Photonic Integrated Circuit
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Lecture No. 19
Waveguide Design-Boundary Value Formulation

Hello everyone. In this lecture let us look at how to design photonic integrated waveguide. So, we have looked at waveguide properties in various aspects and various configurations. The fundamental or it is a very basic waveguide structure is a slab waveguide. So, we looked at a sandwich of layers. So, we have a core waveguide and then surrounded by a lower refractive index medium which is called cladding.

So, we looked at an infinitely thin waveguide. So, that means the light is confined vertically by index contrast and laterally it extends. So, we have a pretty good understanding of that but in practice we do not use this kind of slab waveguide. We use a rectangular type of waveguide. So, that means you have both x and y. So, you have boundary both in x and y. So, the confinement in the vertical direction is by the index contrast we have, and also in the transverse direction you also have index confinement.

So, in this case how do we understand light propagation? So, so far we had the z axis along the z axis how the light propagates, but now we are going to turn it and then say is the z axis it inside or outside the screen but then you have x y region to try to understand how the light or the field is going to be there. So, we already looked at how the field profile would look like and we also understood the propagation constant associated with the transverse and the longitudinal direction, but now everything is going to change.

So, now we are going to look into both x and y aspects. So, we made our problem reasonably simple in the earlier lectures by having y axis infinite, so there is no boundary. So, now we are going to put a boundary to x and y so when you put a boundary how are we going to understand this and more importantly, this is the fundamental structure that we use in integrated optics.

In all the circuits we will have some sort of rectangular structure, not just in photonic ICs, but also semiconductor lasers, for example. So, the light confinement in semiconductor lasers is through this two-dimensional confinement. So, you have vertical x and horizontal y, you know the other way around as well, but you have confinement both in x and y directions so that means there is a boundary there.

So, it is important for us to understand how one can define the conditions, the propagation conditions, and also the field profile. So, how my electric field is going to look along x and how my electric field is going to look along y. So, if you remember in the guided-wave discussion we took doh e or doh h over doh y to be 0. Because there is no boundary at all so y axis is nothing to say but in this case that is not going to be. So, here there is a boundary there so the tangential component of e field and h field there should be a continuity boundary condition now.

So, those are all the new addition that you have to consider when you are talking about two-dimensional structure now. So, this was earlier considered to be a very hard problem to solve, it is still not trivial to solve for this field in a rectangular geometry. It is not trivial at all if you want to do it in analytical sense, but there are some approximations you can do. You can take this two-dimensional case and make it one-dimensional so you can flatten along x or flatten along y in order to make this problem reasonably easy to attempt.

So, the pioneering work was done by Mercantile so he come came up with ways of looking at this two-dimensional structure and how one can define the boundary conditions because the problem here is the boundary condition. When you have two interfaces, you do not have any problem, it is nicely defined, but then the moment you put another interface it becomes tricky. So, you might be wondering what is tricky about it. We will see that in a minute. Let us look at integrated optical waveguide design now. Let us jump into it.

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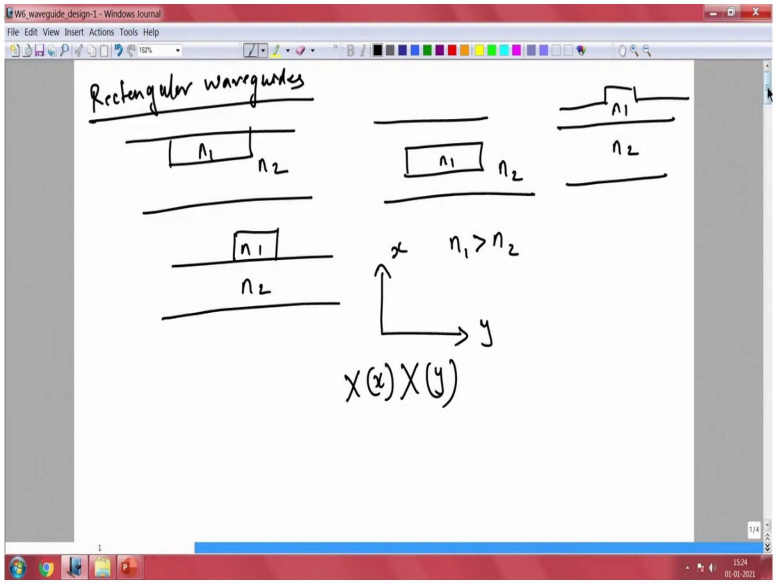

So, any rectangular waveguide, so we saw in the introductory classes the different type of waveguides so we could have a very simple, let us say something like a buried waveguide like this so there is n1 and there is n2. This is one type of structure and the other type of structure is completely buried where this is n1, this is n2 and you could have another type of structure where you could have n1 and n2. There is one more where you could have something like this, so n1 in all cases is greater than n2. So, these are all different types of rectangular waveguides. These are all different types of rectangular waveguides.

So, what is very difficult about these structures? So, I mentioned something about the boundary condition. So, boundary condition for these waveguides are not that clear, it is coupled, why is that? That is what we want to understand here. So, before that everything is now in two-dimension you see here so we have y and we have x. Because we have a boundary in y now the field would also be similar. So, you will have a field along x and a field along y.

So, this how our field is going to be so that means that you need to find a solution along x direction and solution along y direction. So, you need to have this in mind. And now let us look at what is tricky about this particular structure. Let us look at an embedded structure which is much easier to look at.

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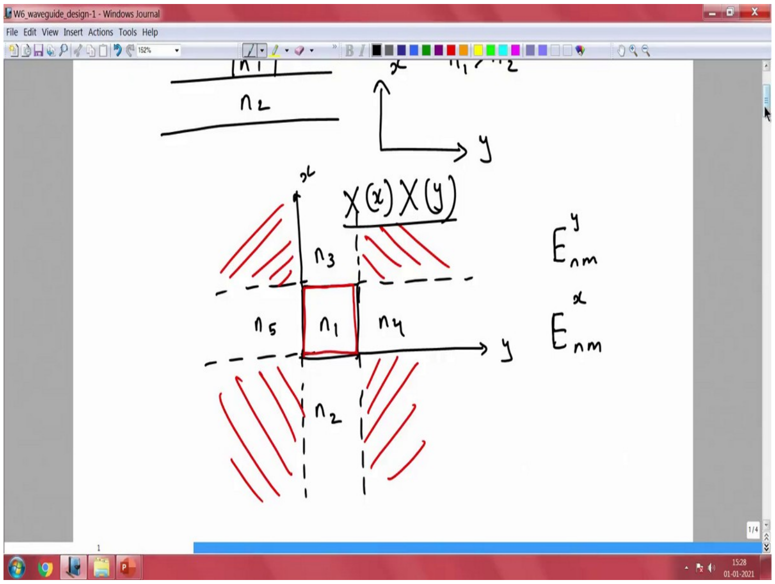

So, I can take a waveguide like this as a certain refractive index and now if I want to dissociate into different components so let me say this is y axis and this is x axis. So, if I look at this then what I see here is I can have region n1, I could have multiple possibilities here, and I could have n2 here and that could be n3, it could be n4 and n5. So, these are all the different regions or different boundaries that we should consider when solving for a waveguide here.

So, if you consider this as my rectangular waveguide but then if you want to put the boundary condition, now we have multiple domains here along the x and along y. So, the problem here is between, if you look at the region n1, n2 and n3, these are all reasonably easy to define the boundary condition. It is straightforward because they resemble the thin planar waveguide geometry similarly n5 and n4 as well so, their boundary conditions are nicely defined but how about the corners?

These corner regions, this region, this region, this region and this region. So, here things become very difficult to define. Why is that, because there in all these four regions, x and y are coupled now. So, you cannot write the boundary condition just for x or y. So, x and y are dependent so the solution will also be x and y dependent. So, this is where the problem gets really interesting and the other important thing to notice here is we have x and y. Because you have x and y, the way we designate any field would also be a little different.

For example, if you have TE mode, so you have an electric field E so it is along y and it will have the mode number m and n, for example. So, this is one mode where it is confined along y direction. This is TE mode, so you have electric field along this y direction and you could also have x along m and n. In this case it is parallel to x axis. So, these are all different notations that we use to designate various fields that you have in this particular rectangular geometry.

So, this is a little different from what we saw in the slab waveguide. So, in order to solve for this you need to find, formulate a boundary value problem. So, this is something that is reasonably straightforward to do, the formulation, at least here so we look at this waveguide and say, so, if you want to have a solution to this, then probably that particular wave should propagate through this with a certain eigenvalue that is beta for that particular solution and you want this solution to be much above the cutoff.

So, we saw the cutoff condition in our slab waveguides. So, you want this waveguide to propagate this mode to propagate through this waveguide so it should be greater than the cutoff. So, let us look at, right now let us ignore the corners. So, we will ignore these corners and we will only consider the uncoupled ones. So, there is no coupling between the x, y, and z components let us say and if that is the case then we can write the wave equation as a scalar form

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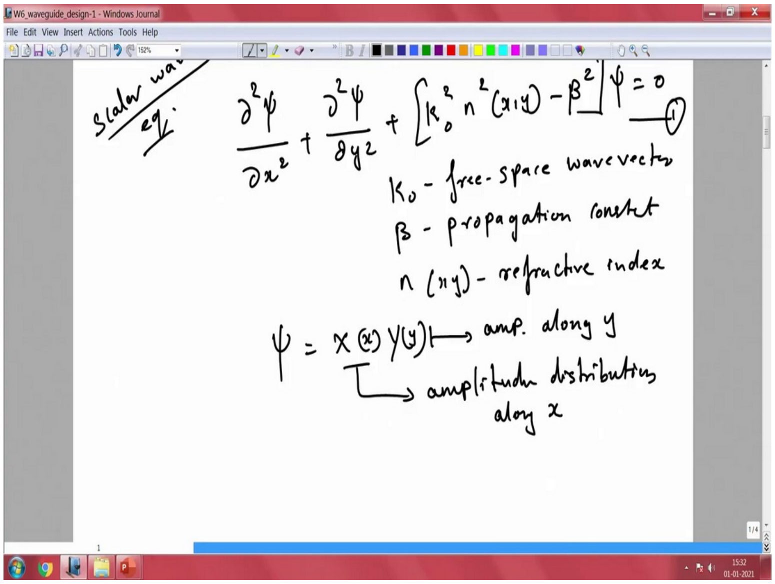

And if psi is your wave, you could write something like

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \left[k_0^2 n^2(x,y) - \beta^2\right]\psi = 0$$

So, this is your scalar wave equation. So, we are writing this because it is uncoupled, let us say there is no coupling in x, y, and z and in this case k naught is nothing but free space wave vector and beta is the propagation constant. But we still do not know what this propagation constant is? It has a certain propagation constant. You do not know what it is because that is what we want to find out and n x comma y is nothing but refractive index.

So, this is similar to what we have seen earlier so there is nothing new in it but this is where we have to start this so we need to start everything with this wave equation. So, now let us look at the solution itself so the psi is the propagating wave here so what could be the solution to this? The solution for this will be as a function of x and y. So, it is rather straightforward to do that is something that we saw earlier, you can do it x of x, y of y where this x of x is nothing but amplitude distribution along x.

And similarly y is amplitude distribution along y so similarly this one is amplitude along y direction. So, this is very straightforward because this is how we see the solution and here again, the solution here, particularly let us say in this particular guided region, this n 1. So, n1 is where you have the light propagating. There you will have a sinusoidal solution. So, you will have sinusoidal solution wherever light is propagating and outside it should have a decaying solution. So, you want to have that sinusoidal solution in n1 but in n2 and n3 regions, you should have a decaying solution let us say for the light that is oriented along x direction.

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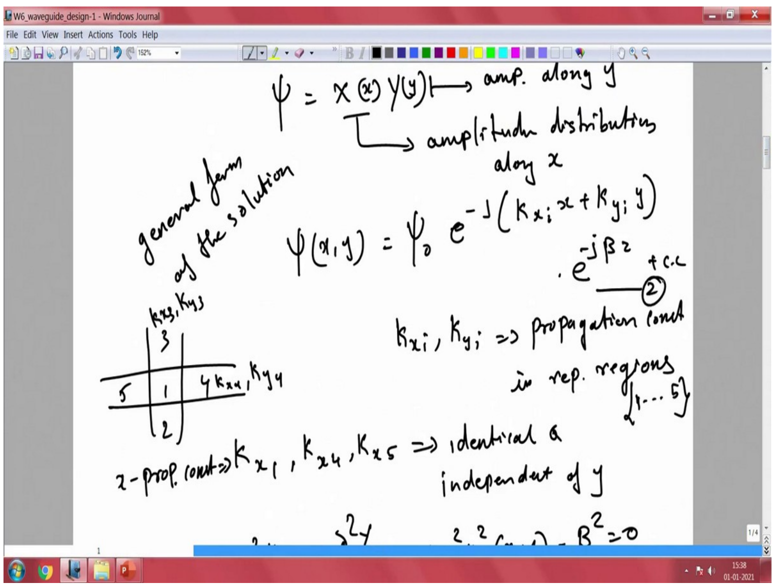

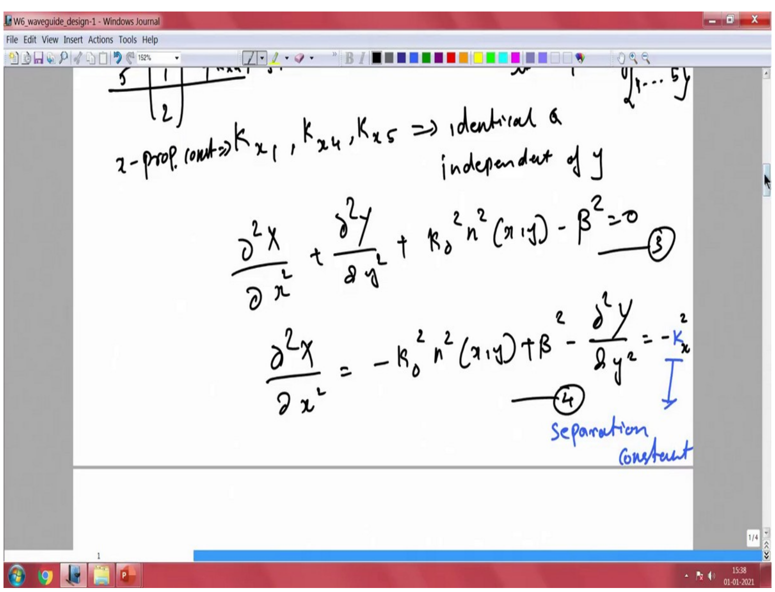

Now your field? So, your general form of the solution, so general form of the solution,

$$\psi(x,y) = \psi_0 e^{-j(k_{x_i}x+k_{y_i}y)} e^{-j\beta z} + c.c$$

So, this is something we have also seen earlier so kxi, kyi depends on which region you are. So, when you take this region 1, 2, 3, 4 and 5, so in this case it will be k x1 and k y1, here it will be k x4, k y4, similarly here k x3, and k y3. So, based on the region that you are. So, this k x i and k y i are nothing but propagation constant in respective regions, 1 to 5. So, this is something that one can take.

So, now the propagation constant along x would be k x1, k x4 and k x5 when you take 1, 4 and 5. And this propagation constant should be identical and independent of y. So, if you are taking along x so your k xi; k x1, k x4 and k x5 so this in region 1, 4 and 5. So, this must be identical, this all should be identical and independent of y. So, this is for x propagation constant. This is x propagation constant so all of this should be.

So, similarly you can do it for y as well. So, this is something you should keep in mind because the wave is distributed between 1, 4 and 5 and similarly if you are taking y direction so 1, 2 and 4 in that direction, y component should all be identical so that is how you will keep that particular energy going through. You remember in the slab waveguide if you want more to propagate, the propagation constant should be identical between the two layers. So, this is something we saw.

$$\frac{\partial^2 X}{\partial x^2} + \frac{\partial^2 Y}{\partial y^2} + \left[k_0^2 n^2(x,y) - \beta^2\right] = 0$$

$$\frac{\partial^2 X}{\partial x^2} = -\left[k_0^2 n^2(x,y) - \beta^2\right] - \frac{\partial^2 Y}{\partial y^2} = -k_x^2$$

$$\frac{\partial^2 Y}{\partial y^2} = -\left[k_0^2 n^2(x,y) - \beta^2\right] - k_x^2 = -k_y^2$$

Again, we are using the same condition here. We have not invented anything new here but the condition is here there are two directions that we have take into account, x and y. So, now we can modify this wave equation that you could do this doh square, x by doh x plus doh square y by doh y plus k naught square n squared x, y minus beta square equal to 0. This is something that we know already. So, we need to do some rearranging here.

So, if we do that rearranging, doh square x by doh x squared, in this case squared, should be equal to minus k naught squared n squared x, y plus beta square minus doh squared y by doh y squared is equal to, we are going to introduce a separation constant here, minus k x square. Perhaps I should use a different color here. Similarly we could, so this is nothing but, this is separation constant, nothing new. Similarly, we can do for our y as well. So, this is equation you can keep this as 2, this is equation 3, and this is equation 4.

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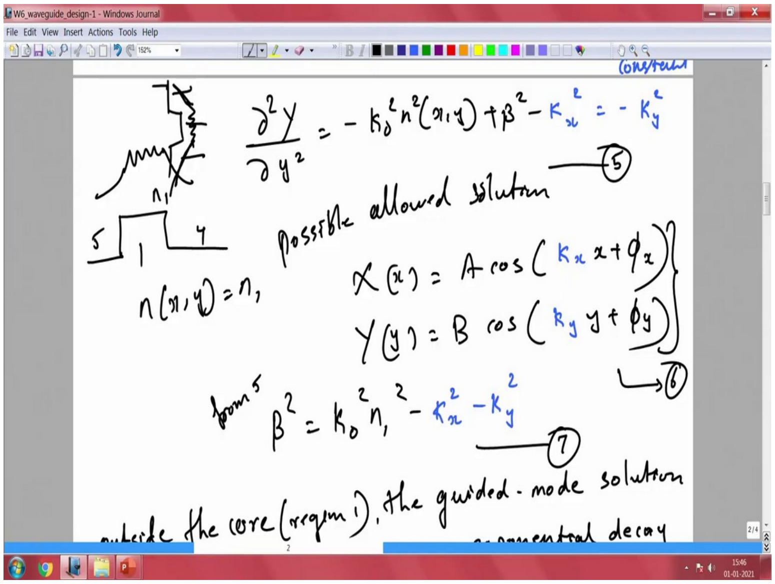

$$X(x) = Acos(k_x x + \varphi_x)$$

$$Y(y) = Bcos(k_y y + \varphi_y)$$

$$\beta^2 = k_0^2 n_1^2 - k_x^2 - k_y^2$$

So, now we can write the same thing for y as well. So, we can solve for y function. So, doh square y by doh square equal to minus k naught square n squared x, y plus beta square so now here I can write this as minus kx squared which is equal to minus ky square. Again this is just a separation constant. So, this is we call it equation 5. So, what we are going to consider here is the refractive index that n x, y that we have. So, this is uniform within the region of interest.

So, here we have 1, 2, 3, 4 and 5 so at least in 1 is where we are interested in because n in region 1 is where we have our core, but this core refractive index is uniform within the region and the rest of the region it is uniform but less than what you have in region 1. So, will use what we call a step index refractive index. So, that means you will have something like this. So, this is region 1, region 4 and region 5 let us say so there is an n1 what we have here. So, this is the refractive index within the region here.

So, now n x comma y will be equal to n1. So, if n 1 is greater than the rest of the region so that is what we have agreed so far, then the possible solution, allowed solution will be of this kind. So, x of x will be A cos in this case k x times x plus phi of x. Similarly for y, you will have B cos k y times y plus phi of y. So, in this case phi of x and phi of y are nothing but face constants that are adjusted to match the boundary condition.

So, if you remember again I want to go back to our simple planar waveguide discussion here. We had reflection from the boundaries. So, we had the face accumulation because of reflection from region 1 to 2 and region 1 to 3 so phi 2, phi 3 for the face matching. So, in this case phi of x and phi of y are similar but then 1 is associated with x boundary and the other one is towards y boundary.

So, now the separation constant that we have, kx and ky, should satisfy, the separation constant should satisfy the boundary condition here that is given by beta, so k naught square, n1 square minus kx minus ky square. So, this is the condition that it should satisfy. This is coming from our equation number 5 itself. So, this is from 5 so this should be satisfied. So, these two equations we call this equation number 6 and this is equation number 7.

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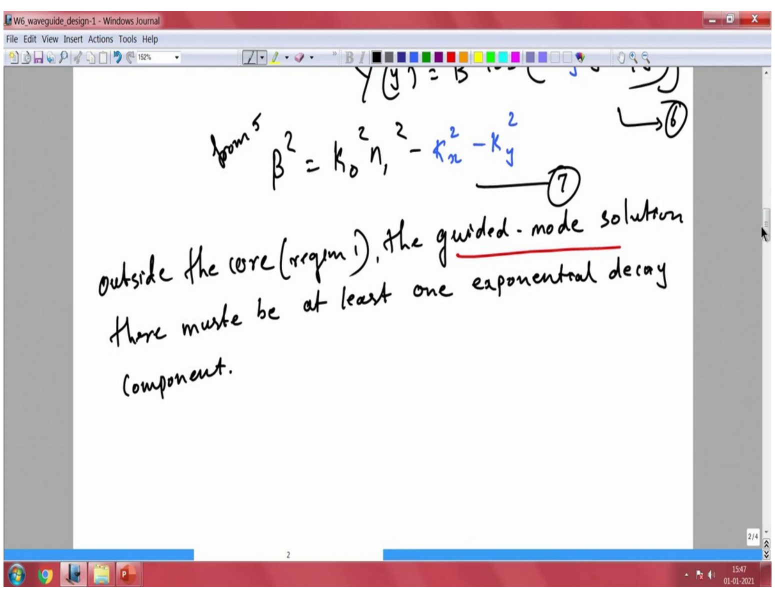

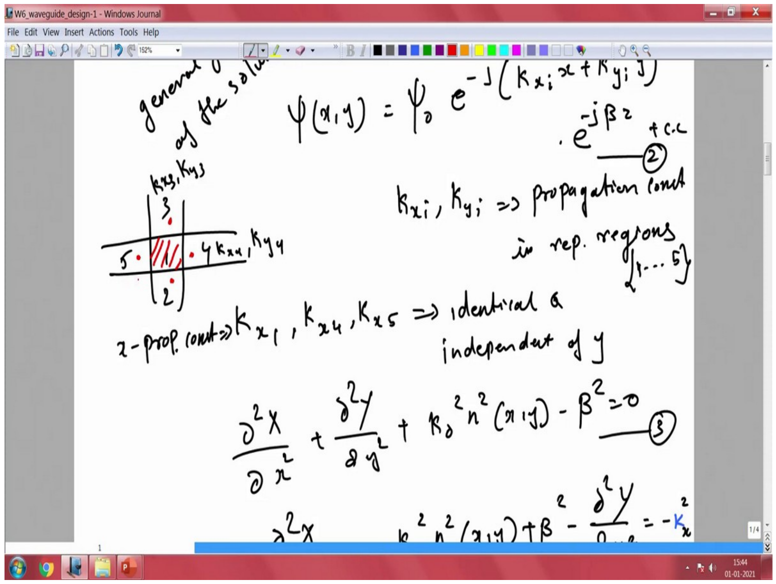

So, now you can clearly see the similarity with the slab waveguide discussion that we had in the earlier lectures but then the only thing here is everything is double now, everything is with x and y. So, the outside the code, the guided mode solution should at least have one component which is exponentially decaying so that is an important requirement in order to confine the light.

So, let us look at the cross section again here. So, this is where you have the light that is getting confined. So, outside all these regions for the moment let us not worry about these corners, the corners we will bring it up as we mature the understanding here. So, all the dotted regions that you see here, so the dotted regions should have at least one component that is decaying.

So, for example, if you take the x component along 1, 2 and 3, all those components should be oscillating in one of the coordinates but then for 2 and 3, should have at least decaying solution in the other coordinates. So, that is very important. Similarly, for 1, 4 and 5 you will have oscillating solutions in let us say in y direction, but in x direction you should have decaying solution for 4 and 5.

So, the required condition here is, so let me do that. Outside the core, in this case, region 1, the guided mode, this is very important, not for all the modes, for the guided mode solution, there must be at least one exponential decaying component. So, this is something important that we should all keep in mind because you have x and y. If you take a simple thin film planar waveguide understanding, it will be all clear because you will have oscillating solution here and decaying outside.

So, that is true in this simple geometry but then you also have a vertical component here along x. So, in this case again you will have a decaying solution here, an oscillating solution and decaying solution. So, as you can see here when it is oscillating along y direction in this case, it will be oscillating here, here and here. But then when you look at x component, it is decaying. So, you should at least have one decaying solution in order to keep this mode in.

So, we will visualize this as we progress but keep this in mind. We will revisit this particular statement later on in the lecture. So, let us consider solution in region 3, for example. So, region 1 is where we have our waveguide propagating, mode is sitting, but let us look at region 3. So, region 3 is where you should have something decaying. So, let us look at region 3.

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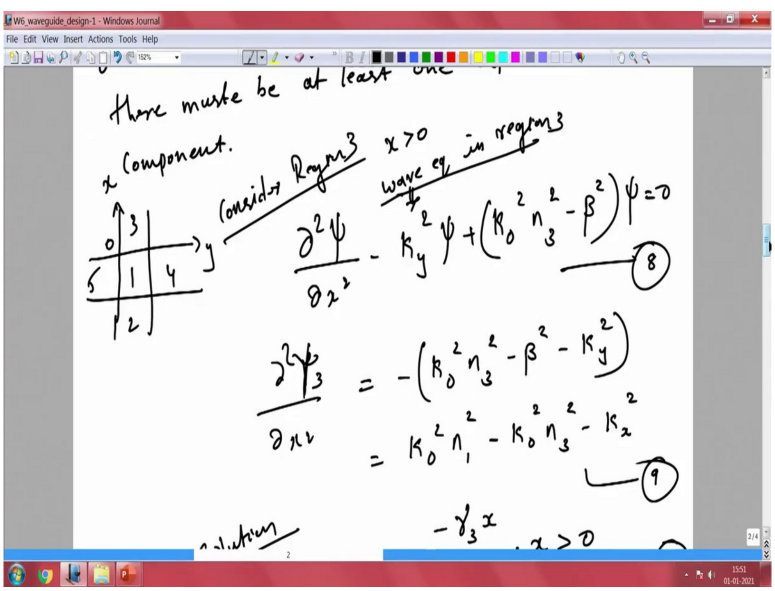

$$\frac{\partial^2 \psi}{\partial x^2} - k_y^2 \psi + [k_0^2 n_3^2 - \beta^2]\psi = 0$$

$$\frac{\partial^2 \psi_3}{\partial x^2} = +k_y^2\psi + [k_0^2 n_3^2 - \beta^2]\psi = (k_0^2 n_1^2 - k_0^2 n_3^2 - k_x^2)\psi$$

So, let me draw this again so that you know we keep this in mind, 1, 3, 4 and 5. So let us consider region 3. In this region where we need to match the boundary condition along y. So, you have your y and you have x so for x greater than 0 if this is there we can say x greater than 0. How are we going to understand this? So, we can right away write the wave equation. So, doh square psi divided by doh x square minus ky square psi plus k naught squared n 3 squared minus beta squared psi equal to 0. So, this is the wave equation in region 3. So, this is the wave equation in region 3.

So, this is what you have. So, now if you substitute the equation 7 here, so you have 7 into 8 now so we are going to do this. If you do that then what you will end up with is the right hand side of this, doh squared y by should be minus k naught squared, n3 squared minus beta square minus ky squared which is equal to we just do some rearranging here, k naught squared, n1 square minus k naught square, n3 square minus kx square. So, this is equation 9.

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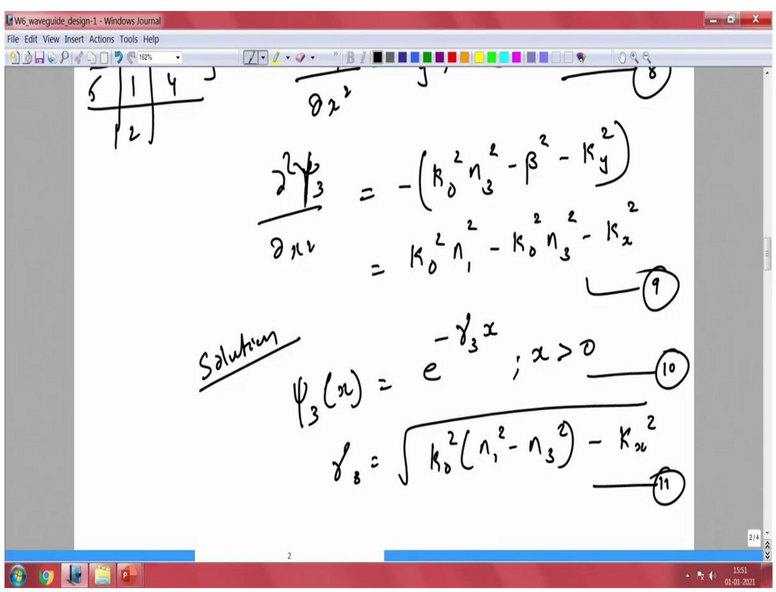

$$\frac{\partial^2 \psi_3}{\partial x^2} = +k_y^2\psi + [k_0^2 n_3^2 - \beta^2]\psi = (k_0^2 n_1^2 - k_0^2 n_3^2 - k_x^2)\psi$$

$$\psi_3(x) = e^{-\Upsilon_3} \quad ; x > 0$$

$$\Upsilon_3 = \sqrt{k_0^2(n_1^2 - n_3^2) - k_x^2}$$

So, this has the solution, so the solution here, let us say this is psi 3, so you have psi 3, around x direction will be, I will put it as x here, e to the power minus gamma x for x greater than 0. So, this is actually the solution to this. So, what is gamma 3 here and the root k naught squared, n1 squared minus n3 squared minus ax squared. So, this is again something familiar for us. We have seen this decaying so how this would decay is because you have an imaginary solution so that means it is decaying here.

So, this is the decay constant gamma that we have seen here. So, now we have seen how this field along x is going to look like. So, we can put a field along x and y now together. So, let us put this, how the field is going to look, both in x and y. so let us go back and look at this particular structure again. So, we just looked at this x component and along y, it should have an oscillating solution. So, all the solution along y will be oscillating for 1, 2 and 3 but then for 3 the y component of the solution will be decaying.

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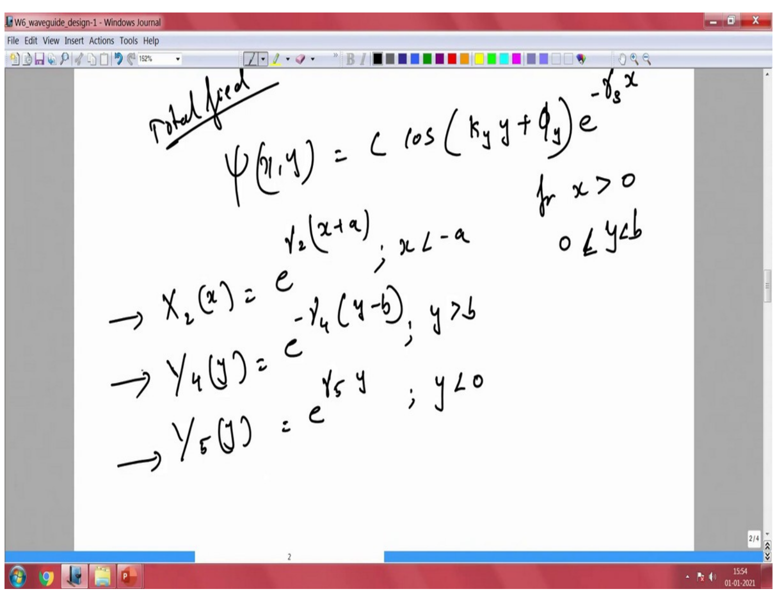

$$\psi(x,y) = C\cos\left(k_y y + \varphi_y\right)e^{-\Upsilon_3 x}\ for\ x > 0; 0 < y < b$$

$$X_2(x) = e^{-\Upsilon_2(x+a)}; x < -a$$

$$Y_4(y) = e^{-\Upsilon_4(y-b)}; y > b$$

$$Y_5(y) = e^{-\Upsilon_5 y}; y < 0$$

So, let us write the total field here. So, total field Psi of x, y now is nothing but some constant c cos k y plus Phi of y so this is our oscillating component along y direction. So, along y direction you will have oscillating for 1, 2 and 3, all the three will have oscillating solution. But then the component that we just found for x, so e to the power minus gamma c times x so this for x greater than 0 and then y between 0 and let us say some with b so we can put b here let us say.

So, within this particular region we have a solution for our field in this region 3. So, we can continue and do this for all the other regions as well. So, the similar set of solutions we can do it for other regions, x and y. So, let us just write for simplicity for 2, 4 and 5 as well. So, 2, 4 and 5 just to give us some understanding on this. So, x2 will be e to the power gamma 2, x plus a for x less than minus a, so a is nothing but the width here.

So, the dimensions are a and b and then you could have y, 4 of y is nothing but e to the power of minus of 4, y times b, this is for y greater than b and then we have y region 5 which is e to the power gamma 5y for y less than 0. So, these are all three different region components for x and y. So, what is gamma 2, 4 and 5? That will follow the similar trend that we had for 3 here. So, this is 3 here so let us do that as well.

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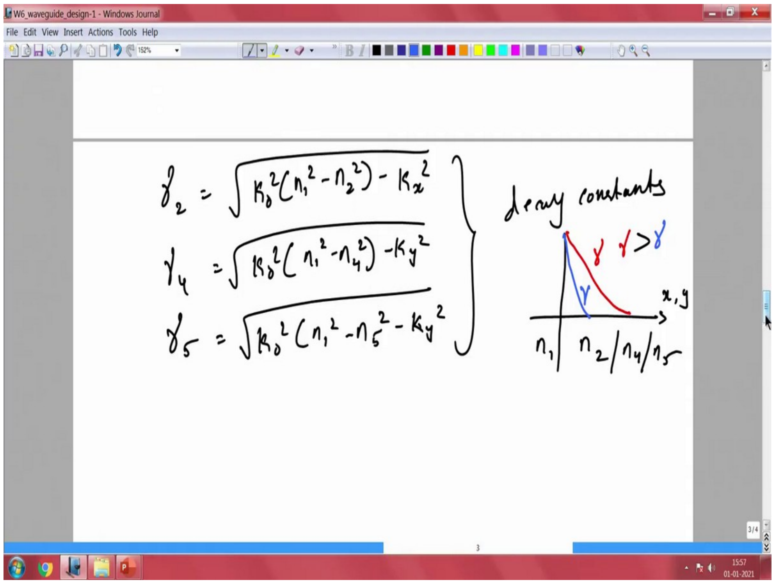

$$\Upsilon_2 = \sqrt{k_0^2(n_1^2 - n_2^2) - k_x^2}$$

$$\Upsilon_4 = \sqrt{k_0^2(n_1^2 - n_4^2) - k_y^2}$$

$$\Upsilon_5 = \sqrt{k_0^2(n_1^2 - n_5^2) - k_y^2}$$

So, gamma 2 is nothing but k naught square, n1 squared minus n2 squared minus kx squared, 4 is root of k naught squared, n1 squared minus n 4 squared minus ky squared, 5 is nothing but k naught squared, n1 squared n5 squared minus ky squared. So, these are all the decay constants. So, this is all nothing but decay constants.

So, as we already discussed in the earlier lectures, this decay that you see from the interface it follows this gamma factor and this gamma strongly depends on the refractive index contrast between the two. It could be n1 to n2 or n1 to n4 or n1 to n5 so difference between the core refractive index and the index that is covering this. So, the higher the refractive index contrast, the lower is the range.

For example you could have this and you could have this. This is as a function of x or y. So, here let us say this is gamma 1, no this will confuse you. Let me pick two colors now. So, there are two gammas here. Red one and then I will pick a blue one here. So now the red gamma that you have is greater than the blue gamma. So, when you have greater index contrast, the difference between the two, then you are also going to make it decay faster.

So, that will ensure that you have high field confinement so this is one of the reason why we try to use high refractive index contrast mediums in order to confine light tightly. So, we will see this on our different waveguide platforms but remember about this.

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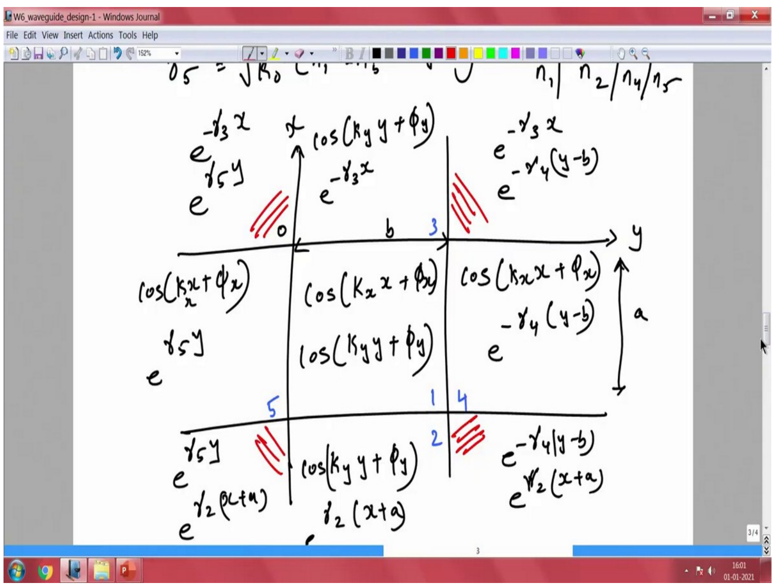

So, now we have just an example of the optical field is in different regions but let us have the two components distributed in all the different regions that we saw. So, let us take this. We will look at the different regions here. This is region 1 and then we have region 2, this is region 3, and this is region 4 and region 5. So, these is the regions that we are looking at.

For the central region, we will have oscillating solution for both x and y so that solution will look something like that, kx, x plus phi x. Similarly, for y, ky y plus phi y. Let us look at 3 which is something that we just saw. So, cos ky y plus phi y and you will have a decay along x. So, that means e to the power minus gamma 3 times x. So here this is y and this is x. So, just keep that in mind.

So, now let us look at y and so y again it is quite trivial that you should have oscillating solution along y. So, cos k y y plus phi of y but now it should decay here, so e to the power gamma 2 x plus a so because this is a and let us say this is b so this is 0. So, this is 1, so let us look at region 4. Region 4 you will have oscillation along x so you can write it here and also I can write this here as well. So, now it is going to be decaying along y direction. So, e to the power minus gamma 4, y minus b and here e to the power gamma 5 times y.

And similarly, for the edges so that is where the tricky part is. Around the edges you will have only decaying solution. So, e to the power minus gamma 3 x so that is coming from here, e to the power minus gamma 4, y minus b. So, you have the y component coming from here, the x component coming from here. So, it takes the boundary or the solution from the neighborhood to do this because there is no clear boundary condition you can apply here. This is coupled.

So, similarly here e to the power of minus gamma 3x and e to the power gamma 5 times y. And at the bottom we have e to the power of minus gamma 4y minus b, e to the power of gamma 2x plus a and here e to the power of gamma 5y and e to the power gamma 2x plus a. So, now the corner regions got only decaying solution. You will not have any field here at all. So, this is what the challenge is about when you are doing analytical solution to this.

So, let us quickly look at how you could solve this boundary value problem once we understand this. So, with this understanding of field, both oscillating field and decaying field that you have inside this region, one should be able to identify where the field is and what is the amplitude of these fields that one could have so those are all the things you can find out with the understanding we have so far.

But the next step is to identify this beta, primarily we are chasing the propagation constant. We need to find this propagation constant from the solution that we have. Right now we have just found how the solution is going to look but then we have to quantify that so that quantification comes from the propagation constant that we have.

So, we have kx and ky that you could find out but then as a complete mode so individual components we have, but then as a whole it has to propagate with one single propagation constant. So, we need to then solve this using boundary value approach. So, let us look at that in the next lecture. Thank you very much.