Photonic Integrated Circuit

Professor. Shankar Kumar Sevaraja
Centre for Nano Science and Engineering
Indian Institute of Science, Bengaluru

Lecture No. 21

Waveguide Design-Perturbation Approach

Hello everyone, let us continue our discussion on the boundary value problem that we are trying to solve. So, in the last lecture, we saw that we have an issue when we are trying to find physically, acceptable propagation constant. We ended up with negative beta, that is not physical at all. So, the, as I mentioned in the earlier lecture, the problem here is coming from the, the corners that we have.

So, we have, we assume that the light is confined, and this four corners are creating problems there, because the boundary condition in this four corners are very hard to define. So, we just took it is all decay, and the four corners are going to decay, and we took that decay from the neighboring blocks. So, that is, that is the best that we could do. But that is, that is not really, accurate enough. So, that is the, that is what, led us to having this negative beta.

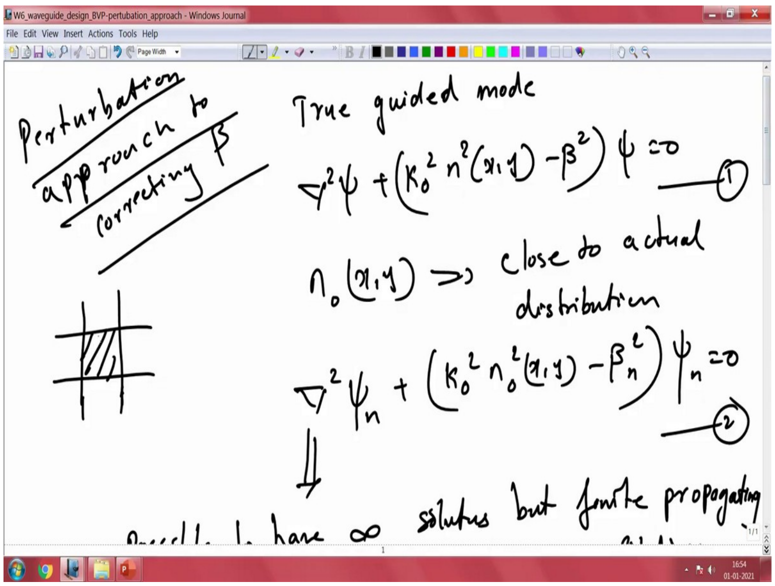
So, is there a way to out of this problem? Of course it is. And that is why we are going to discuss this particular approach here, which is called perturbation approach. So, in perturbation approach, what we do is, we are going to take the actual system and try to move to a comfortable, definition and try to work from there. So, what do we mean by that? So, we take a solution, psi, let us say, but then, this is the exact solution, let us take psi, but then I said, let us approximate that solution to some psi naught.

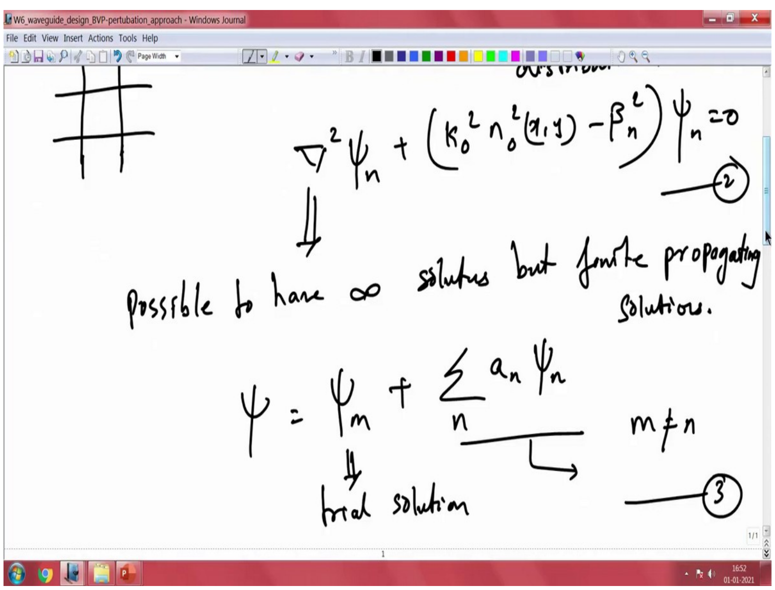
It is not exactly the same, but it is approximation. Similarly, we have a refractive index space that we take n of x comma y is so far we have been using, but then how about, taking an approximation of that space itself, so we are going to approximate that to some n naught x comma y let us say. So, that space may not be exactly the same refractive index contrast and refractive index distribution. But it is close enough, let us say, but how close it is? It is close enough for us to get a solution.

And from that solution, we will slowly migrate to an exact solution. So, that is called perturbation theory. It might look, a little confusing, what, what do we mean by that? It is like approximation. So, it is like best guess scenario. So, if you do not know where to start, you just, put your finger somewhere, and then say, I will start from here, and then move in the direction that minimizes my error. It is basically a mini, error minimization problem altogether.

So, let us look at how we can formulate this particular problem and how we can approach and solve this negative beta problem, at the end, we want to solve this, but we will see whether we will be able to completely get rid of this or make it better, let us let us look at that.

(Refer Slide Time: 03:48)





$$\nabla^2\psi + [k_0^2 n^2(x,y) - \beta^2]\psi = 0$$

$$\nabla^2 \psi_n + [k_0^2 n^2(x,y) - \beta_n^2]\psi_n = 0$$
$$\psi = \psi_m + \sum_n a_n \psi_n \quad ; m \neq n$$

So, the approach that we are going to take as I mentioned, is perturbation approach, approach to correcting beta. So, this is what we want to do. So, we want to take this perturbation approach, so that we can correct this beta. So, let us start with, our true mode. So, what we call the true guided mode. So, what is that mode? That is given by our function so you could do it this way. So, this is our actual, the wave equation. So, that, that has the solution, psi. So, the psi is our actual mode here.

So, where your refractive index is nx gamma y, so this, this x comma y is this what is causing the problem. So, because of this high, the boundaries are very tightly defined, what if we reduce that boundary? So, let us make it a little bit weak or the distribution, something that we could, work with. So, we take a modified refractive index, which is, close to actual, close to actual distribution. So, we can start with that, and the other, once we do that, then your wave equation should also be modified.

So, there you have, the solution instead of psi the exact solution that we have, it is not going to be the exact solution that we have, it is going to be something else. So, we need to take that into account, so it is not any more the exact solution, but it is some other mode that you are going to get. So, there are stuff that, this is going to be. So, this is how it is going to be. So, you have slightly perturbed or disturbed system.

There will be infinite number of such solutions. So, when you are going to change this, you can, you can easily argue that this is not going to be the exact solution that you are looking for, of course, it is not, but then the probability of having such solution is really large. So, then there are large number of solutions that you would get. There is infinite number of solutions that is that is that is, that is the right way to say it, there are infinite solutions, but then there are only finite propagating solutions, there are finite modes that are guided.

So, here we could have possible to have in finite solutions, but finite propagating solution. So, this is a very important understanding, that we should have. There could be in finite solution, we should not be worried about the infinite solution. But what we are considering here is, the subset of it, which is the propagating solution, those propagating solutions are finite. But most of these waves are going to be unguided or gray radiating modes.

So, based on this completeness of the mode, and any reasonable distribution of, electromagnetic field, that we have can be expressed by using superposition of the modes, which is something that we already saw in one of our earlier lectures. So, we can write the exact form.

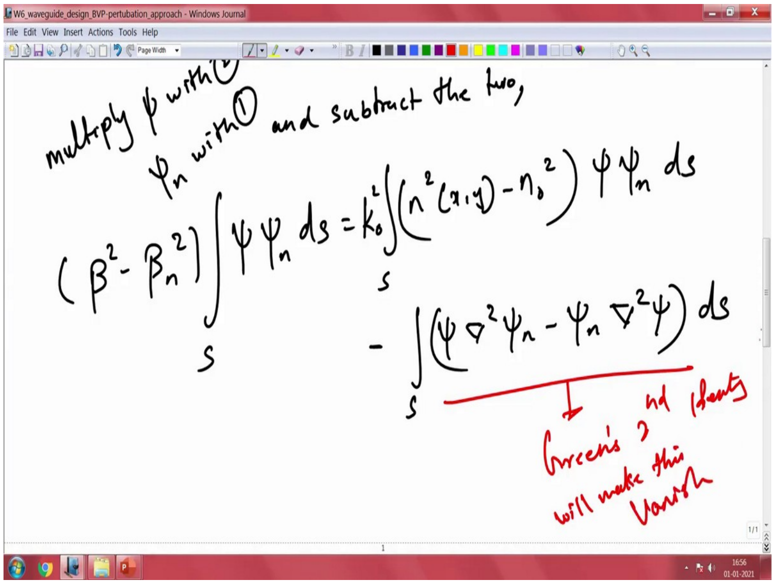
So, let me say, psi here is the exact form of the, of the mode that we have, we could have it as a combination of, linear combination or superposition of, all the modes, or all the fields that you could have. Let us say we will start with something, a trial solution. So, this will be our trial solution for this particular mode, plus summation of n number of modes with their own amplitude and their field distribution.

You have to watch out here on the, on the indexing. So, I used m for the trial solution, and n for the, rest of the solutions or the other solutions, the other modes that are there. So, m here is not equal to n. So, in the, in the summation. So, in this way, the summation represents the total perturbation on the solution that we have. So, they need not be equal. So, you take a trial solution, you are psi m, and then the rest of the things are other modes and you can put it all together and to get close to what you, what you want.

So, you can make this summation infinite and then make it go on forever, but we will have some sort of approximation. So, that we do not go in circles. So, we could simplify this and then look at these modes and not just these modes, you can also look at the ortho, orthogonality of these modes. So, orthogonality property could be used in order to take the subset of this, finite number of modes, but yet they, they are, they are going to be larger number. To make it manageable, we just try to make this, handleable by using this propagating modes and orthogonality property.

So, when we take this solution and then multiply, let me number this equation. So, if we multiply equation 2 with psi and multiply equation 1 with psi, psi n let us say. So, we will end up with a system that has both an exact solution and also I know the perturbed or the trial solution in this case.

(Refer Slide Time: 11:35)



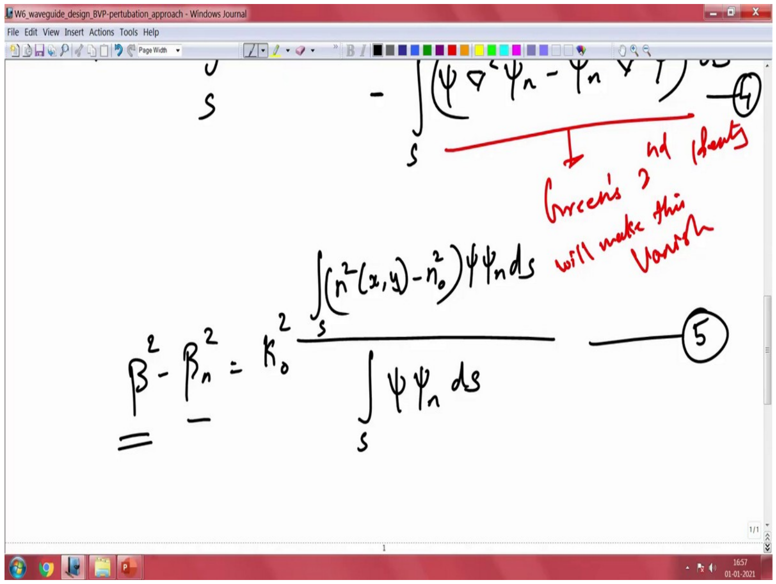
$$(\beta^2 - \beta_n^2)\int_S \psi\psi_n\, ds = k_0^2 \int_S (n^2(x,y) - n_o^2)\psi\psi_n\, ds - \int_S (\psi\nabla^2{\psi_n}^2 - \psi_n\nabla^2\psi)ds$$

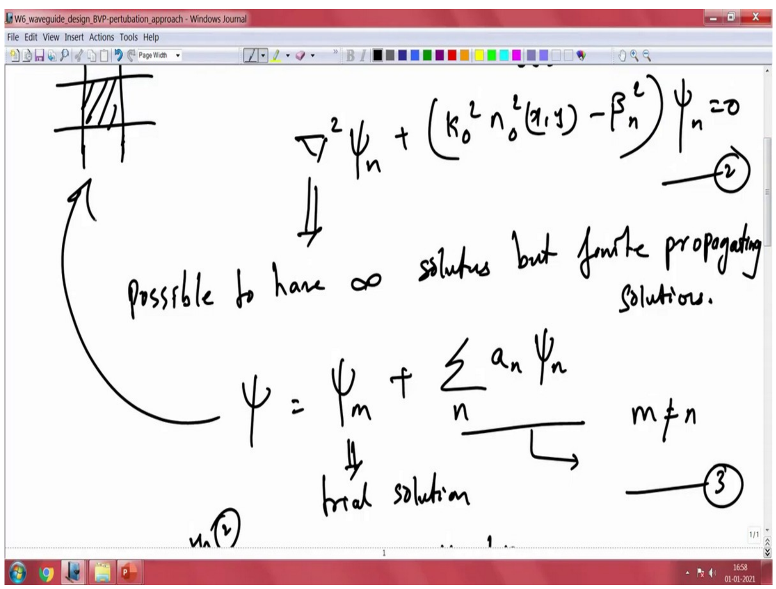
So, what we will do is multiply psi to equation, with 2 and then psi n with 1. So, if we do that, and then we can subtract these two, so we multiply this and subtract and then we need to integrate it over the surface. So, why are we doing this, to find the field in this particular region? So, that means, you need to integrate this, these two equations that we have, one is exact solution, the other one is a perturbed solution. So, you want to put it together and then integrate it over this region, over the surface that we have here, along x and y. So, this is what is going to give us the correct solution.

So, in that case, let us look at the exact solution beta squared and then you have the perturbed one integrate over the surface psi, psi n over the surface that we have, will be equal to k0 squared, we can take it out because this has no implication on the integral here, x comma y minus n naught square, d, psi psi n ds minus this is just basically, multiplying 1 and 2 and then subtracting, I have not done anything special there, you can just do it yourself and check it out.

So, this is what you will end up with so, multiplying it and subtracting the two you will end up with such a, such a thing. So, you can quickly look at this. So, you will quickly notice that this will be gone. So, this will vanish, because of the second identity from Green’s. So, so, Green’s second identity, make this vanish.

(Refer Slide Time: 14:40)





$$\beta^2 - \beta_n^2 = k_o^2 [\frac{\int_s (n^2(x,y) - n_o^2)\psi\psi_n \, ds}{\int_s \psi\psi_n \, ds}]$$

$$\int_s \psi\psi_n \, ds = \int_s \psi_m \psi_{n'} \, ds + \sum_{m \neq n} \psi_{n'} \psi_n$$

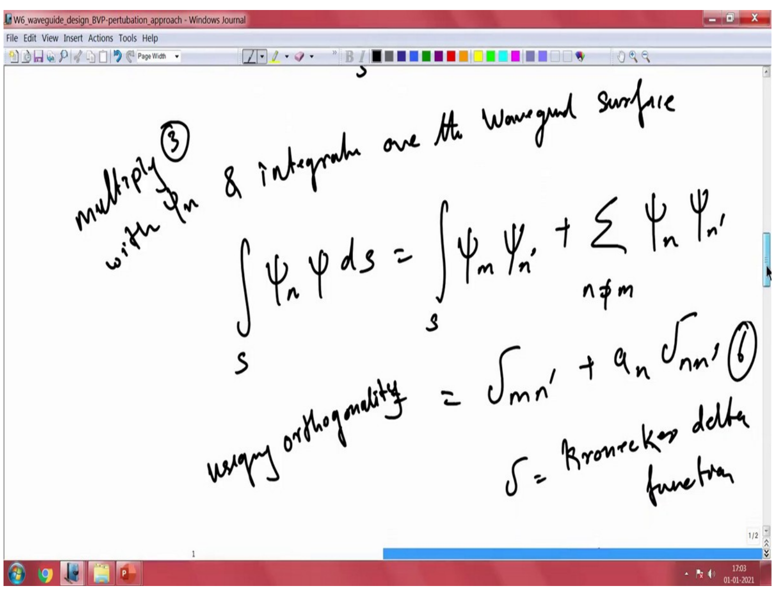
$$= \delta_{mn'} + a_n \delta_{nm'}$$

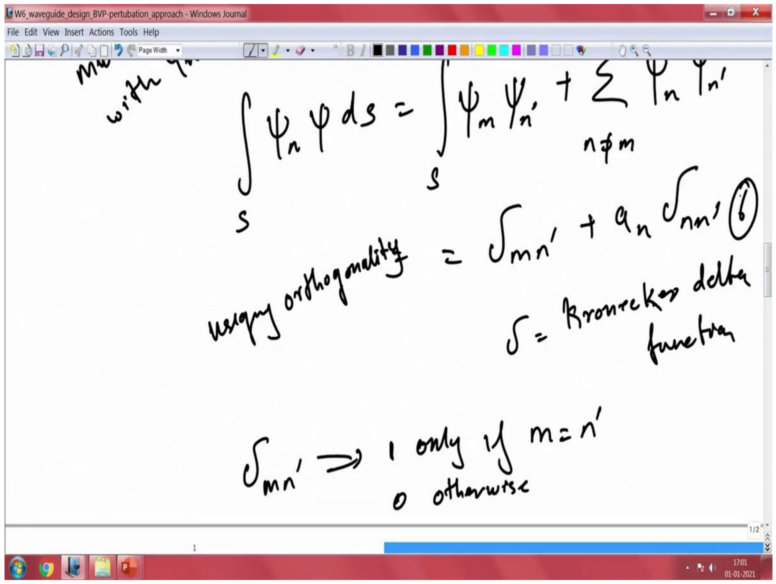
So, now we have a much simpler expression for the exact beta. We are doing the perturbed beta, k naught square, and it is going to be a pretty large integral here, n squared x comma y minus n naught squared psi psi n ds, or the surface psi n ds. So, this is equation four and this is equation 5. So, this gives us an expression, to determine the difference between the actual beta that you are looking at and, and the beta n.

So, this is the simplified or the perturbed or changed wave guide cross section that you are looking at. So, there is a perturbation that you have applied. So, what is the difference between these two? So, this is what we are trying to optimize. So, you want to reduce this, this difference and that will yield you the, the accurate solution here. So, in order to get the complete description, we must evaluate this this particular equation here. So, the equation number 3 here and find how the distribution is.

So, that means, we need to integrate this in the surface or the x and y area that we have for this waveguide. So, along this waveguide. So, we need to look at how this particular distribution is going to be in this particular region, between the exact solution and your perturbed or modified solution.

(Refer Slide Time: 16:38)





$$\beta^2 - \beta_n^2 = k_o^2\left[\frac{\int_s (n^2(x,y) - n_o^2)\psi\psi_n\, ds}{\int_s \psi\psi_n\, ds}\right]$$

$$\int_s \psi\psi_n\, ds = \int_s \psi_m\psi_{n'}\, ds + \sum_{m\neq n} \psi_{n'}\psi_n$$

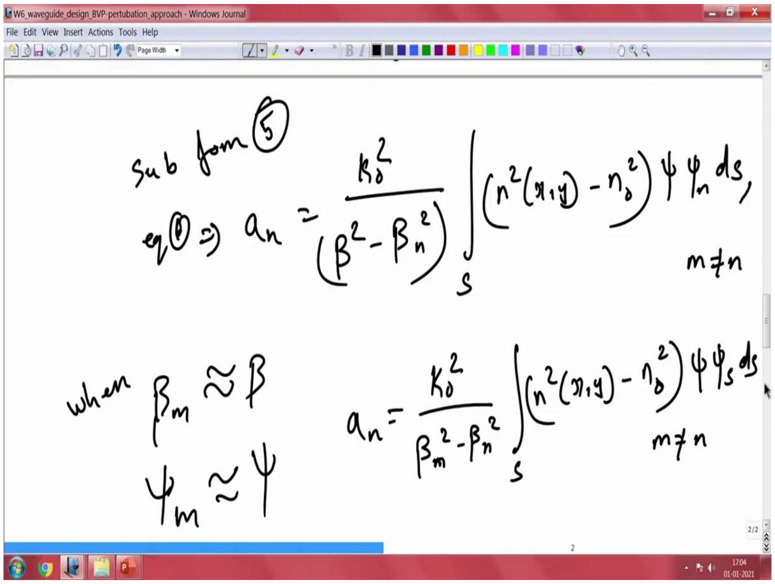
$$= \delta_{mn'} + a_n\delta_{nm'}$$

So, there we are going to multiply, multiply the equation 3 with 1, and integrate over the waveguide surface, the same thing we did here. So, we had subtracted the two and probably I should add here and integrate over the area. So, this is what we are going to do again. So, that means psi and psi ds will be equal to the surface integral, psi m psi n plus, I will n dash here, n not equal to psi n to psi n dash. So, this is going to be interesting, because it depends on the nature of these two modes, the orthogonal nature of these, these nodes, we can use the orthogonality.

You think orthogonality, we could write this as delta m n dash plus a of n delta n dash. So, delta we all know this is nothing but, Kronecker delta function so, this delta is Kronecker delta function. This, for some of you this might ring a bell that we have seen this when we discussed about orthogonality and orthonormality of the modes that we have. So, this will go to unity, this gamma mn dash. This will tend to unity, only if m is equal to n dash. So, this is, we all know this, then it will be ortho normal node.

Otherwise this is all going to be 0. Else, so 0 elsewhere, or 0, instead of elsewhere I would say 0 otherwise. So, this is, this is the condition for this. So now, whatever the integral equation that we had here, that equation 5 and we will make this as equation 6.

(Refer Slide Time: 20:04)



$$a_n = \frac{k_0^2}{\beta^2 - \beta_n^2} \int_s (n^2(x,y) - n_o^2)\psi\psi_n \, ds \; ; m \neq n$$

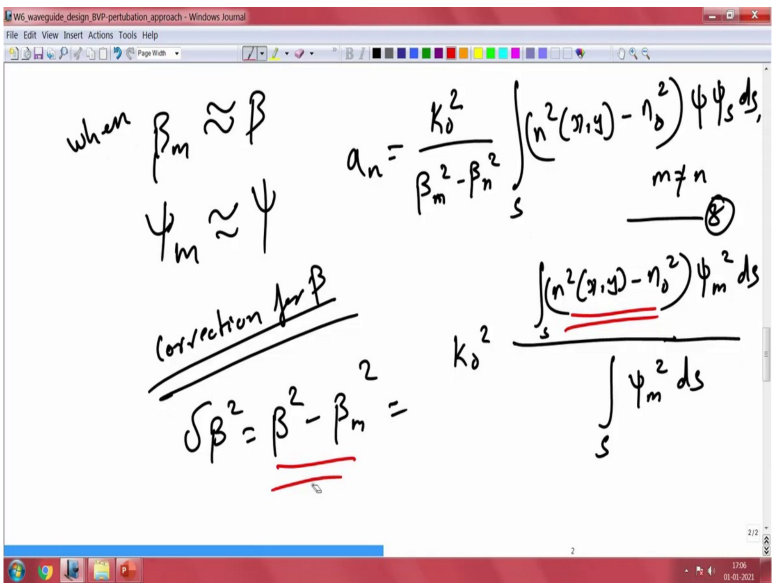
$$a_n = \frac{k_0^2}{\beta_m^2 - \beta_n^2} \int_s (n^2(x,y) - n_o^2)\psi\psi_n \, ds \; ; m \neq n$$

So, now we can look at equation, 5 and then look at this, in we can do some modification, that will help us in. So, substituting from 6 from equation 6 here a n, so that is what we had, let me check a substitution from 5, from 5, equation 6 becomes beta n square n square x comma y minus n naught square psi n ds. So, here m is not equal to n so, that is something that you should keep in mind. So, this is the linking equation.

So, this will be calculated in terms of a n. So, that is to create a superposition that, that resembles the, the actual mode. So, the actual mode is profile is psi. So, you need to recreate that particular mode and that is how we do this. So, now, it is generally sufficient, to retain this zeroeth order solution, where your psi is very, closely resembles one of the modes here. Let us say the psi m or psi m in this case and also beta.

So, beta resembles or beta is very close to be, beta m let us say. So, then we can replace psi n psi m, and beta and beta m here. So, when beta m is very close to beta and again phi m, very close to psi then your a n will be written as k naught square, beta m, let me look at this, beta m squared minus beta, s n squared x comma y minus and n naught square, psi psi s by s ds and here again m is not equal to n. So, this is equation 7 and this is equation 8. And now the correction, so this is just on a n, so, correction for beta now, so, this is the final equation that we are going to write for the beta.

(Refer Slide Time: 23:44)



$$\beta^2 - \beta_n^2 = k_o^2 [\frac{\int_s (n^2(x,y) - n_o^2)\psi_m^2 \, ds}{\int_s \psi_m^2 \, ds}]$$

So, that is basically beta square minus beta m squared. So, that is the difference that we have in other case delta beta square can be written as k naught squared, integral n square x comma y minus n naught squared psi m ds divided by. So, the, this is the general equation that can apply to any kind of waveguide. So, you can apply this to any waveguide, that will, with which you should be able to find the actual propagating beta by applying this approximation. The correction to the beta is, is proportional to the index difference that you see here.

So, that is very, very interesting to notice here. So, your correction that you are applying, that you are looking for is directly proportional to the index difference that you, that you have here. So, the when you have the, index difference to be very small that means you are very close to your, the actual cross section or index that you have physically, then you are, the difference in your beta will also be lower. So, in the corner regions where the index difference from, from the assumed solution.

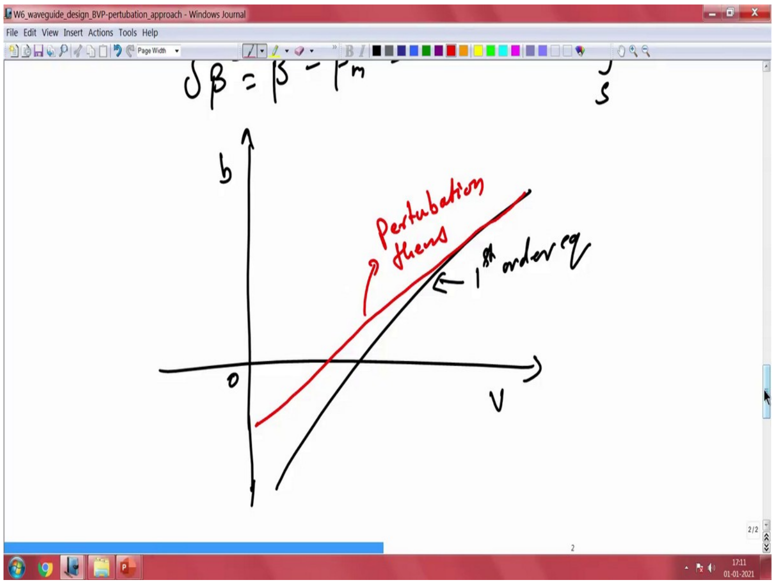
It should also be noted that, to get the true description of the mode field. So, the radiation modes should also be considered here. So, we should consider the radiation modes and the superposition of those radiation modes as well. So, this is something that I briefly mentioned when, when talking about this particular relation. So, when you take this, the trial solution and also add all the superposition of possible solutions, you could also add, the radiation solutions as well here in order to firm up your progress towards the actual solution or the true solution.

And the other thing to notice here is, the, the in, if you look at the final equation the correction for b, it depends on the refractive index, contrast that we have and also your field that you, that you have. So, the overlap integral, of the index, aim of the refractive index is not going to significantly affect your beta, this also depends on your field that you have. So, you can add your multiple modes here that, that I was talking about, that are propagating through. The more the number of modes, the better it is.

But when you are when you are doing, a finite set of modes, it is better to take the guided mode instead of just taking all the modes. So, you can indeed add all the radiation modes that you want, to make the solution much finer, but then at the end of the day, there is only limited resource you will have, in processing this. So, including the radiation modes is not going to significantly affect or influence your beta calculation.

So, it is good to just consider only the propagating modes while, considering everything is good similar to four year expansion that you have including all the coefficients will give you perfect reconstruction, but in a resource limited scenario, particularly in this case, it is not going to affect at all. So, if it is not going to significantly affect your beta, why bother? So, we do not have to take that into account.

(Refer Slide Time: 28:22)



So, now, the question is, have we made our solution any better? So, that is something that we want to look at. So, if you remember initially we had this v number, v parameter with respect to b, we had a solution like this. So, we had negative solution. So, what we call first order equation. So, now, we have made some progress, we have perturbation theory based approach. So, that we just saw here, whether we will have a better solution. Unfortunately, we can make it better, but not, to the level that we want.

So, when it is outside the cutoff, you will have similar solution, when it comes close to the cutoff, it still does this. So, this is coming from perturbation theory. So, one can actually find this, a numerically but for the understanding sake, this, this qualitative approach is good enough. So, what you see here is, still you will not be able to achieve, positive beta, the when you are coming very close to the cutoff. So, this is again a problem, we have not solved this problem completely, entirely, we have not solved. We have made it better, but not solved.

So, that is going to be the topic of discussion for our next lecture. So with this, we have made some progress. Instead of just blindly finding a beta, it is a blind find in the first order, now, we have some kind of approximation, we said, we will not just find beta, but we will disturb the system. We will make some fine changes in our system, that means, I am going to take all the possible fields that is available, and then put together perturbed solution state that I should be able to later on optimize the error and find out reasonably accurate beta.

So, that was our approach, but it turns out, even that approach, you will not be able to get away with this negative beta. So, you, you still stuck with this beta, when you are very close to your cut off. So, when you are far away from the cut off, that means there are many mode traveling, it is all fine. When you are coming very close to the cutoff, so you have to really take a note of this, because when you are close to the cutoff, that means there are not many propagating modes there.

So, if you remember the point that, that I mentioned, that includes all the possible solutions, when you are talking about superposition. So, you have exact solution, you have trial solution, along with a trial solution, add all possible solutions that exists, that may include also radiating mode, to start with, but then later on, we said, we do not have to include all the radiating modes, because the integral is primarily driven by the index contrast. So, you are not going to get much from including the radiating modes, we can leave out the radiating modes and have only the propagating mode.

But when you are coming very close to the cutoff, the number of propagating modes are going to reduce. So, when the number of propagating modes are getting reduced your trial solution and you are number of other solutions that you have, that other solutions is reducing now. So, when the other solutions are reducing from the trial, then getting to your exact solution is also, creating a difficulty here because similar to our four year reconstruction, the more the number of coefficients you have, the better the reconstruction.

In this case, because you are close to the cutoff, you do not have that many solution to help you out, or guided solution to help you out. So, that is the reason why even with this perturbed approach, you are still, better but not, coming to a real solution to start with. So, how are we going to solve this, it looks like we are going in circles, we are trying, we are making some progress, but then we are intensively limited by the approach itself, that there are not many solutions available.

So, how are we going to solve this problem? We take an approach similar to the slab approach, which is called the effective index method, which we will see in the next lecture. Thank you very much for listening.