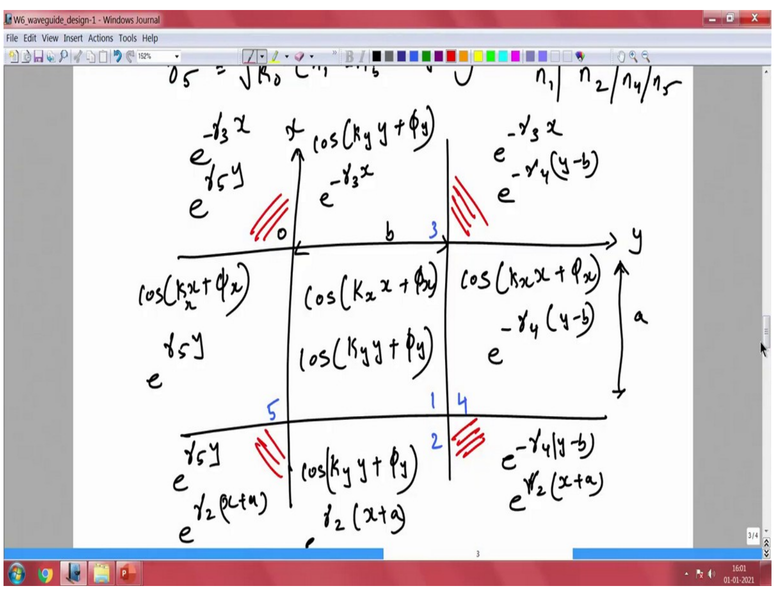
Photonic Integrated Circuit
Professor. Shankar Kumar Selvaraja
Centre for Nano Science and Engineering
Indian Institute of Science, Bengaluru
Lecture No. 20
Waveguide Design-BVP Solution

Hello, welcome back to this discussion on waveguide design. In the last lecture, we just formulated the boundary value problem. So, we have just set the stage with, the different regions that we have in a rectangular waveguide. So, now, how are we going to solve this and find out the necessary parameters, out of this whole solution, including, the type of profile that we are going to have and also importantly, the propagation constant of a particular mode.

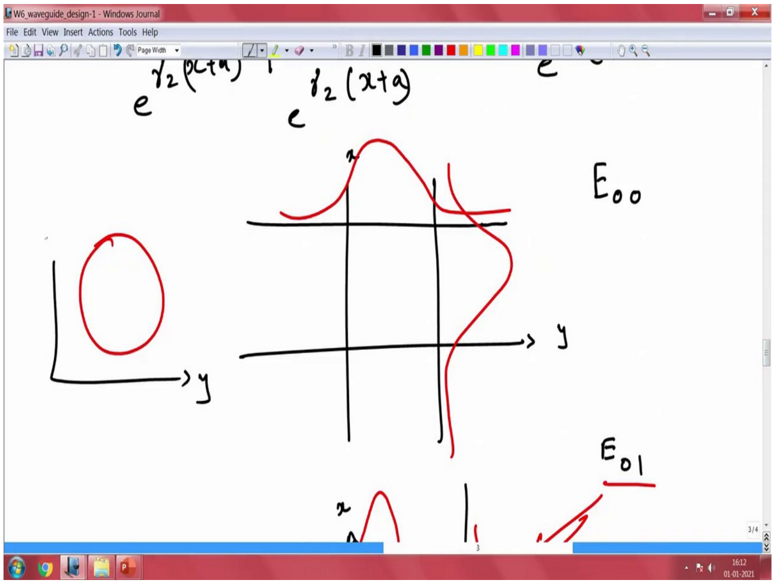
So, let us just start with understanding how this mode profile was. So, we discussed about oscillating and decaying solutions. So, let us look at how we can represent this mode and following that discussion, we will look at the solution to this boundary value problem. So, let us start with how the solution or the field would look like.

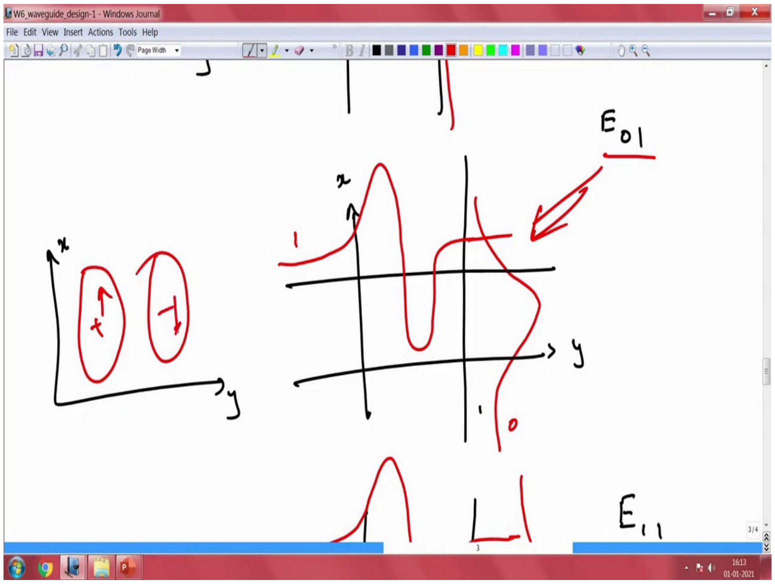
(Refer Slide Time: 01:37)



So, this is this is where we, we left in the last lecture. So, there are different regions that you can, use, we can define with, with both oscillating and decaying solution. So, let us look at how this, this solution is going to look like.

(Refer Slide Time: 01:58)



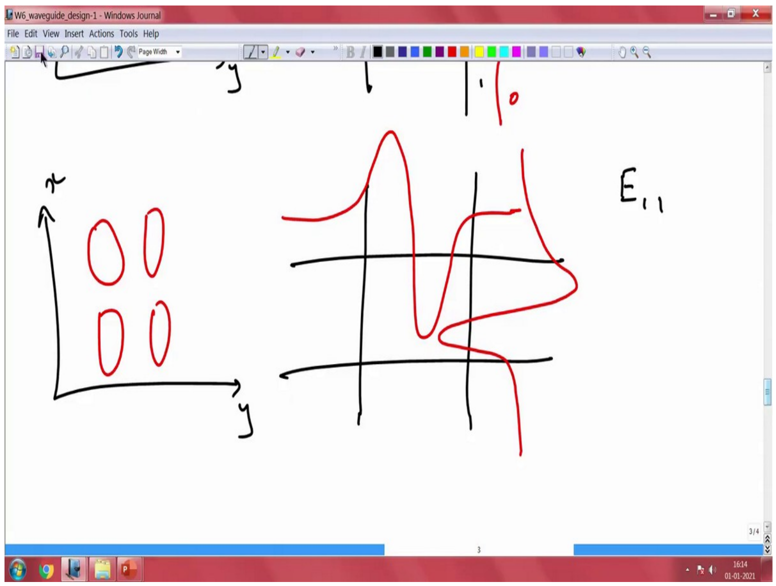


So, for example, you take a simple waveguide system like this. So, now, when we say the electric field, so the electric field has this fundamental mode, that is the first order or first normal mode, that is the zeroth order mode. So, how will the zeroeth order mode looked like, something like this. So, we have seen that in our 10, planar waveguide geometry. So, when there is a electric field 00, so, x and y, along x and y.

So, we know this is a y and this is x, so, the field would look like this. So, you have 0 here, so, that means you will have like this, and along y direction, it will also be like this. So, this is E 00. So, now, let us look at E 12 let us say. So, this is different, let me, let us make it a little easy, let us make it 01. Because 12 is going to be a little crazy drawing it here. So, 01 so, along, again, let us put the directions here, this is, this is y and this is x.

So, along x direction you have 0 nodes. So, that means your field will be like this. So, along y direction you have one crossing, so that means you will have, so this is our E 01. So, this is our E 01 mod, where in y along x direction we have 0 crossing, so that is your 0 and this 1.

(Refer Slide Time: 04:38)



So now, just last bit, to make things a bit more fun. Let us say E11. So, if you have E11, so that means you should have one node crossing for both x and y. So, that means you will have, here and here as well. So, x and y will have both here and here. So, now, let us look at, this is both an x and y direction, but then you have to put this together, in order to have the intensity profile, or the electric field combined E x and E y. How would that look like?

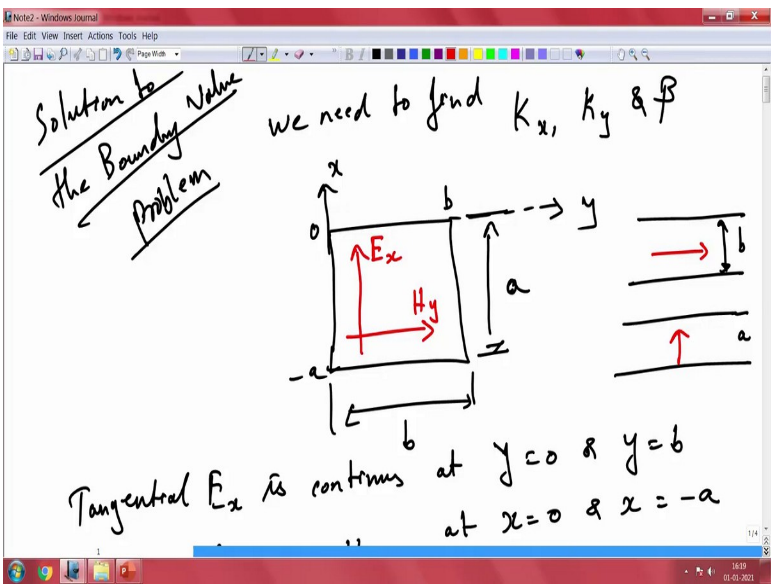
So, let us look at E 00, now. So, when you take E 00, it will just look like one single blob, because it has no crossing, it is just a Gaussian distribution. So, you have Gaussian both in x and y, if you put together it will be a three dimensional Gaussian as you see here. So, this is how it will look like. But then if you look at 01 this is going to be little tricky, along the x direction so, there is no crossing at all, but then along y direction we see a crossing.

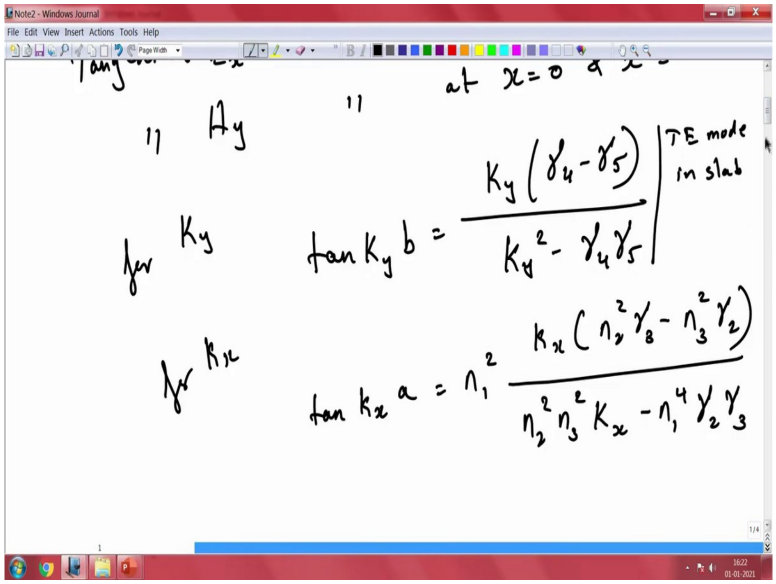
So, that means, let us, let us put the axis here, so this is y and x. So, here y and x. So, here we will have two blobs. So, this is how it will be. So, when you move along x you will not see any crossing. So, there is no crossing. So, when you look at the, along y you will have a positive, negative if you want to call it. So, this is the field up and this is field down. So, positive and negative.

So, let us look at E 11 this is more interesting. So, you have one crossing in x and one crossing in y. So, how can we look at this? So, we know how the single crossing is going to be, in this case you will have 1, 2, 3, 4. So, you will have four crossing, so, when you move across y, you, you see a crossing here, I drew it too far. So, you will have one crossing, when you move from, along x you will also see one crossing.

So, this is how you could characterize your modes. So, how the energy is going to be distributed. So, without understanding, let us move into solving this, we want to find what our propagation constants are going to be. So, let us look at that now.

(Refer Slide Time: 07:57)





$$tank_y b = \frac{k_y(\Upsilon_4 - \Upsilon_5)}{k_y^2 - \Upsilon_4\Upsilon_5}$$

$$tank_x a = n_1^2 \frac{k_x(n_2^2\Upsilon_3 - n_3^2\Upsilon_2)}{n_2^2 n_3^2 k_x - n_1^4\Upsilon_2\Upsilon_3}$$

So, what we need to do, is to, to complete the solution we must first determine. So, this is, the solution to the boundary value problem now. So, what we need? We need, we need to find k x, k y, and beta. So, these are all the things we need. So, we need to use the boundary conditions, connecting different regions. So, it is, that is why it is called boundary value problem. So, let us look at the system that we have in hand right now.

So, we have taken this very simple rectangle, let us say. So, it has a certain width b, and it has a certain height a. Now inside this you have field propagating. So, let us put both E and H field here. So, I can make, this is our H field along y and we have E field along x. So, this is, this is our field. So, now, in this particular waveguide that we have, we need to look at the boundary condition.

So, we must make sure that there is a continuity in the elect, the tangential electric field at y equals to 0. So, you have this is y and this is x. So, we should make sure that we have continuity here. Both at, at y equals 0 and also y equals b. So, we need to have, so let me write those continuity, all you need to make sure is the tangential component of Ex. So, Ex is continuous here and also here. So, that continuity one should maintain.

So, that is, that is one thing and again for Hy field, we should have the magnetic field continuous at x is equal to 0 and x equal to minus, minus a. So, this is minus a and this is 0 and this is b. So, let us put those conditions here. So, so, we should make sure that tangential field Ex is continuous at y equals to 0 and y equals to b. So, we should make sure that this thing happened. Similarly, we should have tangential magnetic component, is continuous at x equals 0 and x equal to minus a. So, this is, this is something that we, know from our basic electro magnetics. So, if you want the energy to be transmitted through these two media, then the tangential component of E and H should be continuous in these boundaries. And the other boundary condition is, automatically taken care and we should also look at the continuity of d and b as well. So, we are just naturally done here.

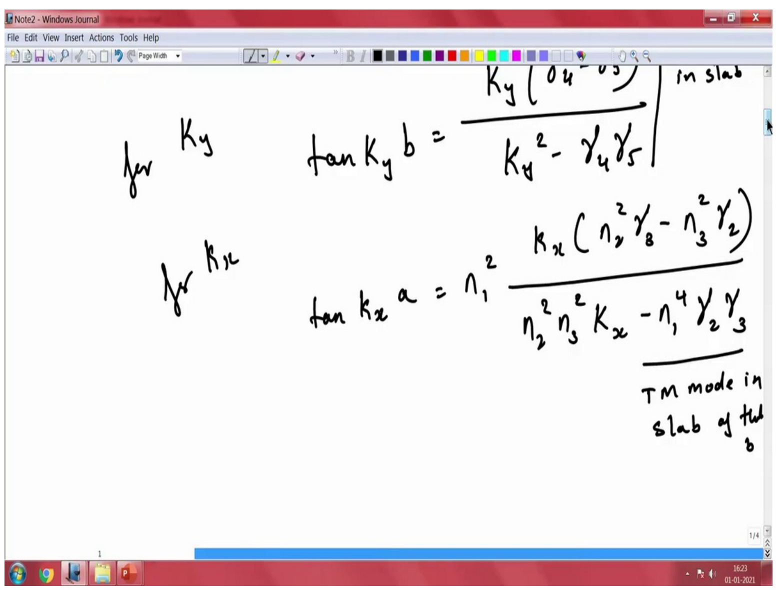
So, if you look at the E field here, so, Ex field looks like a TE more in a slab waveguide. So, there are two things you can look at. So, let, let us just spend some time in looking at this. So, there are two, two ways to look at it. So, one is a thin slab of thickness b or a thin slab of thickness a. So, these are all two possibilities. So, when you take a thin slab of thickness b, then, your electric field would look like a TE mode with thickness b. So, this is looking in this direction. So, you will see your Ex field is like this. So, it will be a TE mode with a slab thickness of b.

But similarly, for the other case, it will look like the thickness is here then what you will have is a tm wave. So, your, your tm mode in a slab of thickness a. So, there is two different ways of looking at it, you can take it and decompose it into x and y and based on their alignment, you can use our simple planar waveguide understanding here. So, one can do that. So, all we are trying to do is trying to understand from our earlier you know understanding of info. So, this is where market ally as proposed like what should be your kx and ky in this case, we are going to directly pick up from what Marcetale has actually proposed and given as a final equation.

So, that is the characteristic equation for ky. So, for ky, he has given something like this ky b which is equal to ky gamma 4 gamma 5 divided by ky squared minus gamma 4 gamma 5. So, this is, this is identical to what we have seen as a characteristic equation of TE more in a slab waveguide. So, this is characteristic equation of that. So, we can use a similar tangential component for h y that involves the interface x and y.

So, you remember this for h y, we should have a continuity here when x is equal to 0 and continuity at x is equal to minus a. So, if you use that and then use this at the boundary for, boundary 2 and boundary 3. So, that is the two other boundaries that we have, then we come up with this characteristic equation for kx. So, for kx, it will be tan kx a, which is given us n1 squared, let me write it a little below and kx a is nothing but n1 squared, kx into squared gamma 3 minus n3 squared gamma 2 divided by n2 squared n3 square kx minus n1 to the power of 4 gamma 2 and gamma. So, this is again characteristic equation for tm mode in a slab waveguide of thickness B. So, I will just mark it here, this is nothing but TE mode in slab.

(Refer Slide Time: 16:23)

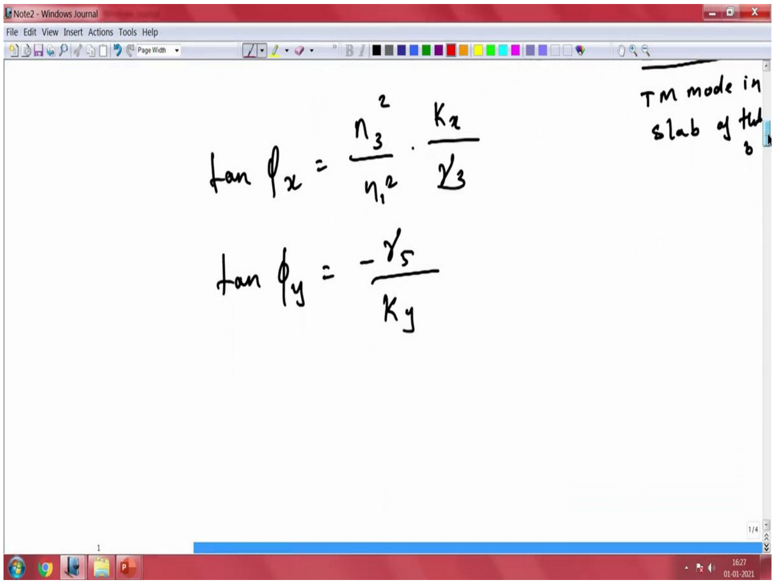


$$tank_yb = \frac{k_y(\Upsilon_4 - \Upsilon_5)}{k_y^2 - \Upsilon_4\Upsilon_5}$$

$$tank_xa = n_1^2\frac{k_x(n_2^2\Upsilon_3 - n_3^2\Upsilon_2)}{n_2^2n_3^2k_x - n_1^4\Upsilon_2\Upsilon_3}$$

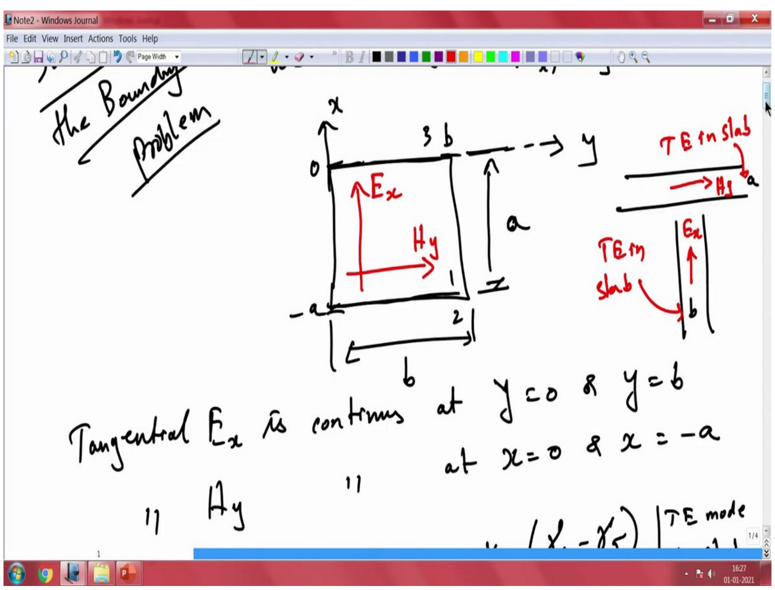
So, this is similar to that and in this case, this is TM mode in slap of thickness b, let us say. So, you can have these two nicely, done for both kx and ky, which would, which would completely describe the mode field. And you could use this for the other regions as well. So, 4 and 5 as well. So, by using that you can come up with this detailed, field distribution on this individual mode, it will be too much to have it here. So, what we are going to do is, we will put it up in, in the note section that you could a have a look at.

(Refer Slide Time: 17:22)



$$tan\varphi_x = \frac{n_3^2 k_x}{n_1^2 \Upsilon_3}$$

$$tan\varphi_y = -\frac{\Upsilon_5}{k_y}$$



So, now, the phase terms. So, now we have kx and ky, but then we also have the phi term here. So, that is, let me go back and then show you where this is, this is important. So, if we look at this equation, so we have this kx, ky so, that is something that we had, we have just found out what is the relation we can do, but then what about phi here.

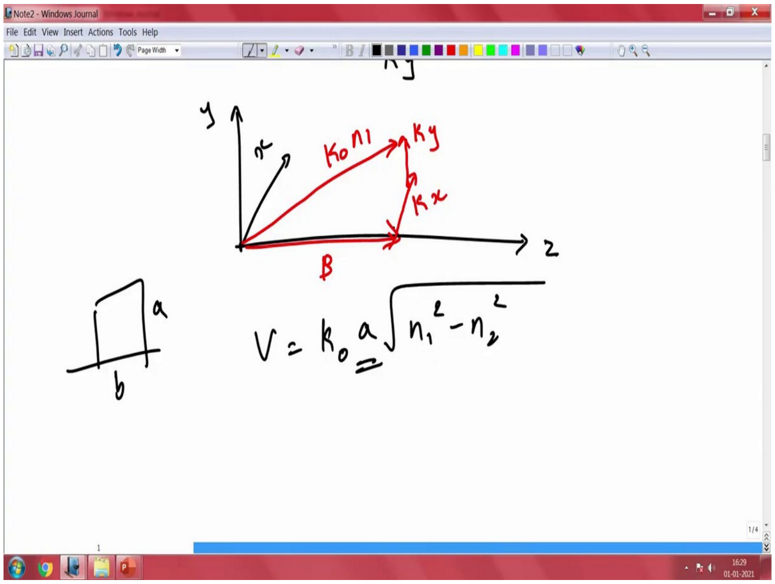
So, we have phi x and phi y, so that is the phase component inside this each boundary. So, that is given by, the following equation. So, tan phi x is nothing but n, at least, in this case n3 squared by n1 squared times k x y comma 3. So, this is for region 3 and 5. So, next thing is for phi and phi y is nothing but minus gamma phi divided by ky. So, you know, we can see that the rectangular modes, is a simple product of two orthogonal spatial modes. So, that is what, we have seen so far.

So, the x dependence of the mode is found by solving it as a slab, here. So, the x dependent is solved, is solved by considering it a slab. If there are no structures, there are no structures in the y direction similarly, y dependency of mode can be, treated by considering the waveguide as a slab with infinite, x direction. So, we are just converting this two dimensional structure, confinement both in x y as to slab thing. So, we had a slab like this and we had a slab like that.

So, this, this was a and this was b. So, we had this Ex in this direction, telling you that it is like, TE mod with thickness a. And in this case, we had the other way around. So, this is hy, and here it is Ex with a, a thickness of b. So, both are TE mode, TE, TE in, in slab. So, here again TE in slab. The only thing is your thickness is b here, here the thickness is a. So, you just oriented it in this two way.

So, we do not have to really, push too much here. So, we can still take this and, and continue solving. So, similar to what we have done for this, the slab waveguides, so the two solutions are coupled now, with the propagation constant beta now, that is what we see, where your kx and ky are nothing but the transverse component. So, let me quickly draw that particular vector diagram here, the three propagating vectors that we all know of.

(Refer Slide Time: 21:12)



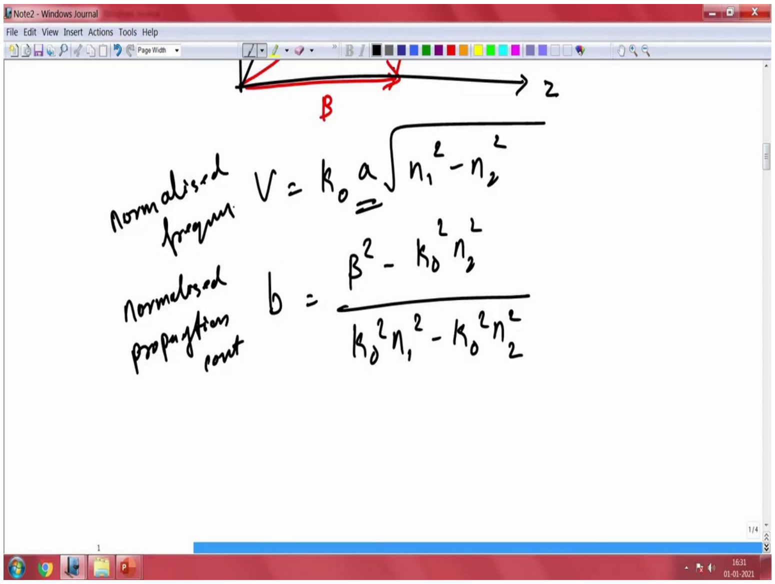
$$V = k_0 a \sqrt{n_1^2 - n_2^2}$$

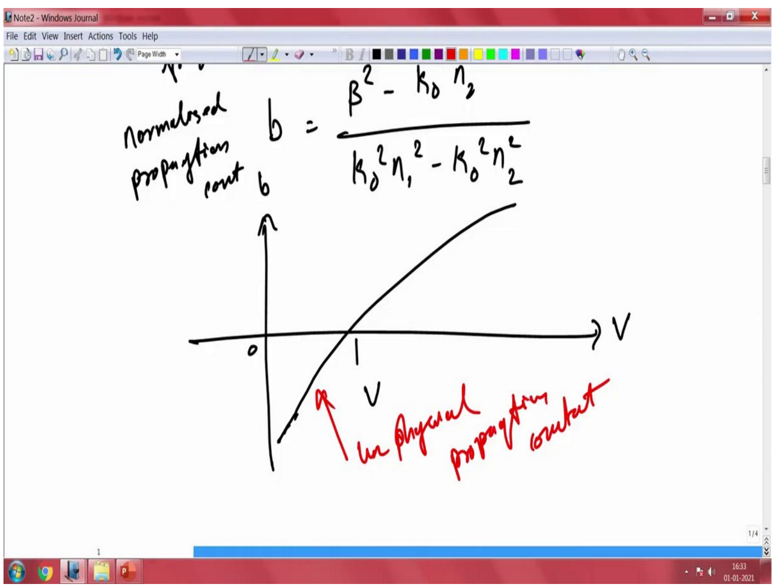
So, this is your propagation direction z let us say, and let us say this is x and say this is y. So, when the wave is propagating, you will have a beta here. So, this is where it is propagating through, we will have an x component. So, we will have an x component here that is kx, over getting through and it will also have ay component. So, you will have y component parallel to y and that is where we have our propagation vector k0 n1. So, this is in a three dimensional space that one could consider. So, this you could use to visualize how things are so, we need beta and then kx and ky should be, we should be able to get, get this thing done.

So, similar to what we have done for the slab waveguide and, and other fibers and other things, one can use the normalized parameter here the propagation constant b to the normalized index b to your normalized frequency v. So, v is our normalized frequency, that we know which we already know, let us say n1 squared minus n2 square so, this is something that we, we already know. So, for smaller dimension, where a is reasonably small, n1 here is this core refractive index and n2 is your cladding refractive index let us say.

So, you could plot v versus b. So, if we plot v versus b, we should be able to find the propagation constant of this. But it is not easy to plot this for, all kinds of geometry. You need to start from somewhere because it has a certain width and it has a certain height. In this case we only have a to it, but in our case we have a certain width and we have a certain breadth in this case, depth and width is what we have. In this case, what we need to do is we have to do some sort of approximation. So, the mode that we consider here, we need to look at how these modes evolve, as if, as an approximation and try to arrive at the exact solution.

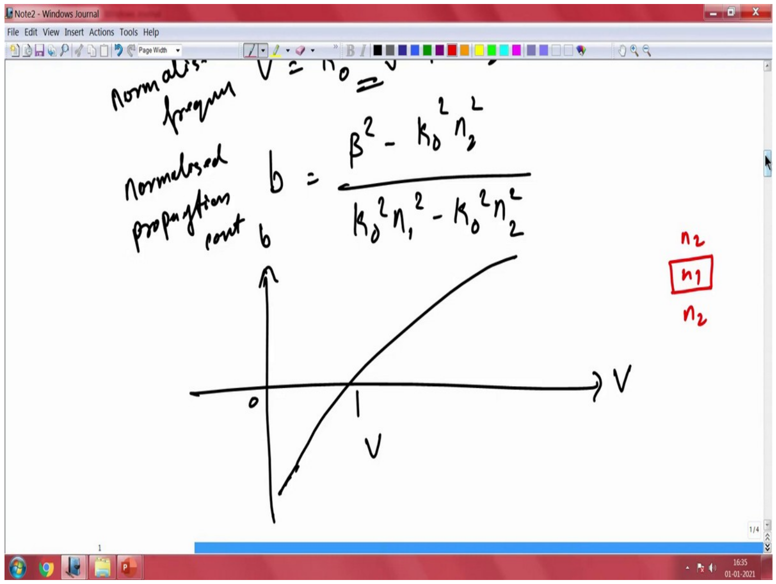
(Refer Slide Time: 24:03)





$$V = k_0 a\sqrt{n_1^2 - n_2^2}$$

$$b = \frac{\beta^2 - k_0^2 n_2^2}{k_0^2 n_1^2 - k_0^2 n_2^2}$$



For example, if we are going to plot this b versus v, and again I will write here b to remind you all this is something we already saw, but again in, you may not remember right away. So, I will write it here. So, this is our v versus, v and b. So, one could write b with a function of b. So, b is nothing but norm, normalized propagation. Normalized propagation constant, and this is, this is normalized frequency here. So, now, if we plot this, normalized frequency, let us say this is your v parameter and this is your b, you will find something interesting.

So, what you are going to find is that your, your curve your propagation, your normalized propagation will have negative values. So, you will, you will see some negative values. So, your b you can, can directly calculate beta from here. So, it, this is not possible, in reality. So, the values that you see here that are negative, it is just not physical to have b negative, when you are considering this structure. You may remember that we have, we discussed about, direction of b whether it is positive or negative, it will tell you whether it is forward propagating or backward propagating.

So, backward propagating we put negative and forward propagating we put positive, but this is whatever we see here it is not physical at all. So, you cannot have sudden change in the direction as you change just the, the refractive index let us say, or the dimension, suddenly by changing the dimension you will not make the, make the wave move forward or backwards. So, that the direction of energy flow will not change, but this particularly happens for case, for a certain v.

So, when for example, when the v number is less than a certain value, you will not, have a feasible solution, you are, you see unphysical or non physical propagation constant. So, this is, this is not, not good for us, because below a certain v this particular technique is not giving us the right kind of solution. So, what to do? Because of this uncertainty, we need to think about ways of improving this approach. So, you want everything to be positive. So, that is what we want and in order to do that particularly, when you have a near cutoff conditions.

So, it becomes really uncertain. So, that is not feasible and the other thing here is for any symmetric waveguide, so for a symmetric waveguide, where we have n1 n2, in this case, you have everything covered similar, refractive index, I should actually do this. So, it is completely filled with n2 and it is symmetric in nature. And if it is symmetric in nature, you, you expect the waveguide to have, at least one guiding mode, same as what we saw in the 10 form.

So, there should be at least one guiding mode, but unfortunately, this negative number, the propagation constant is not giving us the confidence that whether the solution that we have here, is, is feasible at all or not. So, this is where we look at, beyond a simple analytical approach that we had so far. So, we need to look at some perturbation approach let us say to correct for this, beta that is being negative here.

So, so far, we looked at how to approach this, this boundary value problem to achieve or to determine, kx ky and beta. But unfortunately, when you look at the beta, we are getting negative values of beta, which is not physical at all, for a given structure. So, we need to find a way to solve this problem, but this problem only exists when you are talking, when you are considering structures that are very close to the cutoff here. So, when the waveguides are pretty large and the refractive index is reasonably low contrast between n1 and n2, then this problem will not erase.

But then when you are having dimensions that are smaller, when the dimensions are reduced, and also when the refractive index is high, refractive index contrast is high to be precise, then we, we find this problem of, non physical beta values. And this is one of the reasons why solving high refractive index waveguides are not easy to be done with, with analytical methods, you need to do with numerical techniques. So, a simple waveguide will be, will be very hard or it is impossible to do it because of the negative beta that we see.

However, one can use, some polished approach. So, one that is already being explored and used is called perturb, perturbed approach or perturbation approach. So, where instead of just trying to find this, this beta that you want, you can start with a certain beta. So, you say, you start with some best guess beta. You start with that best guess beta and then try to approach to the, the exact solution. And that is done along with the field overlap and, and field expansion as well. So, that is something we will see in the next lecture. Thank you very much for your listening.