Photonic Integrated Circuit
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Lecture No. 02
Wave Optics Review

Hello, welcome to this lecture series on a review of electromagnetic waves. So, electromagnetic waves are class of you know waves that defines a broad spectrum of waves that we know it. It covers from RF to optical and then beyond. So, there is a there is a very strong foundation that one use in order to build understanding of light using this electromagnetic waves.

So, electromagnetic wave theory evolve from understanding light as a ray and later on as a scalar wave and then evolved into electromagnetic wave theory and then later on more advanced topic like photon optics or quantum optics. So, this is a evolution. So, electromagnetic wave theory is basically a limit of scalar wave theory and scalar wave theory is a limit of Ray theory. So, this is all backward compatible.

So, you can explain few of the light phenomenon's using ray optics and then there are few things that you cannot explain with ray optics for example interference and that is where we started to look at light as waves. So, thanks to Huygens who came up with this postulate that light is a wave. So, once you define light as a wave, then you we were able to explain interference between two light waves and colours and so on. But then later on the theory got even more interesting when Maxwell proposed that it is not a very simple scalar wave, light consists of electric and magnetic fields.

So, these are two different energies together, which was never thought before. So, this was coming from understanding of RF waves in free space. So, this is a merger of people who are studying electricity and magnetism and people who are trying to understand light. So, when these concepts were brought together, there was a revelation that you could explain all the phenomena of light and properties as well using this comprehensive theory of electromagnetics.

So, in simple terms electromagnetic theory actually helps you to deconstruct a wave and define it as a electric field and magnetic field and how they interact in a medium and what will happen to this electric and magnetic fields as you propagate through isotropic or anisotropic medium and also it relates external influence on to the electromagnetic wave in this case light waves. So, all these theories could be understood using very simple formulation proposed by Maxwell. So, before going into the electromagnetic theory, let us first look at what a simple scalar wave theory is so that we can build some understanding already.

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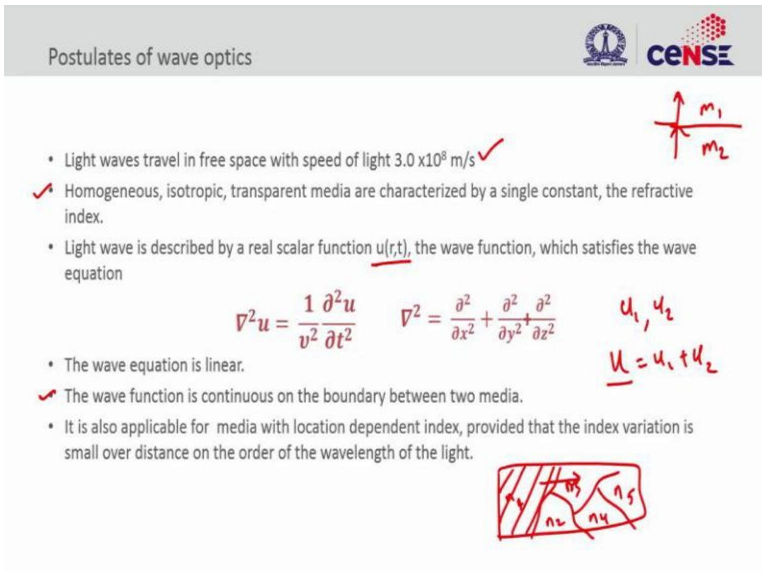

So, let us first look at the wave theory, the postulates of wave theory which applies to electromagnetic theory as well. So, first thing that we postulate is light waves travel in free space with finite speed. So, in this case, it was measure that it travels at speed very close to $3*10^8$ meters per second. So, this is the speed at which light travels in free space. So, free space in this case could be air and vacuum.

And then homogeneous isotropic and transparent medium, or characterized by a single constant in this case a single optical constant which is called refractive index. So, any material that is homogeneous are also isotropic in nature. So, homogeneous and isotropic I think there is a clear distinction between these two. So, isotropic meaning uniformity when it comes to direction as in whichever direction you move in space you do not see any difference. So, that is isotropic. Transparent medium that means the light should propagate through the medium.

So, either wave should be able to propagate through the medium without loss let us say and if that is the case then you know, you could define this material using a constant a single constant and that is called refractive index. So, the light wave that is traveling could be, you know represented by a scalar function. We are still in wave optics or scalar wave optics so it could be represented by a function u(r,t). So, r represents the position dependence and t represents the time dependence and which is called the wave function and this wave function should satisfy the wave equation.

So, the wave equation here is $\nabla^2 u = \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2}$. So, this is basically your wave equation. So, if u is a solution, then it should satisfy this condition or this wave equation and this wave equation is linear. So, if it is linear, then you have all bunch of conditions. So, the condition for superposition also applies here. So, if u1 and u2 are two waves. So, let us say u1 and u2 are two waves, then $u = u_1 + u_2$, also a solution. So, you can apply all the linear dependencies on this wave equation.

So, the wave function is continuous on the boundary between the two medium. So, if you have two media, medium n m_1 and m_2, so the wave function should be continuous between these two alright. So, this is what scalar wave optic postulates. So, there is no discontinuity there is a discontinuity in the media, but then the solution the wave function should be continuous.

So, you can earlier we saw homogeneous, isotropic and transparent medium. But then what happens if the medium is not homogeneous. So, that means you can have local index differences. So, if you take a material where the density of the material varies whether we can apply this certainly we can do it so we can take use of the boundary condition that we have here and this condition.

So, you could take a material where you might have slightly different refractive indexes n_1, n_2 n_3, n_4 and n_5. So, this is a really a worst-case scenario where you have different refractive index (()) (08:30) perhaps very small change. So, the refractive index actually changes over distance, very small difference so that the ∇n is very very small, very close to 0 let us say it is not 0, but very close to 0. So, they are very very small difference very small difference over over distance that you travel.

And if that is the case, then you can represent this material with changing refractive index that is what we say within this region it is homogeneous but then you go from one medium to the other medium we know that the wave function is continuous. So, that means energy should be able to travel from region 1 here to region 3 or region 2 in whatever direction they propagate here. So, even you do not have a homogeneous medium, you can still use the wave theory here. So, that is what the postulate tells us here.

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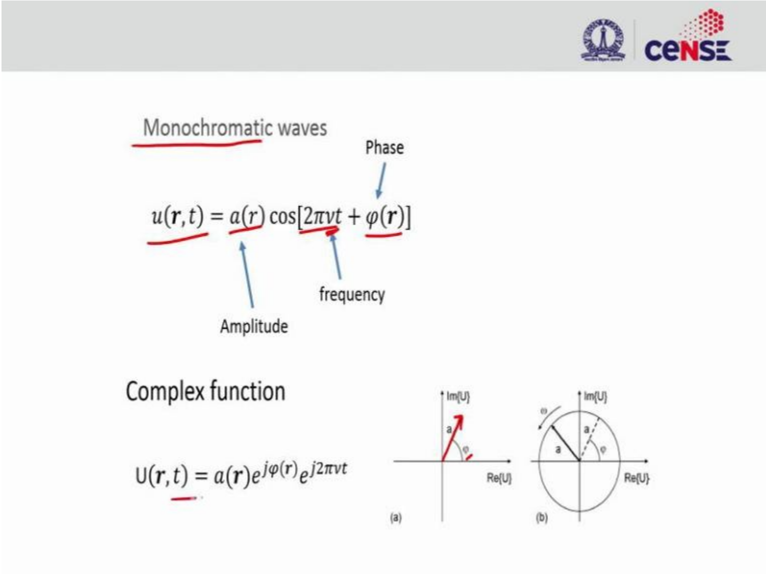

So, let us look at how one can represent this waves is a very simple form of representation. So, u(r,t) could be represented as a complex amplitude with an oscillating term. So, here $u(r) = a(r)\cos[2\pi \nu t + \varphi(r)]$. So, this is the frequency factor, so how the wave is oscillating and this is the phase at what how quickly the phase is changing and in the complex representation one can look at this as this is the complex amplitude that we have here and it has having a certain amplitude and phase φ here.

So, when it rotates with a certain angular frequency, $2\pi\nu$ here, you will get a wave that is coming out of this system. So, here it is represented as r you so that means it can be as a function of any orientation the wave could in x, y in any plane that you want. But what essentially this is simple wave tells us it is oscillating at a certain frequency, only at as a single frequency that is the reason why we call this as a monochromatic. So, it has only single frequency and this is how we represent it in a complex form.

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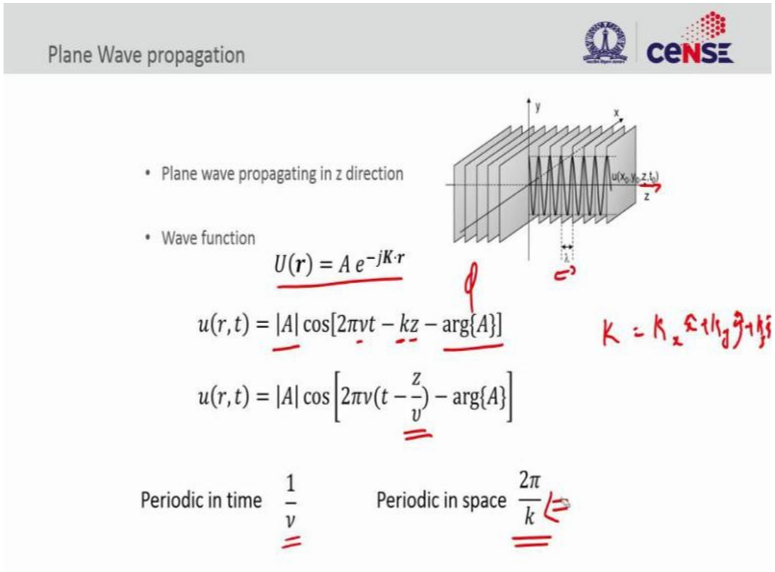

And the another very simple way of understanding this wave is by using a primitive wave that is a plane wave. So, in this case plane waves are nothing but the phase front of the waves are large planes as you can see here. So, this is something that you might have encountered earlier. So, the way that we can represent a very simple plane wave is through this complex amplitude A with your phase factor e^{-jkr} so where K is your wave vector and this K could be represented as $k_x \hat{x} + k_y \hat{y} + k_z \hat{z}$ so, this is the unit vector that you have plus Kz times z. So, this is all the unit vectors you have.

So, you can represent it as earlier so there is a complex amplitude here $\cos 2\pi\nu t$ that is the frequency term and this is your wave vector k along moving along the z direction. So, as you can see here the wave is moving along z where you are oscillation as you can see here and argument A is nothing but your phase, so what is the phase factor that you have? So, now looking at this you can rearrange this if you would like and when you do that you see an interesting term here. So, this is how the wave actually beats with respect to time. So, that is the frequency of oscillation.

And this is the time pre-period of the wave and then in space it travels with $\frac{2\pi}{k}$ as the period so that is what lambda is. So, this is basically the wavelength of light in free space, so, $\frac{2\pi}{k}$, so k is your wave vector. So, one can use a very simple plane wave to understand how these waves would propagate and how to understand whether this wave is propagating in a certain direction or not. So, this tells us that the wave is propagating along z direction.

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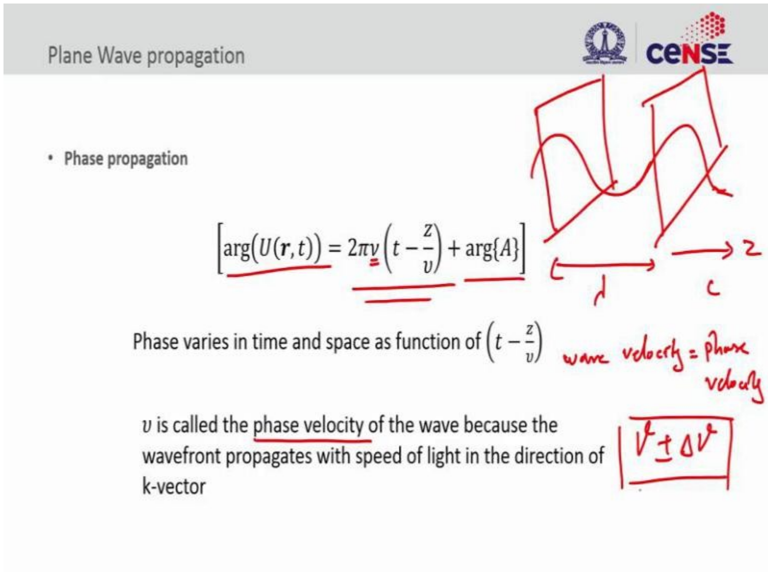

So, we talked about the amplitude, but what about the phase? So, that is the argument U(r,t). So, phase is a very important property of light and here in this case the phase is represented as a function of time and this is argument A. So, this is how we represent the phase and it is, it varies as a function of time and space if we phase is not constant, it is varying as the wave moves along any direction and also with respect to time because the wave is oscillating you will also see a change in the phase. So, this change in the phase as a function of distance A what you call phase velocity. So, as you move along its changes the phase, so this is called the phase velocity.

So, if we just looked at this wave that is moving along this and then there was a plane, so there was a plane defined at each. Let us, say this is the for the plane wave and we represented this as λ. So, as you can see the wave is moving as a plane wave and it is moving with a certain speed so we know this speed of light. It is moving at speed of light C, but then you also see here the face is also changing as a function of distance as you move along the phase is also changing. But at what speed that to phase changes?

So, that is actually same as the speed of light in this particular case. Because you are talking about a single frequency. So, for a single frequency or a monochromatic wave your speed of light equals to the phase velocity or in this case the speed at which the wave moves equals to phase velocity. So, the wave velocity equals to phase velocity. So, this is what happens when you have a monochromatic wave. However, later on we will see that if the wave is not monochromatic, for example, if you have $\nu \pm \nabla \nu$which is the case in reality, this is not going to travel with a single velocity.

So, because each frequency got its own a phase change because it is a function of frequency here. So, that is the reason why you will not be able to represent a group of wave or a pulse that is propagating using a single wave velocity that is where you have group velocity. So, this moves as a group of waves. We will see that later, but for this understanding of a monochromatic wave, the wave velocity would be equal to a phase velocity.

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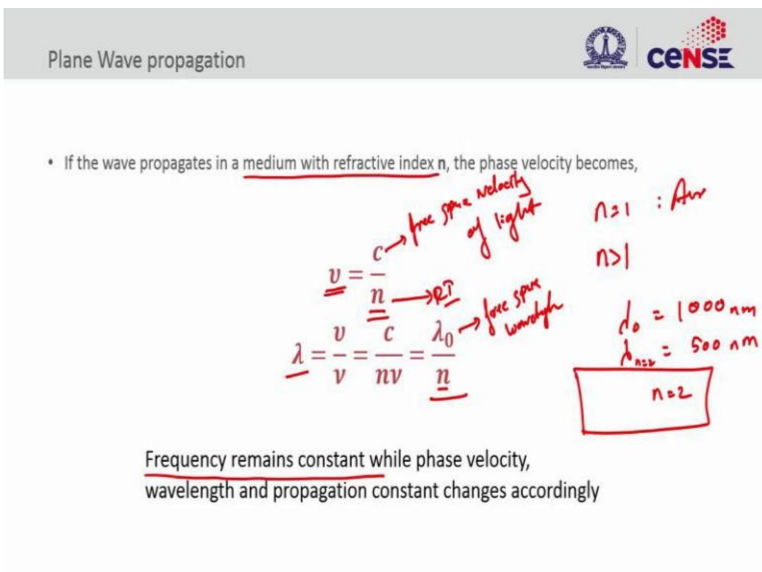

So, how fast they move? How fast these waves move in a medium? So, the medium is now characterized by this refractive index n. So, the refractive index has a role to play in the speed both in the wave velocity and phase velocity. So, now since we are discussing monochromatic wave, there is no difference between phase velocity and wave velocity. So, let us look at the phase velocity how it is profile. So, now the phase velocity is now reduced let us say depends on the refractive index of the medium.

So, C is the free space velocity of light. So, now n is the refractive index. So, when the refractive index of the medium is greater than one which is the case for any dense medium because n equals to 1 for air, but then for any medium any solid medium n is going to be greater than 1. If that is the case then you are phase velocity will be always less than the free space velocity. So, that is the reason why light moves slower in any denser medium. So, if you have higher refractive index medium, then proportionately the velocity are going to be lower.

So, what happens to the wavelength of light, will it change? Of course it will change because once a speed goes down your refractive index will also reduce the wavelength of light. So, λ_o is your free space wavelength. So, now lambda which is the wavelength in the medium is reduced by the refractive index that you have. So, just to give you an example for a free space wavelength of 1000 nanometres let us say so this is in free space, this is an air. So, now what happens if it is propagating through a medium, so it is now going through a medium that has a refractive index of 2 let us say.

So, now the refractive index in that medium so your wave length will be only 500 nanometres. So, you remember the refractive index or the other wavelengths of light is material dependent. So, you cannot use the free space wave length when you are talking about light propagation in a medium. However, the frequency remains constant. So, the frequency of light will remain constant because frequency of the light is time dependent not space dependent and that is the reason when you put light into any medium, the wave length will change but the frequency will not change.

And that is the reason why your colour will not change for some of you who are wondering when I change from one medium to the other medium should I see a colour change. So, obviously no, you will not see a colour change. You will only see a change in the wavelength. For some reason we associate colour with wavelength that is for all for practical purpose, but physics tells us the colour of the photon depends on the frequency not on the wavelength. So, wavelength we use it for our own understanding and for practical purposes not necessarily for actually related to the colour of the photons.

So, the frequency remains constant while phase velocity wavelength and propagation constant write the K that we saw 2π or K is the wavelength. So, if the wavelength changes, the propagation constant K will also change. So, the only thing that remains constant is your frequency when you go from one medium to the other or you know traveling in any medium for that matter alright. So, this is a plane wave propagation in uniform medium let us say and that brings us to end of how to understand a very simple scalar wave propagation.

So, there are a couple of things that we understood in this section. One is any medium that is transparent, isotropic, homogeneous in nature we can represent that material using an optical constant and that optical constant is a refractive index and this refractive index will change the nature of or some of the properties that we derive from light. For example, the wavelength of light will change the lambda will change as a function of refractive index and what is the next thing that will change the propagation constant. So, the propagation constant will change because you are wavelength is changing.

And the next thing it is going to change is the speed of light itself. So, in a denser medium light will feel harder to move so it will slow down, compared to light that is traveling in free space. You can consider this as a kind of resistance. So, when you have a denser medium light is to travel slower and when we have a rarer medium less dense medium or in free space it will move relatively faster.

So, that is all fine and then we also found that there you can represent a monochromatic wave using a complex amplitude and a phase and you cannot randomly choose any wave function for this. So, when you choose that solution when you choose a wave equation, it would satisfy the wave equation that we just learned. So, that is $\nabla^2 u = \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2}$. So, it should satisfy this wave equation. So, only those solutions are the waves that could propagate through the medium.

So, we saw plane wave as an example, which is a simple representation of waves and there are limitations whether this is really existence because plane waves are basically mathematically they are good and it helps us to understand basic optical functions but they are not real they are they do not exist in in nature let us say. So, there is always a section of wave that we can consider as plain way, but a plane wave does not exist. The reason for that is it is a plane, so that means the amplitude of this wave exist everywhere. So, that is what plane means.

So, there is no boundary so plane extends to infinite in both the direction and if that is the case, then the amplitude should be infinite which is impossible. So, that means it should carry infinite energy which is simply impossible. However, if you put a boundary condition, then it makes sense. But then the plane wave definition does not allow as per that (())(25:04) it is always an infinite plane. So, that is the reason why we use plain waves to understand phenomena's has like reflection, refraction, interference very well because it is the phase front is uniform there is no change in the phase front.

However, in reality it is basically an approximation does not exist. So, I think with this basic understanding we know a little bit about waves now and what are all interesting things that one can understand use this to understand waves in medium. So, with that this particular wave scalar wave optics is done. In the following lecture we will look at electromagnetic representation of these waves. Thank you.