Photonic Integrated Circuit
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Lecture No. 22
Waveguide Design Effective Index Method

Hello everyone, so, let us finally look at, is there a better way to find the propagation constant that is close to reality? And not just mathematically given out by this calculation, so it should be physically realistic as well, like non negative propagation constant for a given cross section. So, there is a method call effective index method, is something that we use to convert this complex two dimensionalstructureto a one dimensional variation.

So, when you say one dimensional variation, we are going in the direction towards slab approximation. Well it is a very good approximation it works amazingly well,it can be rather straightforward very quick, to look at the structure and also get too close to, what one would get with a numerical methods were of course, numerical methods are straightforward if you have, solvers, there is a FVM, FDTD or a finite difference method, there are many solvers that are available out there.

So, one can use that numerically solve this, but then analytically making it gives you a, a quick understanding of your design, you do not want to, spend time injust getting your bright mode numbers or whether this TE or a TM mode that exists to run this whole numerical method there, you can quickly analyze this and for effective design, this is a good starting point as effective index method is. So, let us look at what this effective index method is.

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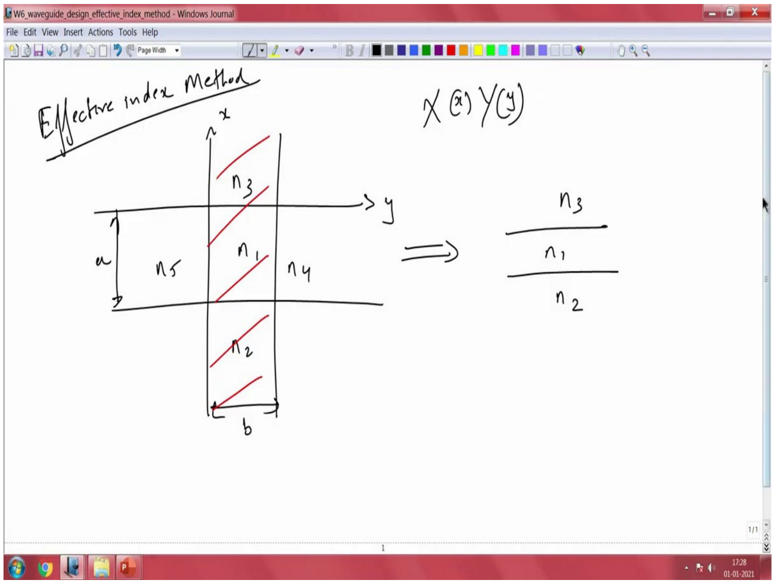

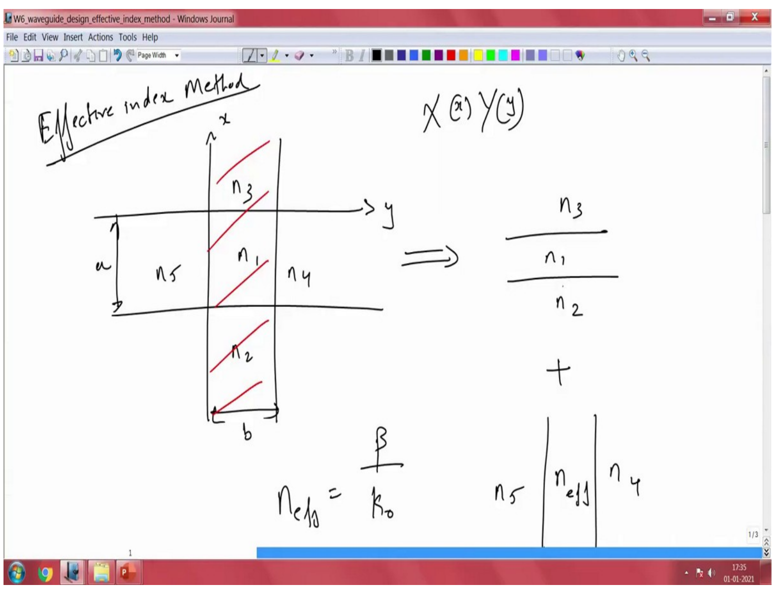

$$n_{eff} = \frac{\beta}{k_o}$$

So, effective index method is very similar technique to what we, saw earlier in this particular section of lectures in, by decomposing the two dimensional structure that we had. So, this is y this is x, we had n1, n3, n2, n4 and n5. So, it has a certain thickness aand certain width b. So, this is what we, we started off with and we had, decompose this into x of x and y of y something of this kind, so, that we could nicely handle the different fields here.

So, the idea here is to, find the propagation constant beta, but also make sure, you have a much more accurate way of finding it without, having numerical errors in this thing. And it has to be rather quick as well, so you want a very simple method for that. So, the effective index method actually converts this two dimensional problem as I mentioned into a one dimensional problem. So, if we take a very simple buried structure like what we have here, so, this is refractive index n, let us say or n1 surrounded by all these refractive indexes around so, it is buried somewhere.

So, the rest of the refractive indices could be, identical for all practical purpose. So, the way that we, we do in effective index is by making, taking just a slab alone to start with. And that is this section, so we start off with only this section, we don not worry about what is outside. So, only we take the central section, that central section, for our all practical purpose, you can make it as a slab waveguide. So, we make it as a slab waveguide, then we know how to solve this. So, once we know this particular slab, then we, we combine it with another vertical structure here, where we, we have n5 and them we have n4, let me, it is not coming out clear, on the other side.

So, we are going to add this with the vertical structure. So, we have n5 and then we have n4 and here what we call n effective. So, the effective index method, so we first stretch the waveguide out long, along the a thin axis what we call, along the y axis here. And then form a simple slab and then you, take it and then put it into the thin slab in the long direction. So, this is, this is again the approximation that we are going to do.

So, this is something that we do, and this effective index, is nothing but beta over k naught. So, while k naught is your vacuum vector, and your beta is your propagation constant. So, this is, this is something much, much simpler to operate and find out. But if you want to look at, whether your mode orientation is all properly done, then we need to discuss about the field orientation when you stretch it and when you compress this.

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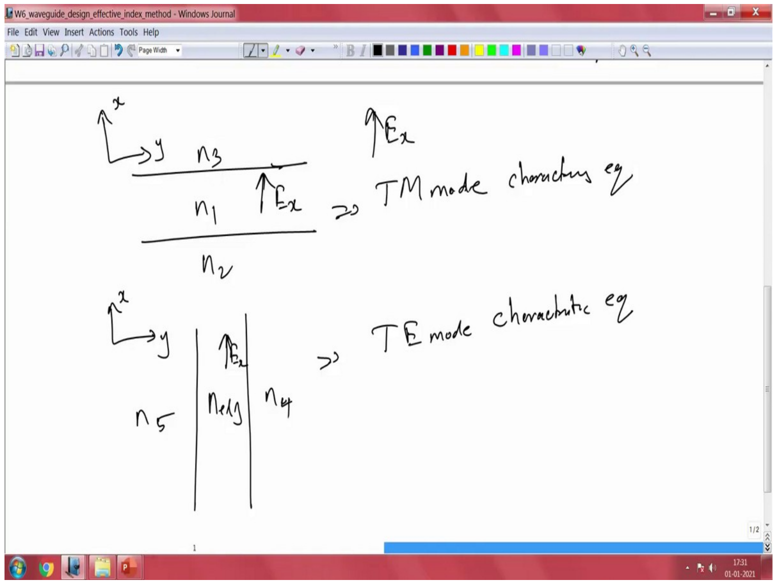

So,when you are considering, the stretched out one, so here we will consider, if the wave is pointing vertical, if you say it is Ex let us say, we say experience, then in this particular case you are talking about TM mode. So, the electric field is vertical here. So, the electric field is perpendicular to the long axis that you have here. So, we need to use appropriate characteristic equation here. So, is TM mode characteristic equation should be used, when you have a TE TE TE TE type wave to start with. Because you have your slab this way. So, the electric field is sitting like this.

So, now you have stretched out so, you have done that and now in this particular case, your E field looking this way. So, this is along the long axis. So, that means we need to use TE mode characteristic equation. So, this is something that one should be careful about, when you are using effective index method that you first know, which direction your electric field is and based on that you have to use TM characteristic equation, when you are stretched along the thin axis and TE when you are doing at the thick axis.

So, this, these two are important to note just for your clarity, this is y and this is x. It is already mentioned here. So, let us look at how one can use this effective index method to find the electric field and also find the propagation constant beta. So, we are going to do this, by using a very simple example.

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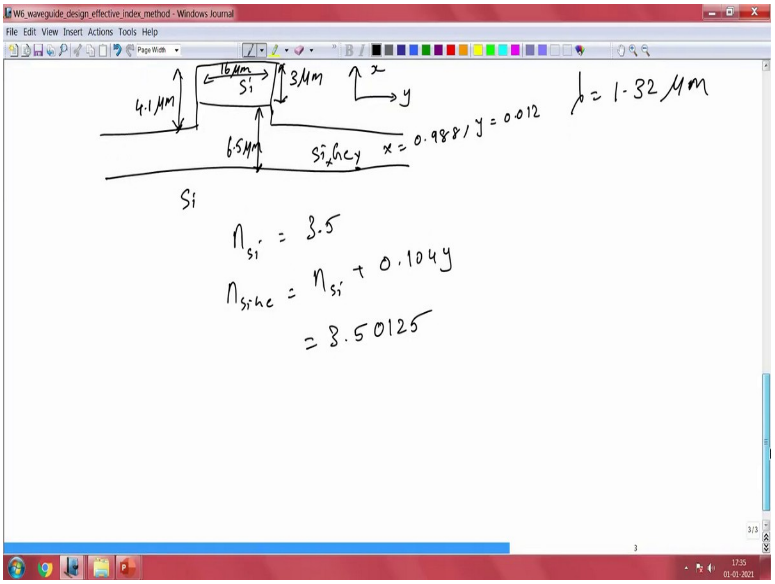

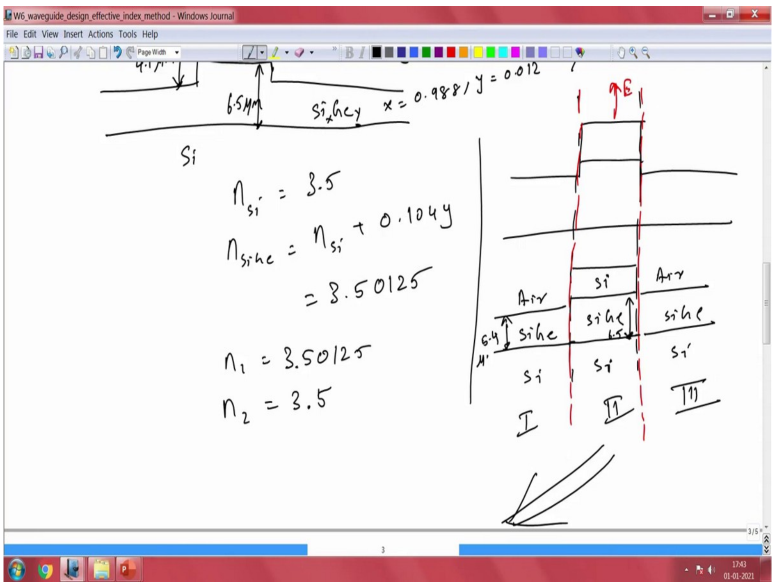

Let us see, so let us take a simple waveguide structure, let us use a rich waveguide structure. So, we have silicon substrate and I have a thin layer ofsilicon germanium in this case and then I havesilicon on top, this is a standard example that I that I have here. Let us do something important here, because silicon germanium can take, a refractive index based on the composition of silicon and germanium. So, let us say here, x is 0.988 and y is 0.012.

So, this is what we have. And the thickness I have here is 6.5 micrometers. And thickness of silicon that I have is 3 micrometers and the total thickness I have here is 4.1 micrometers. And the width of, of this particular structure is 16 micrometers. So, this is all my y and this is my x.

So, this is the structure in hand now, so we have a very simple rich structure and we want to find out the propagation constant of this. The another important parameter that we, we need to know is at what wavelength because we need to know the lambda here.

So, here the lambda is, say 1.32 micrometers, so this is the problem with us now. So, the refractive index of silicon is 3.5 and refractive index of silicon germanium we have, it is silicon plus 0.104 times x. So, the x whatever we have there, so what is the percentage? The x is nothing but, what is the percentage of germanium, in this case y here. Let me, let me put it as just directly y. So, this ends up as 3.50125. So, this is what we have.

So, we have the refractive index of silicon, we have the refractive index of silicon germanium, we know the wavelength. So, that is all we need from this particular problem. So, now, we need to decompose this whole rich structure.So, let us try to deconstruct that. So, we have, let me do it here. So, we have a structure like this. So, what is the first step in effective mode, effective index method?

We have to stretch it out or we have to segregate into different slab sections. So, let us divide it into 3 slab sections now.So, here I have silicon, silicon germanium and air, in this section, I have silicon and then silicon germanium and then silicon, and on the other hand I have air, silicon, germanium and silicon. So, these are all the 3 sections I have. Section 1, section 2, section 3, very simple segregation. So, I can use a different color here. So, this is, this is rather straightforward. So, we can quickly do this using simple slab approximation.

So, let us look at the first, this the sections here, the various sections here. So, the height is also important, 6 and a half and so, on we will we will use that as and when required, but we need to look at what is the characteristic electric field here. So, this is going to be vertical, this k is, it is going to be vertical electric field. So, that means, in the central section it will be TM mode. So, TM mode is what you will look at.

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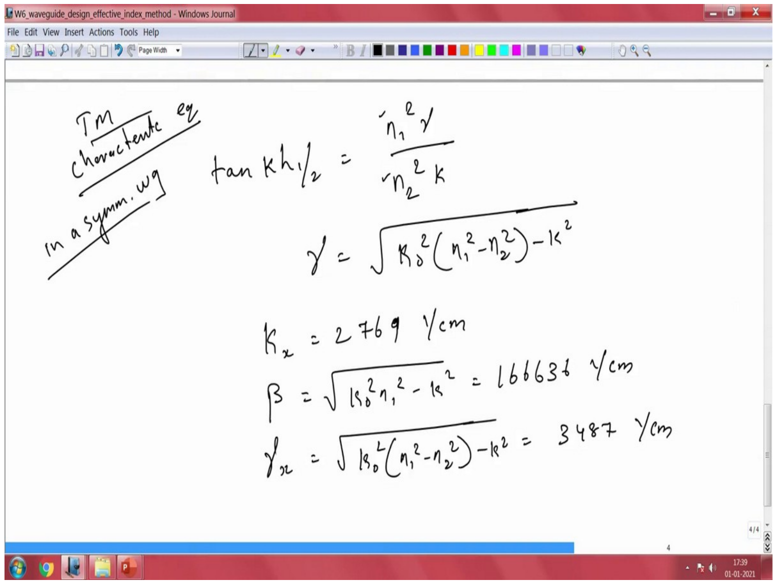

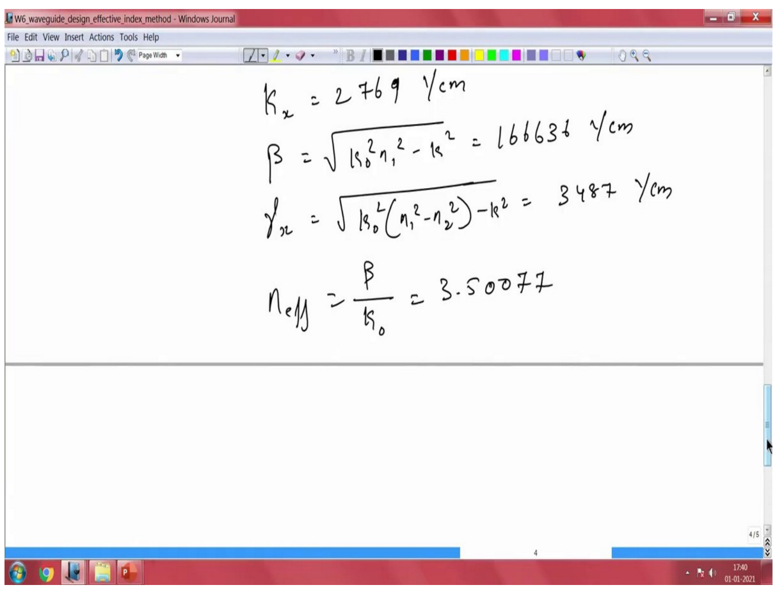

$$tankh_1/2 = \frac{n_1^2 \Upsilon}{n_2^2 k}$$

$$\Upsilon = \sqrt{k_0^2(n_1^2 - n_2^2) - k^2}$$

$$\beta = \sqrt{k_0^2 n_1^2 - k^2}$$

So, that means, the TM characteristic equation is, that we have looked at. So, this is n1 n2. So, this is that characteristic equation of TE, TM mode in a symmetric waveguide. So, now, we need to plug in the values. So, we know n1 so, n1 is 35, 3.50125 and the surrounding n2 is 3.5. So, that is refractive index we have. So, n1 we know, n2 we know. So, what is gamma? So, gamma is nothing but root of k naught square n1 squared minus n2 squared minus k squared.

So, using very simple numerical technique here, so we could find that your kx is 279, around off per centimeter beta is k naught squared n1 squared minus k squared which is equal to 16636 per centimeter, and if gamma x is root of k naught squared n1 squared minus n2 squared minus k squared is 3487 per centimeter. So, this is what we have from the relations that we currently have. So, we have numerically found that k X beta and gamma x for this particular section, beta is known, so, how about n effective, we need, we are interested in calculating the ineffective, for that particular section. So, n effective is nothing but beta over k naught. So, that is 3.5007 so, that is our n effective.

So, you can apply this for, the other structures as well. But now, we have to apply the TM mode characteristic equation of asymmetric mode. So, this is for the, the central section, the central section, that is section 2. So, we applied this for the central section, why we use symmetric waveguide? I think probably, you would quickly understand because you have silicon and silicon here. That is why I used symmetric equation for this particular section. And the effective index is also found.

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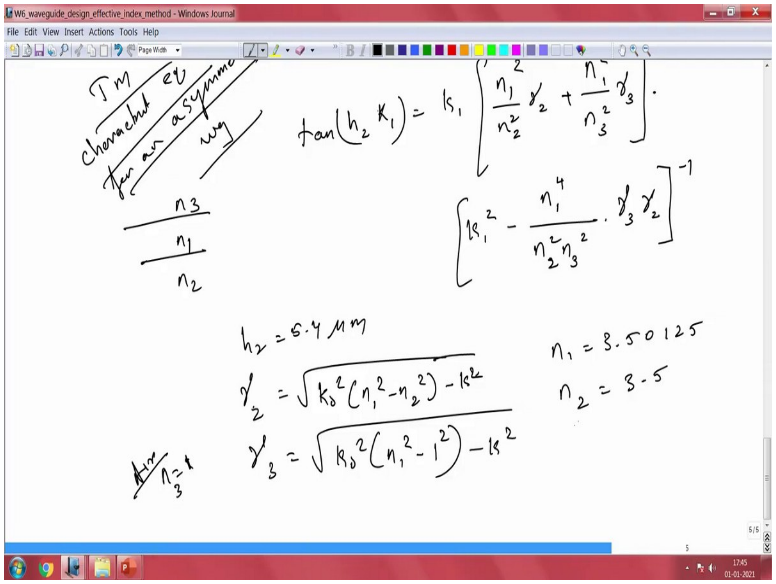

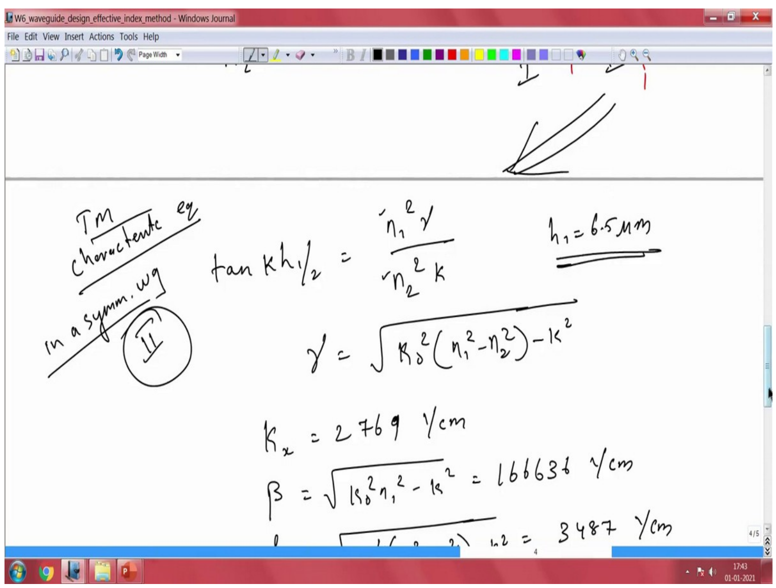

So, next let us look at TM characteristic equation, for a, for an asymmetric waveguide. So, what is asymmetric waveguide here, that is our shoulder section, air, silicon, germanium silicon. So this is asymmetric, you see, so, that is why we are going to use asymmetric relation here. So a symmetric relation, you remember that it is a little complex equation here, k1 equal to k1, n1 by n2. Let us say n1 n2 and n3.

So, this is asymmetric. So, n1 square n2 square gamma 2 plus n1 squared by n3 squared gamma 3 into k1 square minus n1 of 4 divided by n2, n3 squared times gamma 32. So, this is the characteristic equation of a TM, asymmetric waveguide.So, now the h2 here. So, h1 in this case was, was the thickness that we had. So, h1 equal to 6.5 micro meters. So 6.5 micro meters is our h1. So, that is the thickness that we had hereof silicon germanium, we are only considering silicon germanium, because silicon is considered as symmetric waveguide.

So, you are only considering this thickness. So, this is 6.5 micron, in this case this shoulder. So, this shoulder height is 5.4 micrometer. So, that is something that you can get it, 5.4 micrometer. So, now you are h2 is 5.4 micrometer and gamma 1, in this case gamma 2 so, gamma 2 is nothing but root of k naught squared n1 squared minus n2 squared minus k squared and similarly, gamma 3 is nothing but the root of k naught squared n1 squared minus 1 squared because n3 here equal to 1, because it is air.

So, on top we only have air. So, the n equal to 1. So, we have this minus a squared. So, n1 we know 3.50125 and then n2 is silicon, 3.5 and 3 is 1, this we saw here. So, we just have to apply these to this particular equation and we will get the following.

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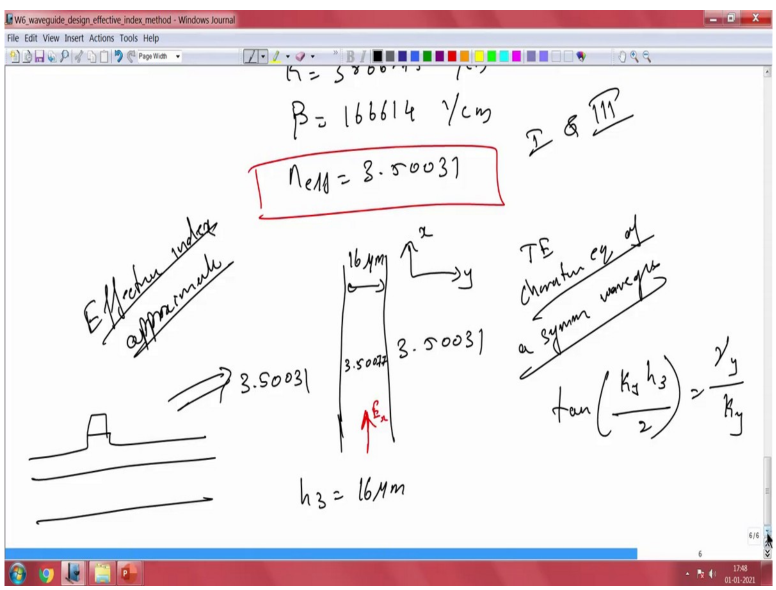

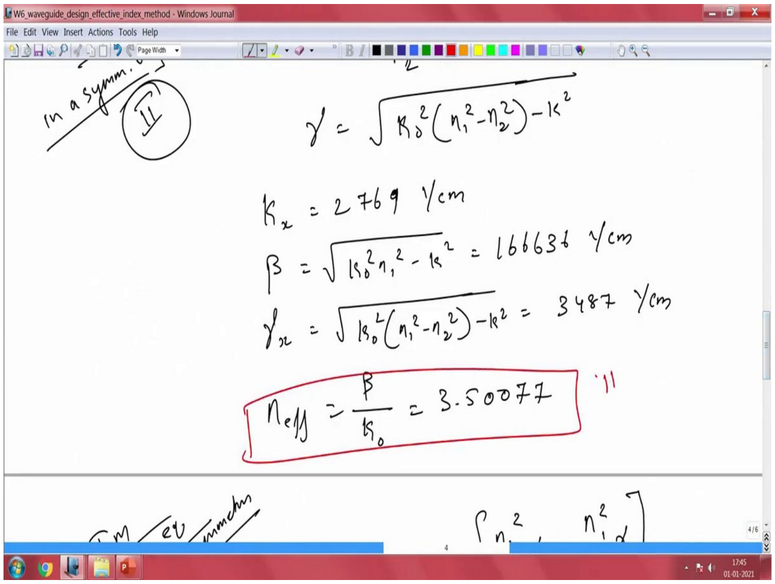

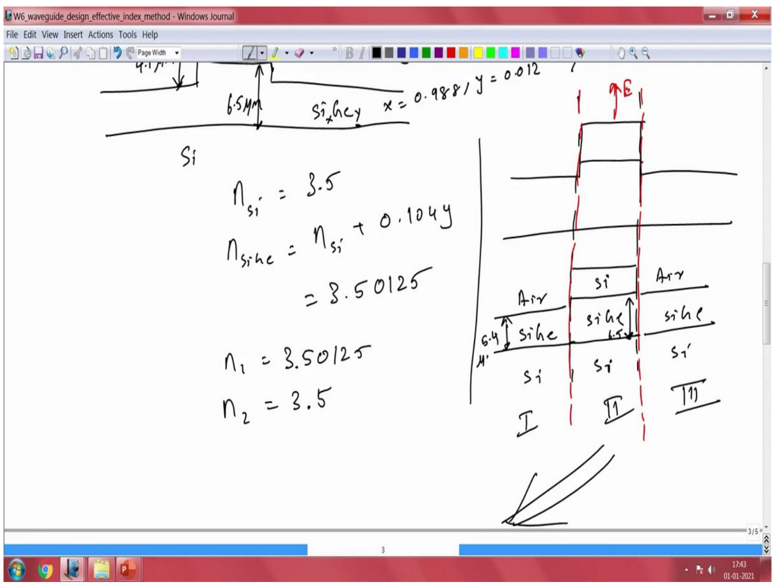

So, k equal to 3866.75 per centimeter and then we have beta which is 166614 per centimeter and n effective from that is 3.50031. So, now, we have the n effective for the shoulders. So, this is for region 1 and region 3 .And now, we have here n effective for region 2. So, we have for region 2 and for region 1 and 3. So, now we can construct our slab now. So, that is going to be vertical in nature. So, we are going to have this vertical one and here this is going to be 16 micrometer wide. So, that is our waveguide width, you see.

So, first step was to look at individualsections and now we are looking at a complete slab here. So, this is 16 micrometer. So, that is our waveguide width here.

So, we take that 16 micron and now the refractive index on these 3 sides we already know so, 1 and 3 is 3.50031 here, 3.50031, and the central section how much was it? 3.50077. So, the central section is 3.50077. So, now we have converted whatever we had into a very simple, attend from configuration. So, this is your effective index approximation. So, whatever we had, you remember this one is now converted to something like this. So, where you have two different refractive index of the shoulder.

So, now we have to look at again the field is going to be looking in this way. So, Ex let us say, along the x direction. So, that is something we should not forget about. So, y is here x is here. So, now, since Ex is parallel to the long axis, we have to use TE characteristic equation now. So, we need to use a TE characteristic equation of a symmetric waveguide. Because of a symmetric waveguide our ky h3 in this case, divided by 2 equal to gamma, y ky. So, this is our characteristic equation here. So, you can plug in the numbers, so your h3 here is nothing but 16 micrometer that is height three.

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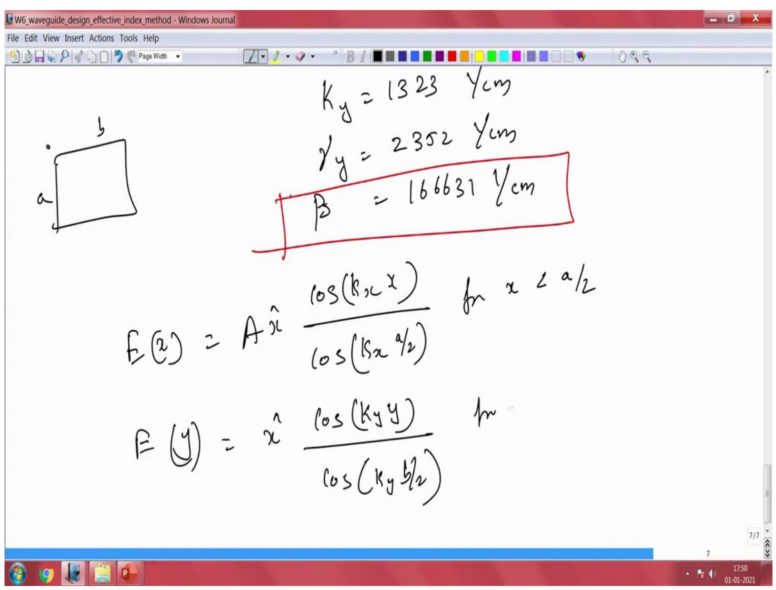

$$E(x) = \frac{A\hat{x}\cos(k_x x)}{\cos\left(\frac{k_x a}{2}\right)}\ for\ x < a/2$$

$$E(y) = \frac{\hat{x}\cos(k_y y)}{\cos\left(\frac{k_y b}{2}\right)}\ for\ x < b/2$$

So, now, we plug in the numbers and when you plug in the numbers and if you do the numerical calculation here, you would get ky will be 1323 per centimeter, gamma y is 2352 per centimeter and beta will be 166631 per centimeter. So, this is this is what one would get. So, this value of beta is the Eigen value for the mode of the waveguide. So, this beta that we arrived is your propagation constant or the Eigen value of the mode in the waveguide. And now, we need not stop here, we have found the beta, but we can go ahead and also, measure the field.

So, we can actually plot the field, how the field is going to look like, because we know, we know the field equation. So, so, the field equation could be easily given for both x and y direction. So, you have a certain intensity a along x let us say cos kx times x divided by cos kx for the dimension, let us say if a is the width, this is a by 2becausefor x greater than, so for a less, less than a by 2. So, we have that a certain, this is b and this is a, let us a say.

And Ey will be x cos ky divided by cos ky b by 2. So, this is for y less than b by 2, because it is symmetric in nature. So, that is why we can do this. And for the other factors. So, you want to know your facts will be Ex E to the power minusx minus a by 2, this is for x value is greater than a by 2, this is all decay now. So, E of y will besome constant E to the power of minus y, y minus b by 2 y is greater than b by 2. So, so, these are all the field equations so, that we have seen earlier lectures on the simple waveguide thin, planar waveguide.

So, if you, or plug in the numbers here, so, you can just put, if we know all of this, so, we know x, k x, we know ky, we need, we know gamma x and gamma y. So, all these quantities are known. So, when we plug this in, what we get is the following.

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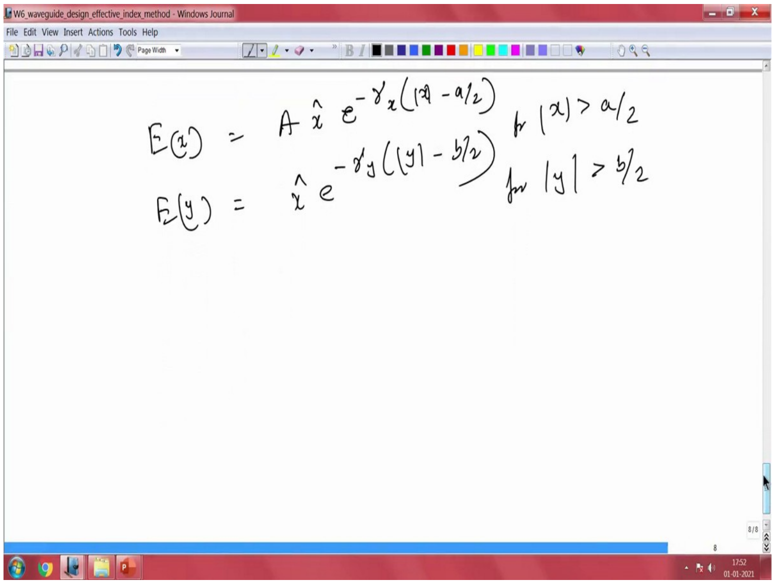

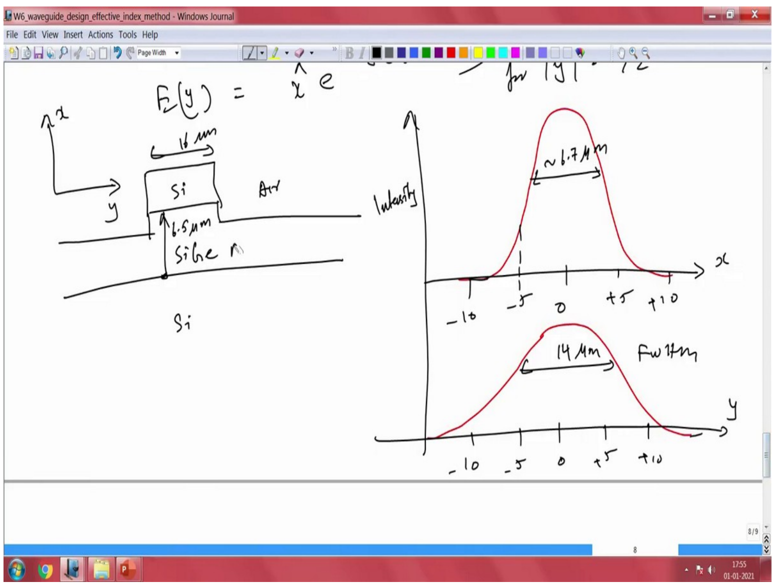

$$E(z) = A\hat{x}e^{-\Upsilon_x(|x|-a/2)}; |x| > a/2$$

$$E(y) = \hat{x}e^{-\Upsilon_y(|y|-b/2)}; |y| > b/2$$

So, let me first draw the structure, so that we can relate our result to our structure. So, here this is silicon germanium and this is silicon and this is silicon and this is air. So, this is 16 micro meter. And you remember this was 6.5 micro meter, so that is what we had. So, if we put all the numbers in, you will get two plots, one is along x and one is along y. So, this is, this is 0.Your x is going to be, the mode is going to be inside the silicon germanium. So, that has a higher refractive index. So, if you plug in these numbers you will get something like this. So, this value will be 5, 10 minus 5, 5 minus 10 plus 5 plus 10.

So, now, let us look at how large is this going to be. So, the width of this, full width half max will be approximately 6.7 micrometers. So, this will correspond to the, the height of silicon germanium. So, the what kind of width? Width we have, it will be closely to that width it will approximate be that width. So, now let us look at the y direction. So, y direction is broad here you see. So, in y direction we have 16 micrometer broad waveguide

And what you, we will get is something like this, where your full width half max will be approximately 14 micrometer, full width half max let us say but then the dimensions are brought here. So, you can see, by looking at this is intensity of course, so by finding out the simple constants that we need to know, we should be able to find out how the field profile will look like. So, if you are interested, you could take this up and, plug it into a small program in MATLAB or Python to simulate this one.

So, this will give you a good idea about the different mode dimensions and so on. So, you can go ahead and then change the refractive index that you have in this material. So, you can randomly choose some material system and then find out, so this will give you a better understanding of the solution that you could get.

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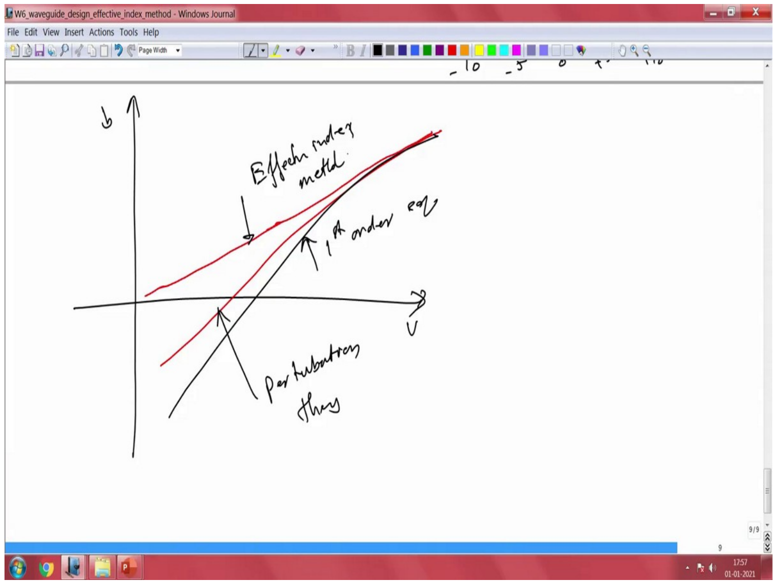

But now, we are finally back to the v number, v parameter with respect to propagation constant beta. If you remember, we started off really bad with this first order and then we were able to make it better with perturbation theory, but then the effective index is well, actually solve this problem. So, you will, you will have a solution that is very close to reality. So, whatever we, actually discussed in theory. So, it will take you to 0 when you go here. So, this is your b and this is our first order equation that we had and this is our perturbation theory. And this is our effective index method.

So, what we have shown in this understanding is that, by using effective index method, we should be able to convert the two dimensional waveguide structure that we had with certain heights, certain width into a flat slab like structure and use the characteristic equation that we already know. So, use that to solve for the effective index. So, effective index is what you are trying to look at.

First of all, first you have to convert those stack into a thin single refractive index. And once you take those refractive index, you can apply that to your, the effective structure. So, instead of individualrefractive indexes, you can convert effective index for column 1, column 2 and column 3. And now you have a slab again, but in this case vertically aligned slab and use the same technique of characteristic equation and find out the beta there.

So, using this you should be able to arrive at a reasonably accurate number for beta. So, from technique to technique it might vary, this is effective index method. We have find a different method to solve this problem, but there, the beta will be slightly different, but not very different, these, these are all very close to numerical accuracy. So, we do not have to go into the details of one method is better than the other in terms of fifth digit accuracy and so, on. But, effective index method works reasonably well for most of the system particularly,systems that are moderate refractive index and so, on. Even with higher refractive index stacks a quick calculation is, can be easily done using effective index method.

So, one important thing to notice here is choosing thecharacteristic equation, look up the electric field, whether the electric field is aligned to the long axis or the short axis. So, along the thin form, which is the in finite axis or the short axis where you have the confinement. So, based on that you should choose the right characteristic equation. So, if you make any errors in choosing the right characteristic equation, whatever beta you get will be wrong, it will not be the right, right beta for that propagating mode. So, make sure you, you choose the characteristic equation TE or TM in order to build your, calculation.

So, basically this is the end of designing your, your waveguide structures, but will not end here, you have not visualized the actual modes and, and fields yet. But we will see that when we talk about different platforms and how the, how you can visualize these modes by using numerical techniques, but for the moment, we have basic understanding of knowing where to check for these parameters, how to approximate these parameters, and what are the boundary conditions one should apply in order to arrive at a reasonable beta that is close to reality. With that, we would like to close this particular section of discussion. Thank you very much for listening