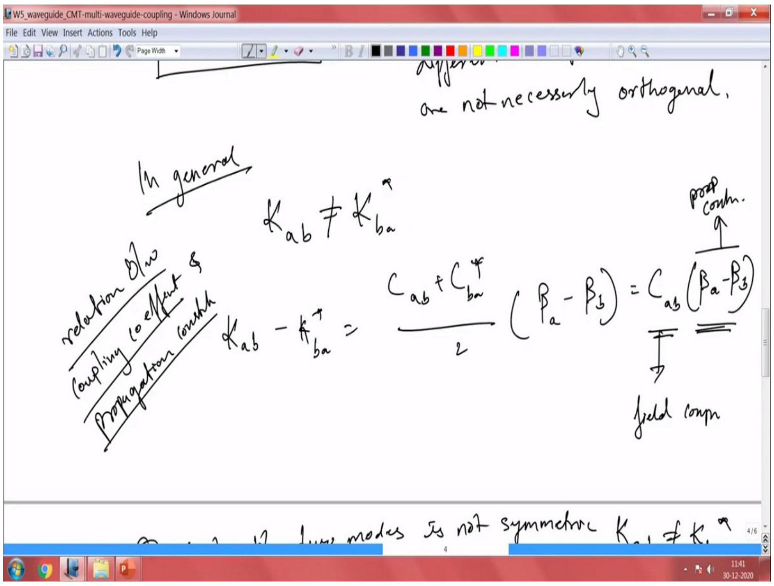

$$K_{ab} \neq K_{ba}^*$$

$$K_{ab} - K_{ba}^* = \frac{(C_{ab} - C_{ba}^*)(\beta_a - \beta_b)}{2} = C_{ab}(\beta_a - \beta_b)$$

So, that is another understanding that we should know. It is not possible to change the to change the coupling between two modes without without simultaneously changing their overlap coefficient, that is C, or their propagation constant. So, this is this is an an important understanding that that one should take from the coupled, the multi waveguide coupled coupling that we have here.

So, spend time on understanding this, the two important message that you should take away is there is an overlap that is associated with this, the overlap coefficient. So, the overlap coefficient is important when it comes to coupling and then you have the coupling constant kappa that that looks at the two E field and the perturbation associated with this.

So, a combination of these two is required to establish the coupling in this case you have the overlap, this is the field component and this is the propagation constant. So, you you need to understand both these, it’s not just the beta. One can change the propagation constant by doing I know interesting engineering, but then the field should also overlap.

So, this is this is where I I talked about field overlap between the the fundamental mode and the first order mode, which will be 0. So,, you will never be able to couple it and in the other case the field overlap between the zeroth order mode and the second order mode, where you will have non-zero, you should be able to couple it there. So, it is not just a kappa, it is also the overlap integral of your field as well.

So, with with with that we have a clear understanding of waveguide coupling right now with a single waveguide and also multiple waveguides. So, let us look at in the in the following classes how we could couple in a single waveguide whether the waves are propagating, co-propagating along the waveguide.

You could also have counter-propagating. So, there could be a scenario where your Ea, the field is moving from left to right and Eb is moving from right to left. So, these two waves are moving in opposite direction. In that case, is it possible to couple light between these two and in the other case both the modes are travelling with each other. So, that is called co-propagating and counter-propagating.

So, let us understand these things in the next lecture, but this, whatever we discussed today, in this section is very important for understanding what we are going to get for this co and counter-propagating. With that, thank you very much.