Photonic Integrated Circuit

Professor. Shankar Kumar Selvaraja
Centre of Nano Science and Engineering
Indian Institute of Science, Bengaluru

Lecture No. 23

Coupled Mode Theory 1

Hello everyone, let us look at mode coupling in this lecture. So, so far we understood, how one can confine light and this confinement would basically result in Eigen solutions. So, these Eigen solutions are termed as modes. And we may want to transfer energy, with one Li single mode. So, that is what we saw in the last lecture. So, we just want to transport light from point A to point B using a particular mode or a particular solution. So, this particular solution is also representative of, what polarization that we are going to use.

So, we have more numbers, in one hand and we have polarization in the other hand. So, a particular mode could be of order 1 or order 0. So, that is a fundamental mode, of a particular polarization. Similarly, we could have a fundamental mode for other polarization so, let us say TE and TM. So, since these solutions are, are orthogonal, let us say, they are Eigen solutions, they do not talk to each other. So, if you want to transfer energy from one spatial mode to another spatial mode, we need to look at, what, what could be the mediating factor here.

So, we cannot just transfer, transfer energy from one mode to the other. We already saw that, why light is very hard to work with, because they do not give out energy that easily. So, here again, the spatial modes that we saw in the last class, they are not going to share their energy that easily, which is a good thing, when you are transporting light, you do not want the mode or the solution to leak out or, give the energy out to other radiating mode or non propagating or unguided modes, so you want them to hold the energy that you give.

So, sometimes this is good, but sometimes this can be really difficult, when they do not share their, the energy. In some instances, you want to transfer energy from one mode to the other mode. In some cases, you want to transfer energy from one polarization to another polarization for example, you want to do polarization rotation, or you may want to do polarization filtering, and this device should be able to discriminate this different polarization. And this particular device should be able to couple light from one mode to the other mode.

So, this is a very challenging task. And from our basic understanding so far, we may not be able to do it that easily. So, we need to understand what would mediate this energy transfer. And that is a whole topic of discussion in the, in couple of lectures starting with this, where we are going to look at the couple mode theory. So, how are we going to couple different mode? So, let us look at it.

We are going to look at this coupling in a single waveguide and waveguides of multiple nature. So, you could have multiple waveguides and a single waveguide with some perturbation. So, these are all the scenarios where you could, mediate coupling. So, we will see why, we, we need such perturbation in the waveguide in order to mediate this coupling. So, let us quickly move into this topic of mode coupling.

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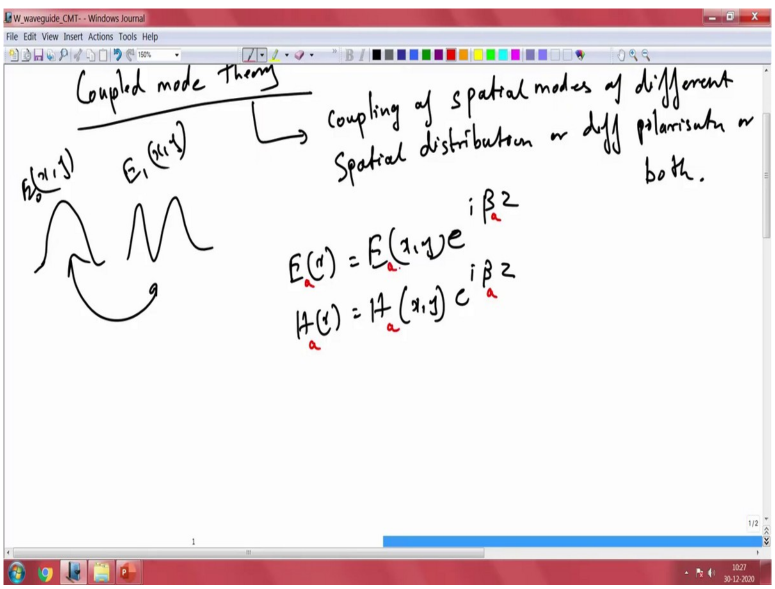
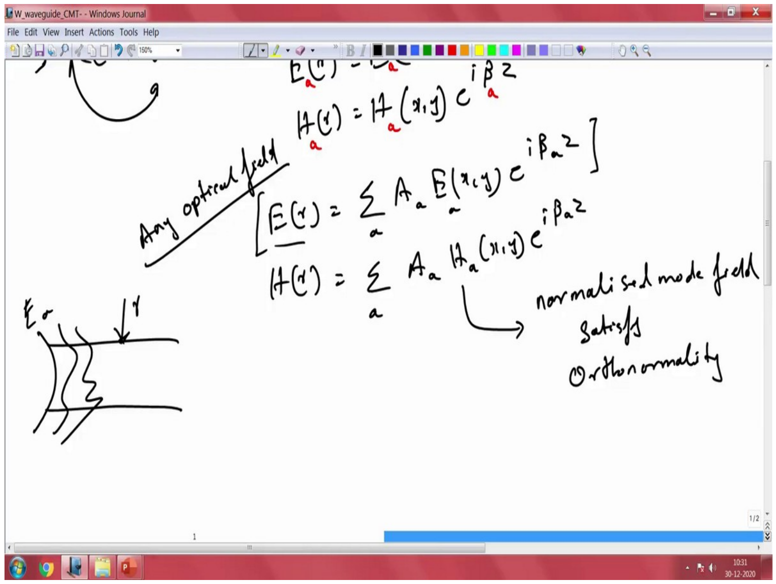


$$E_a(r) = E_a(x, y)e^{-i\beta_a z}$$

$$H_a(r) = H_a(x, y)e^{-i\beta_a z}$$



$$H(r) = \sum_{a} A_a H_a(x, y) e^{-i\beta_a z}$$

$$E(r) = \sum_{a} A_a E_a(x, y) e^{-i\beta_a z}$$

So, the coupled mode theory is what, this deals with coupling of spatial modes of different spatial distribution. So, it is coupling of spatial mode of different spatial distribution, or it could be different polarization, or it could be both. So, this is, this is exactly what we would like to do. So, it can be some arbitrary shapes. So, you can start with, a simple spatial distribution of some A of x comma y something of this kind and then we have another wave, let us say E0 E1 x comma y.

So, how do I transfer energy between these two or even it is possible at all to start with. So, what are all the conditions we need to transfer this energy from one to the other? That we have, we have seen earlier that, these, these are all Eigen solutions. So, we should keep that in mind, that is going to be a fundamental to this whole discussion. So, the normal modes, what we mean by normal modes are the fundamental, the Eigen modes here could be represented by using a simple spatial distribution of this kind. So, this is what a very simple spatial distribution of this mode.

So, this is for electric field and this is for our magnetic field. And since this is for a particular mode, we will give it an index. So, we can give, let us say some a. So, it is going to be for a particular mode. So, the normal modes, given by this, this E, E of r and H of r, are the characteristic solution of the Maxwell. So, these are all nothing but the solution of the Maxwell. And let us try to expand these modes. So, how this mode is going to look like. So, the mode expansion could be done by using a linear combination of.

Similarly, one could look at. So, this is nothing but, any optical field in the waveguide, could be just represented by a combination of multiple fields here. So, here the summation a is nothing but putting all the Eigen solutions together and making a resultant field here. So, the Er is, is nothing but the electric field at a particular position. So, you have a certain waveguide let us say, at a certain position r, you want to look at how the, the electric field and magnetic field is going to look like?

And in this case, it is going to be affected by all the possible solutions that we have in the system. So, you have multiple solutions here, there is Ea here. So, there are multiple solutions that we have, these are Eigen solutions, they are all going to be show up in this particular location here. So, that particular field is nothing but, combination or linear expansion where you, if you want to call it, of this individual fields you have. So, here Ea is nothing but your, your normalized mode field.

And this would satisfy the orthonormal relation. So, this is nothing but the normal, normalized mode field or the Eigen field that we all know and this should satisfy the ortho normal, ortho normality let us say. So, they are all, orthogonal. So, that is something that we know from the Eigen solution. So, this, this summation over all discrete indices that we have, the all the discrete mode that we have here of the guided mode, and one can integrate it over the continuous index and that would result in the radiation and evanescent mode all together.

So, when you do this combination you will know what are all the mode there including, the radiation and evanescent modes, all the possible solutions that we, we have. In, in a very simple ideal waveguide, where these mode are defined as normal modes, that means they are Eigen mode here, they do not couple with each other. And one can expand this a, the constant here as a function of x, y and z in that case. So, that is the spatial distribution of the field here. So, one thing that we should understand from this simple representation is that, when the mode are normal, when, when the mode are normal in the system, they to not couple with each other.

So, that is something that we should look at. So, what is ortho normality? So, the, the normal modes are orthogonal in nature and can be normalized. So, when you normalize it becomes ortho, orthonormal. And, and they, and these fields will have its own intensity, what should be the intensity of this particular mode, that is traveling?

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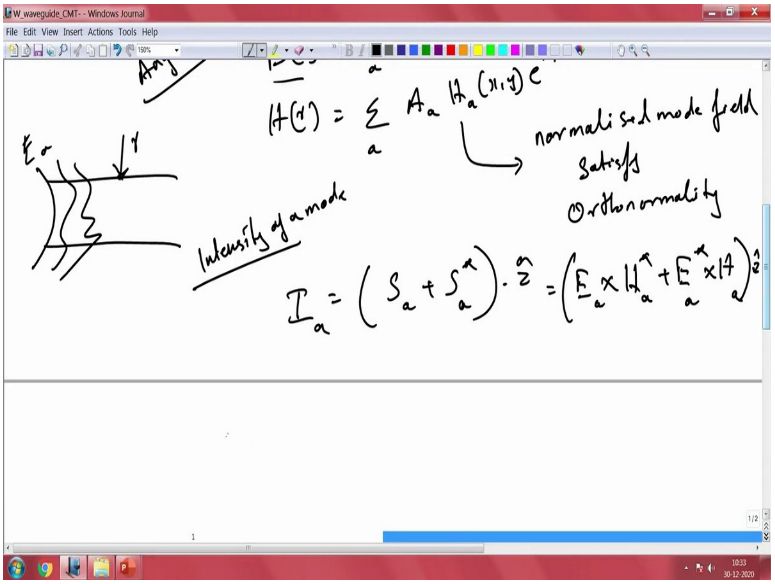

$$I_a = (S_a + S_a^*).z^* = (E_a x\, H_a^* + E_a^*\, x\, H_a).z^*$$

So, intensity of a particular mode, so intensity of a mode could be different as I of a let us say, and that is given by our pointing vector, S of a, plus S of a star. And this is moving in z direction. So, you could have z cap, in other terms, this is E cross H. So, E of a cross H of a star plus E of a star cross H of a. And again, z's hat. So, that is energy flowing along the z direction. So, this is the intensity of a waveguide mode, this is how we defend, this can be a function of x and y. So, this will be a function of x and y and this is, this is actually the intensity. And the power that, that you have how, this is the intensity.

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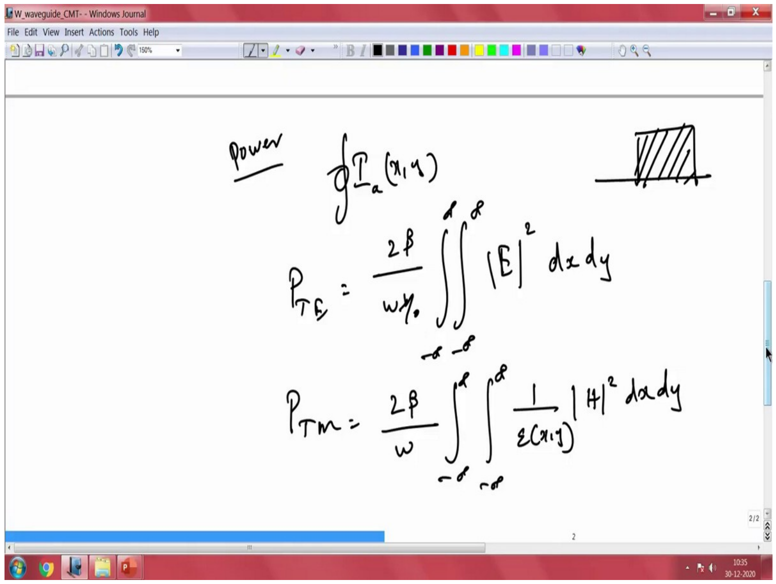

$$P_{TE} = \frac{2\beta}{\omega\mu_0} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} |E|^2\, dxdy$$

$$P_{TM} = \frac{2\beta}{\omega} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left(\frac{1}{\in (x,y)}\right) |H|^2\, dxdy$$

So, what is the power? So, power is nothing but, so if you want to have power, so, power is nothing but integrating, your over x and y, in the region, wherever you are looking at. So, if your waveguide is looking like this, so then this is the power that is being carried the cross section, of your waveguide. So, this gives you the intensity of your waveguide. So, let us look at, how one can look at the power for TE waveguide and TM waveguide.

So, power for TE waveguide is given by a double integral, both x and y so, minus infinity to plus infinity. So, this is the region that we look at, so dx and d and nothing but the E field. So, to beta over omega mu 0 is the constant here. So, similarly, for TM, the power that is propagating through is omega double integral 1 by epsilon x comma y, it is h square dx dy. So, this is how the power can be calculated. So, by just integrating it, over this xy space that we have.

So, but in a loss less waveguide, the field, the mode fields have the orthogonal, orthonormality solution. So, the orthogonality relation is something that we should, we should also know. We can find whether these modes are actually orthogonal or not. So, for example, we mentioned this

Ea. So, there are set of solutions that we have. So, we can actually find whether these modes are orthogonal. So, that property need to be understood.

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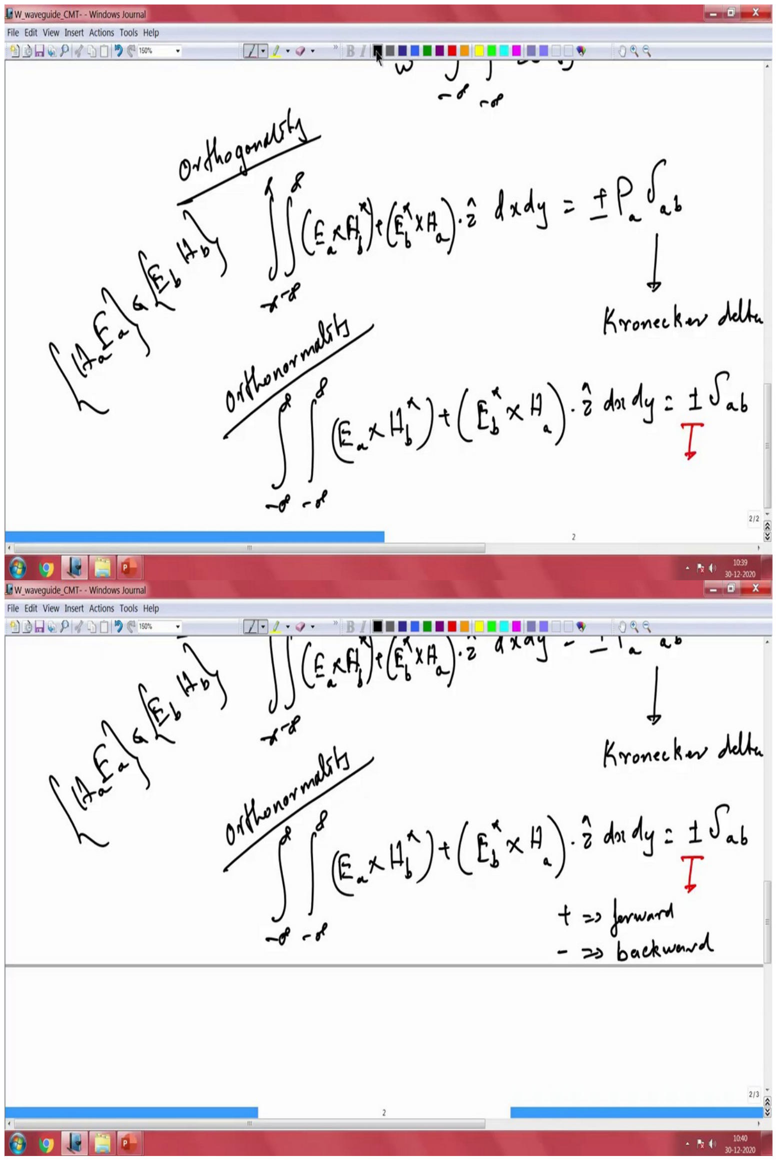

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (E_a x\ H_a^*) + (E_a^*\ x\ H_a).z^*\ dxdy = \pm P_a \delta_{ab}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (E_a x\ H_a^*) + (E_a^*\ x\ H_a).z^*\ dxdy = \pm \delta_{ab}$$

So, how do we find the orthogonality? It is a, basically the cross product is how we do that. So, we can take the space here, dx dy, just nothing but Ea times Ha star plus Ea star cross Ha. Ha Times z hat dot product, dx dy and this should be equal to something, what is that? That is the delta function. So, plus or minus P a times delta a let us say. So, here you need to have orthogonality between two different modes. So, what you see here is the self orthogonality.

So, this is, this is something that you could do, but, what you are looking at is between two different waveguide mode, in that case your a here has to be, let us say a and b and here b and a. So, now, the, the delta function here is nothing but a b. So, this is, this is how we can find out whether these two mode that we are talking about, the Ea mode and Eb mode, that the field that we have the, a mode and b mode, I should say Ha and Hb as well. So, these are all the two modes we are talking about. So, these more fields, whether these fields are orthogonal or not. So, that can be given by this simple relation.

So, what is, this delta? This delta is nothing but our Kronecker delta function. This is nothing but Kronecker delta, good. So, now, the mode fields can be normalized. So, now, we have the, just the orthogonality. We want to look at the ortho normal. So, orthonormal is basically normalizing, you will not see a big change here, except the power is, is gone in the next relation.

So, orthogonality we saw. So, ortho, ortho normality is exactly the same equation or rather same relation Ea times Hb star plus Eb star times Ha dot z dx d y is equal to plus minus delta ab. So, this is the auto normality relation here. So, you may notice that, there is a plus minus at the right hand side. So, what, what is the significance of plus and minus here? So, it is just a delta function.

So, why do we have to put whether it is plus delta or minus delta. So, plus sign and minus sign shows that direction of propagation of this particular mode. So, in this case this plus and minus tells us the direction of propagation. So, normally we use plus for forward and minus for backward, so, the forward and prop, backward direction could also be found by this, this delta function.

So, the electric and magnetic field of a particular mode, it is a particular mode a can be represented by this normalized mode fields. So, one this Ea and Eb, this is just a normalized field. And in one can, take it to, much more elaborate representation, we say Ea and Eb and this,

this is just a mode representation. If you remember we had this mode number M, a similar way, it is nothing but mode, mode M here, but since we are talking about two different mode, we took a and b.

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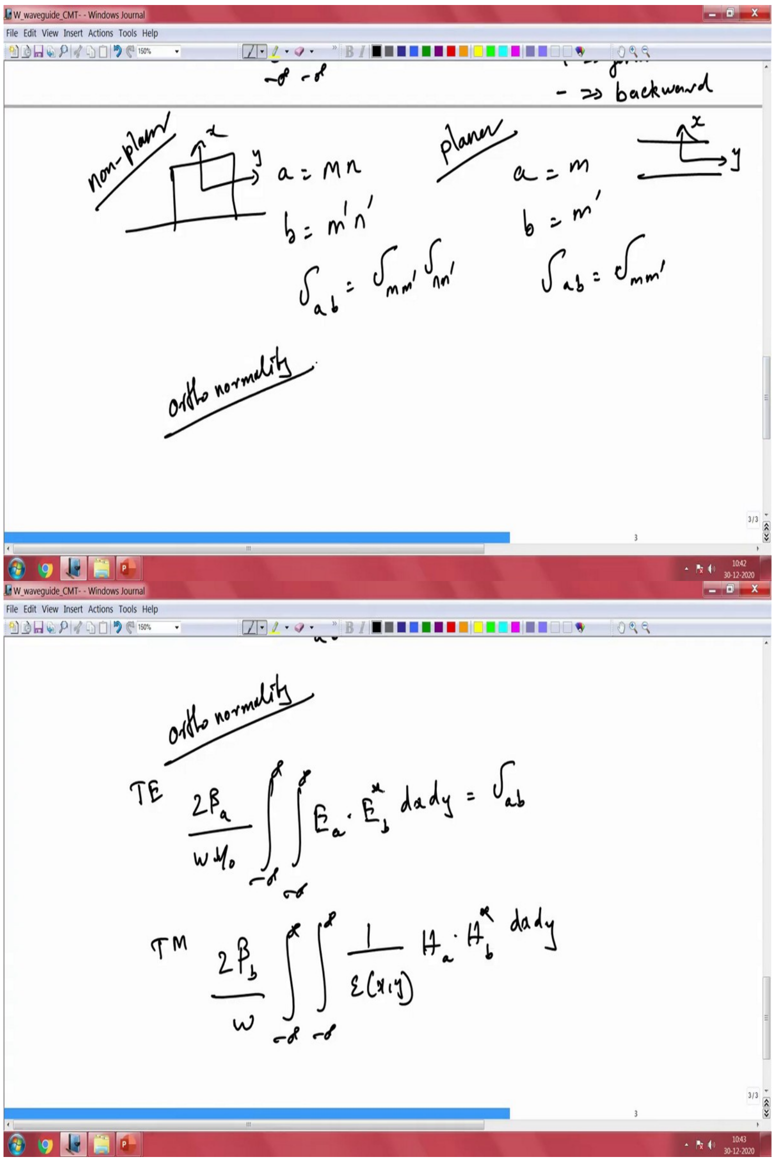

$$P_{TE} = \frac{2\beta}{\omega\mu_0} \iint\limits_{-\infty-\infty}^{\infty\infty} E_a x\, E_a^* \, dxdy = \pm\delta_{ab}$$

$$P_{TM} = \frac{2\beta}{\omega} \iint\limits_{-\infty-\infty}^{\infty\infty} \left(\frac{1}{\epsilon\,(x,y)}\right) H_a x\, H_a^* \, dxdy = \pm\delta_{ab}$$

But then this M is not a single number, if you are taking a non planer waveguide, for a non planar waveguide, so this a would become m n let us say. And then the b would become m dash n dash let us say. But for a planar waveguide, so this is non planer, so for a planer waveguide, so your a will be m and your b will be m dash let us say. So, rather straightforward and your delta here will be a, a b and you will have delta m m dash delta n n dash, but in this case your delta a b is nothing but delta m m dash. So, these are all two mode.

So, this is something, the convention that one should know, because we are talking about x y system, where you will, your light is confined both in x and y direction. However, in planar, we do not have, a boundary along y axis. So, it is only along x axis. So, there is no mode definition along y. So, it is just m. So, this is something that you should keep in mind when, when you are representing a mode. So, x, x y or y x depends on how you see it.

So, let us look at the, the orthonormal relation for a TE mode than a TM mode. So, for a ortho normality, for a TE mode. So, let us look at the ortho norm, we saw the general case here, let us look at a specific case for TE mode here that is given by 2 beta a over omega 2 mu 0 Ea dot Eb dx dy which is given by delta ab. And now, for TM waveguide, so this is 2 be, 2 beta b over omega 1 by epsilon x comma y Ha dot Hn star dx d y, forgot to do this, equal to delta ab. So, this is the orthogonality relation or orthonormal relation that shows us that power cannot be transferred between these two mode in a linear and lossless medium. You can see it just ends up with a, a delta function there is no power transfer here.

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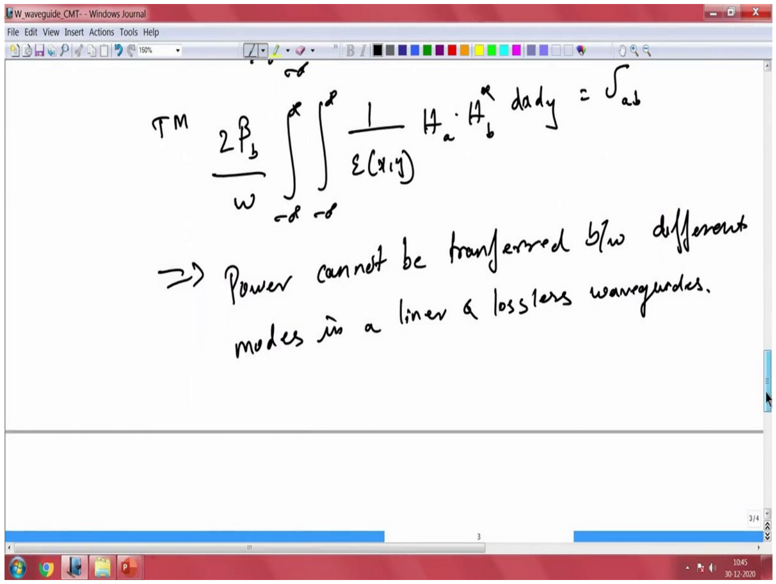

So, this implies power cannot be transferred, between different mode in a linear and loss less waveguide. So, this is simply not happening because of this orthogonality that we just saw, between two different modes. For anisotropic or lossy waveguide, the orthogonality condition for these modes, would have a different form. So, we will look at how we are, we are going to couple that.

So, that is the crux of, of understanding the orthogonality between these mode. So, now, we have established that, a normal solutions, the Eigen solutions that we have in the waveguides will not transfer energy, (())(24:53) it, we cannot transfer the energy without doing something special. So, that is something that we understood so far. But now let us look at how we can create, some mediation, in order to transfer this this energy. We need to come up with some ways of coupling it, because we cannot leave it this way, it will not be useful, for us to manipulate the light in the waveguide.

So, we need to understand and try to come up with a strategy to couple this. We need to expand these, these waves and then see how we could couple this. So for that, we, we can do this by creating some perturbation. So, when you have a waveguide, without any perturbation, that means it is a uniform waveguide, without any optical perturbation. So, the wave is not seeing any discontinuities in the system. So, your epsilon x y is, is uniform across z, when it is propagating in along the z direction, you are not seeing any, any difference at all.

In that case, it is nearly impossible, or it is impossible to couple between these two mode, because there is no perturbation in the system, and they stay orthogonal. So, that is what we have seen. But if we want to make these mode to talk to each other, we need to create some disturbances, we need to create perturbation in the system. And what this perturbation is going to do, we will see that shortly, the whole idea of creating perturbation is trying to meddle with the propagation constant of these waveguides that, that we have.

So, right, right now they are propagating at beta, beta a and beta b, let us say. But now, if we are going to have some perturbation, it is not going to be beta a and beta b. So, because of this perturbation, this perturbation, you can understand it like, having the waveguide narrow down, or you can make it broader or bringing another waveguide closer to the system. So, that is a two waveguide system, or even in a single waveguide system, you could have some roughnesses, change in the waveguide, these are all constitute to perturbation.

And when you have this perturbation, your propagation constant will change, you are not anymore, traveling with beta a and beta b between these 2 wave guides, you have beta a dash and beta b dash. So, the propagation constant changes. So, we have seen that in our earlier lectures that your beta depends on your waveguide geometry as well. So, when there is a small change in the dormitory that is what we call here perturbation and that perturbation is going to affect your propagation constant.

So, let us look at why we intentionally do that the perturbation here and change this propagation constant and take that and then try to couple this. So, we will see how that perturbation is going to help us in coupling this.

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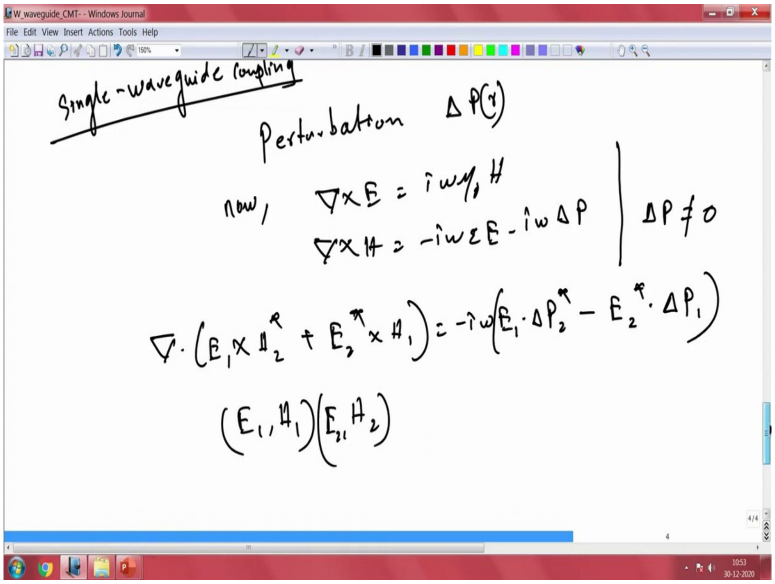

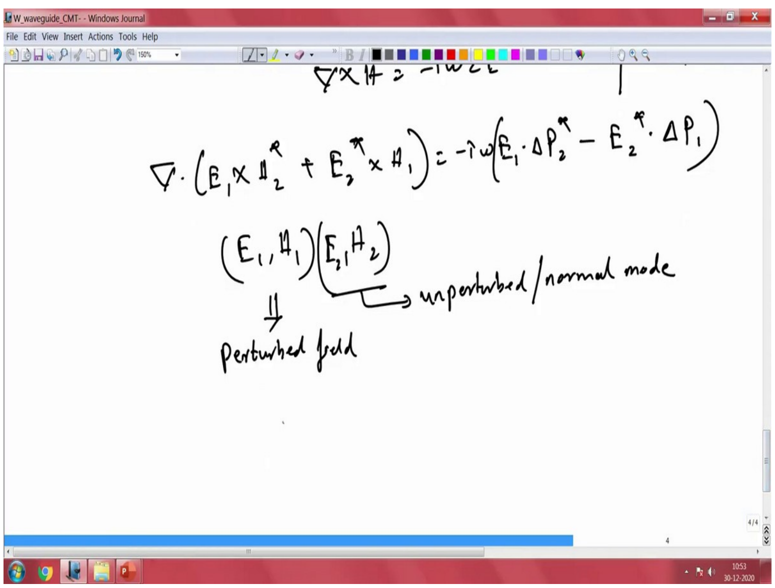

$$\nabla x E = i\omega\mu_0 H$$

$$\nabla x H = -i\omega \in E - i\omega\Delta P \;\; ; \; \Delta P \neq 0$$

$$\nabla . (E_1 x\, H_2^* + E_2^*\, x\, H_1) = -i\omega(E_1 . \Delta P_2^* - E_2^*\, x\, \Delta P_1)$$

So, let us look at that. So, when a facial, spatial dip, dependent perturbation to a waveguide, when we do that, the mode becomes, a non ideal anymore. So, the normal mode, is not normal anymore. So, there is some disturbance that you create, that is, that is going to disturb this relation that we, we just saw. And, once we have this disturbance, then it is possible to couple this.

So, let us look at how we could do that. So, one can expand the mode in terms of normal mode, So, any mode, here, that is what we saw, let me go to, here. So, any optical field, any optical field in the waveguide could be represented as a linear, combination of orthogonal or normal mode here. So, this is similar to your four year series, so you can make any kind of signal by using your Fourier components. So, the same thing applies here as well. So, each four year component is nothing but your mode here. So, the normal mode. So, even in this case, we should be able to, use the same strategy that even the perturb system could be expanded in this form.

So, let us take a very simple, a single waveguide coupling. So, let us look at, so let us consider a coupling between the normal mode, and in a single waveguide configuration. And we create some spatial dependent perturbation. So, that is, the disturbance here, the perturbation, this perturbation can be represented as perturbation to the polarization here, is represented here at certain frequency. So, this is the perturbation that we have.

So, with this perturbation your Maxwell equation will be, so now, with perturbation your Maxwell will be slightly modified. So, the field in the perturbed waveguide is now, governed by these two relation between E and H, where your delta p is not equal to 0. So, that means the perturbation is not equal to 0. So, when, when you make this perturbation to 0 then it becomes, a simple unperturbed normal waveguide.

So, now, we have added some perturbation into the system and then see how your Maxwell equation is going to help us to find this coupling. So, now, we can, relate the two Maxwell equation, del dot let us say, there are two fields E1 and E2, plus E2 star into H1 equal to minus I omega E1 dot delta P2 star. I will tell you in a bit what this E2 is minus E2 star dot del P1. So, here E1 comma H1 and E2 comma H2, these two are our fields, those of the perturbed, this is nothing but perturbed, this is the perturb field let us say.

And here we have this one is unperturbed or in other words, this is the normal mode. So, now, what we are trying to do is we are taking one of the mode here, and try to perturb it with a perturbation delta P.

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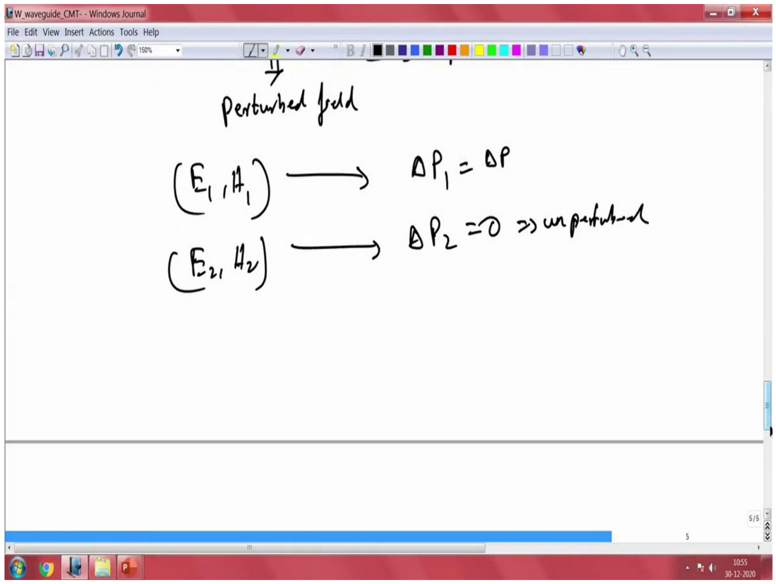

So, the perturbation associated with E1 E2, E1 H1, perturbation is delta P1 and for E2 H2 it is delta P2, but then our definition here is the perturbation for E1 H1 is delta P1, but then we have defined E2 H2 is going to be unperturbed, we do not want to perturb both the field at this point of time, we just want to see what happens when I just perturb only one. So, since we are not perturbing the second field, your delta P, so delta P becomes 0 here.

Because, because this is unperturbed. So, only you have delta P1. So, since we have delta P1 just 1. So, we could call this as delta P if you want, for, for the discussion. So, if you have this, then you can substitute this equation in the integral form that we already know here the, the summation that we saw. So, let us try to bring in the integral form here, across the cross section of the waveguide. So, we are going to look at how this perturbation is going to show, show up when you are going to find the field in a given cross section.

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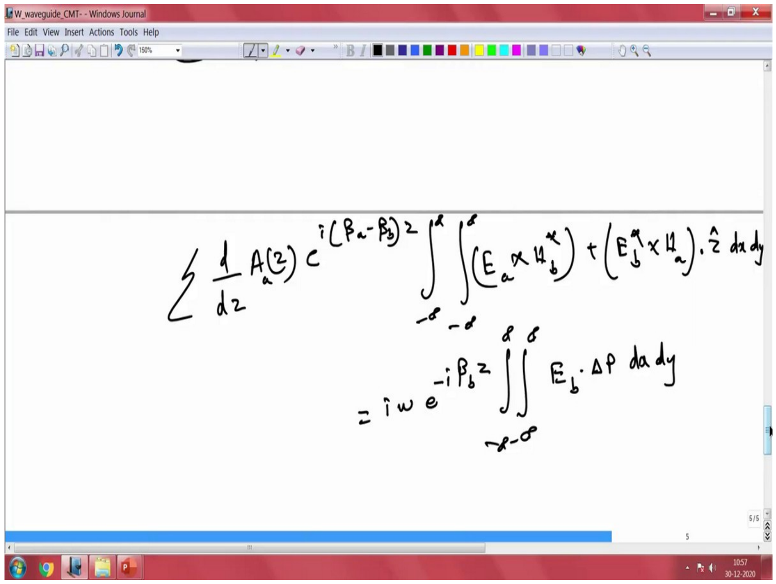

So, let, we start with the summation, you are going to have all the fields here, along distance dz. So, a times z, so this is your mode, that you are looking at, epsilon a let us say, E to the power I, beta a minus beta b z. So, this is the field that you have and this is the overlap that you are looking at, the cross product Ea times Hb star plus Eb star times Ha times z hat dot is at dx dy. And this is equal to i omega E to the power of minus I beta b, time z.

So, you can go back and look at, why we are having this. E times b dot delta P dx dy. So, if you integrate the field over, the waveguide that we have along the cross section, this is what you would, would arrive at this earlier, the hand side was just a delta function. So, you did not have anything special there, but now because of your delta P, so because of your delta P we now got the effect of delta P on your Eb. So, now, there is a sort of non-zero quantity on the right sides or rather, a coupling quantity on the right side. Well, so this is, this is how your, but you can apply your ortho normality here.

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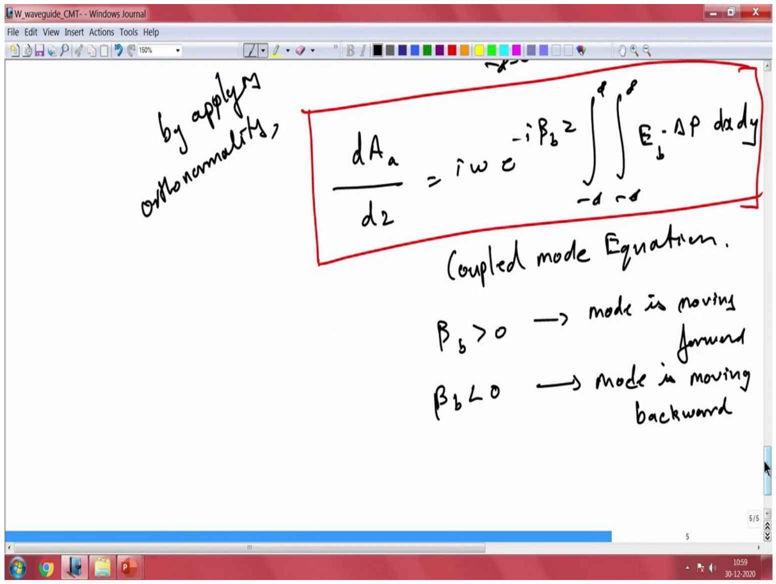

So, by applying orthonormal solution here, so orthonormal condition, by applying ortho normality, we can make, simplify, dot delta P dx d y. So, now, again here, you have the beta quantity here, we will just mention that in a bit. So, this is how, your, your coupled equation is going to look like. So, and this is basically what we call coupled mode equation. So, here the sign of beta, it tells you whether it is moving forward or it is backward.

So, beta B if it is greater than 0, it is, mode is moving forward. If beta b is negative, then mode is moving backward, this is forward and backward propagating. So, the result can be used for coupling, cost by any kind of, spatial disturbance that you have. So, you can take the waveguide here and, and we should be able to look at how the coupling is going to look like when you have any spatial disturbance.

This, the spatial disturbance that we talked about, the delta P can be a perturbation, polarization due to the effect of non linearity for example. So, that will create optical interaction at, at certain frequency that you, that you have. So, one could take it even further and then find how this delta P could be, implemented.

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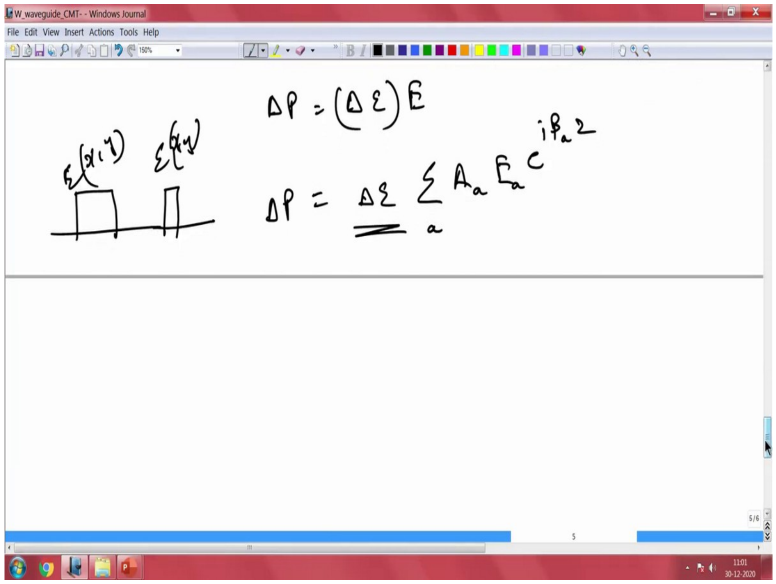

So, far we just mentioned you could have this delta P. So, how one could implement this delta P? So, it is basically change in the, in the dielectric constant. This is something we already saw. So, one can look at this as again a combination of fields that you have, across. So, you look at the, the field variation here, this is (())(40:17) epsilon. So, this delta epsilon is what is going to bring this perturbation. So, the perturbation, so far we just it is delta P, but now you, we are defining that this delta P could be brought in by changing the, the dielectric space here.

So, that means you can put a, a change in the shape or size, going from here to here. So, there is a, a change in your epsilon x y. So, this change will create this coupling. One can do that. And, this coupling, that we saw here, so let me also expand that.

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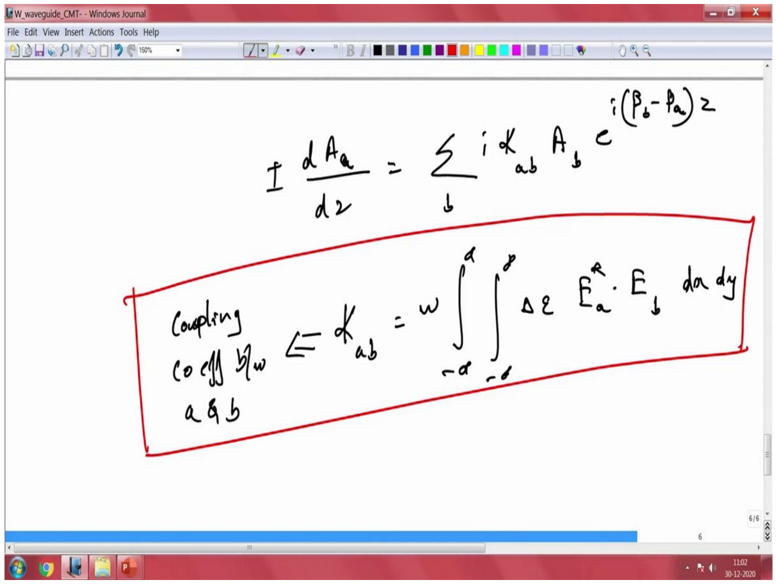

So, that then we will have the comprehensive, what we call coupling coefficient. So, we can have the field here, how the field varies as it propagates through, as a function omega of b i kappa ab A times b e to the power i, beta b minus beta a times z. So, this is actually how one can expand the coupling, between the different mode as it propagates through the system. So, now the kappa a b could be given as, delta epsilon E of a dot E of b dx dy. And this, is nothing but coupling coefficient, between a and b.

So, this is a important relation that we should keep in mind, when we talk about coupling between two different, mode or two different configurations. So, so far we looked at coupling between the two mode. So, why it is not possible, we establish that just to summarize, the reason for that is the orthogonal solutions. So, they do not talk to each other, they will not transfer energy. In order to make them talk to each other, we need to create a perturbation, and this perturbation can be done by creating change in the dimension, for example.

So, you can create, change the dialectic space. So, when you create the dialectric space, change you should be able to create a coupling between these two. And these coupling strongly depends on how strong your, your difference is. So, we just looked at this, with a single waveguide configuration. We will see how this can be done for multiple mode or multiple waveguides in the next lecture.