Photonic Integrated Circuit
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Lecture No. 18
Guided Modes in Symmetric

Hello everyone. Let us continue our discussion on this waveguide parameters. So, we looked at waveguide field, how to arrive at the field inside the core and outside the core and so on so we have a better understanding now on that. So, going forward we may want to define certain parameters to characterize the modes and also structure as a whole. So, we want to capture the elements that we have discussed so far and try to create parameters out of it and these parameters could help us or give us a much easier way or a simpler way to quickly calculate or quickly characterize whether this particular structure or particular dimension or a particular combination of core and cladding refractive index will guide any light or not.

So, we do not want to run a complete numerical solution to understand whether the light will be guiding here or not. So, we have very simple ways of doing that and we call those as waveguide parameters and it is much easier to do. You can analytically do that to find out whether there are any possible solutions to this and we are going to look at that, what are all those parameters.

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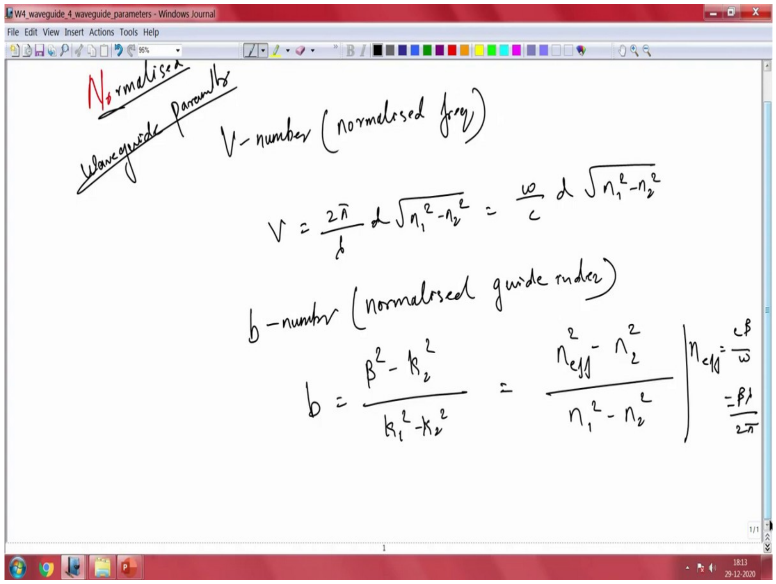

So, we call that as normalized waveguide parameters. It is what we call those and the more property of waveguides are commonly characterized using this normalized parameters. We call that as v number or normalized frequency. It is called v number, in other words it is called normalized frequency and this captures the dimension, the wavelength and refractive index everything together.

So, let us look at that so

$$V = \frac{2\pi}{\lambda} d \sqrt{n_1^2 - n_2^2}$$

You can also write

$$V = \frac{2\pi}{\lambda} d \sqrt{n_1^2 - n_2^2} = \frac{\omega}{c} d \sqrt{n_1^2 - n_2^2}$$

So, there is another way of doing it. So, your d here is the dimension, how big or how small, n1, n2 gives you the refractive index of the cladding and the core, and lambda is the wavelength. So, now you can quickly understand that this v number captures the optical property, the wavelength, the dimension, the geometric property, all this into one number. So, it is very handy and we can use this to understand whether we could propagate light or whether this particular medium is useful at all if we are going to design.

And the next thing is propagation constant. We know the propagation constant and we have a normalized guide index using this propagation constant. So, it is a the is called normalized guide index. It is called the b number, it is called normalized guide index and that is given by

$$b = \frac{\beta^2 - k_2^2}{k_1^2 - k_2^2} = \frac{n_{eff}^2 - n_2^2}{n_1^2 - n_2^2}$$

You can actually convert this into refractive index.

So, now you can look at this so n

$$n_1 = \frac{C\beta}{\omega} = \frac{\beta\lambda}{2\pi}$$

So, this is effective refractive index we already saw in the earlier lectures. So, using these two numbers we should be able to understand or do a quick assessment of how the waveguide is going to perform or whether we have any guided modes or what we should do in order to get that.

So, this will be different for both TE and TM. So, for TE mode how one could do it and then for the TM mode how the solution is going to be. So, we need to find the eigenvalue equation that is in terms of the normalized frequency so we need to do that. So, the way to get that is by equating this.

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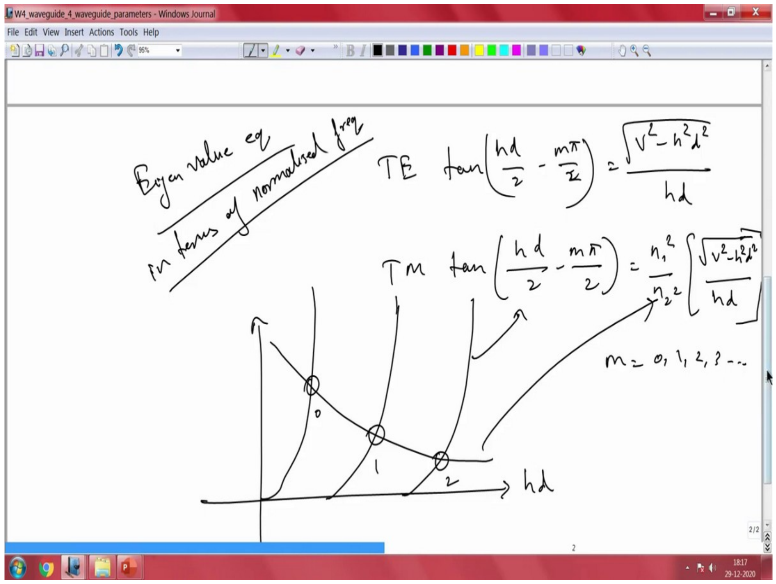

So, the eigenvalue equation in terms of normalized frequencies and that for TE, waveguide is given by

$$\tan\left(\frac{hd}{2} - m\pi\right) = \frac{\sqrt{V^2 - h^2d^2}}{hd} \quad \text{For TE}$$

$$\tan\left(\frac{hd}{2} - \frac{m\pi}{2}\right) = \frac{n_1^2}{n_2^2}\left(\frac{\sqrt{V^2 - h^2d^2}}{hd}\right) \quad \text{For TM}$$

So, here m is 0, 1, 2, the mode number. So, this is the eigenvalue equations that you could get. And you can solve for this. So, you can allow values of h d here and this would give you the value of waveguide parameter v for both TE and TM.

If you just solve for it, say when you can do that really analytically here as a function of hd. So, what would be your x and y axis. So, this is a tan function. So, you can imagine the tan function is going to go like this. So, now if you are going to solve for this, then you are going to the right hand sides will cut across something like this. So, now this will give you, the solution here. So, the intersection point is the solution. So, this is the left hand side and this is our right hand side and this gives us all the solutions.

So, this is 0, this is 1, this is 2 and this is nothing but m equals 0, m is equal to 1, m is equal to 2 and so on. So, by using this you could find how many modes are present in here. So, how many solutions one can have using this very simple equation that we have. So, you can just do this in simple MATLAB or Python whichever you like to find out how many possible modes you could have. So, these modes that are there are going to be propagating modes and that is the reason why you have this Eigen solution.

So, let us look at the difference or what we can learn between the v number and the normalized guided index. So, if the v number is very small, right if the v number is very small then the guided light means they are very close to the critical angle and the effective index is very close to the leakage let us say. So, let me go back here, yes.

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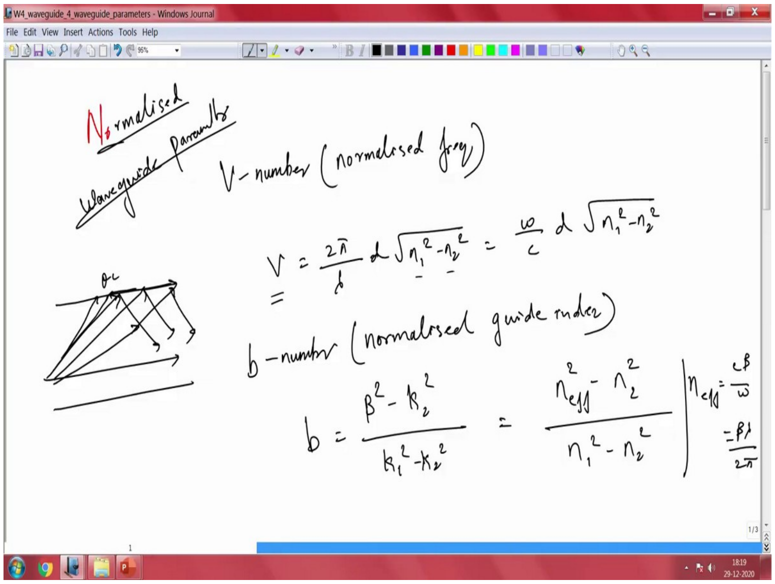

So, if the v number is very small in this case, nearly 0 that means your refractive for that particular mode, your refractive index is very close to n2, the effective index of the mode is very close to n2 and that could lead to leakage. So, the wave that penetrates deep into the cladding and it becomes leaky. So, when the v number is very small. So when the v number increases, now you have rays that are starting to evolve. So, you have a certain angle here and this is your critical angle, let us say at theta c.

So, when you say your v number is very small, very very small v number, so that means you are very close to the critical angle so you are just there and probably outside as well. So, now when the v number increases, then you have the solutions that are propagating now. So, you are slowly trying to make the wave propagate through changing this. And when you have a very large v number that means you are propagating through the waveguide core in this case. And that is how you can compare your v number with your wave propagation in this case.

So, let us look at the ways to find how many number of waves and something called cutoff. So, we saw when you do this you have this intersection that gives you your solution, but what if you do not have any solution. So, that is what we call cutoff. There is no solution or there is no propagating mode that is available or propagating solution available. So, that is what we call mode cutoff.

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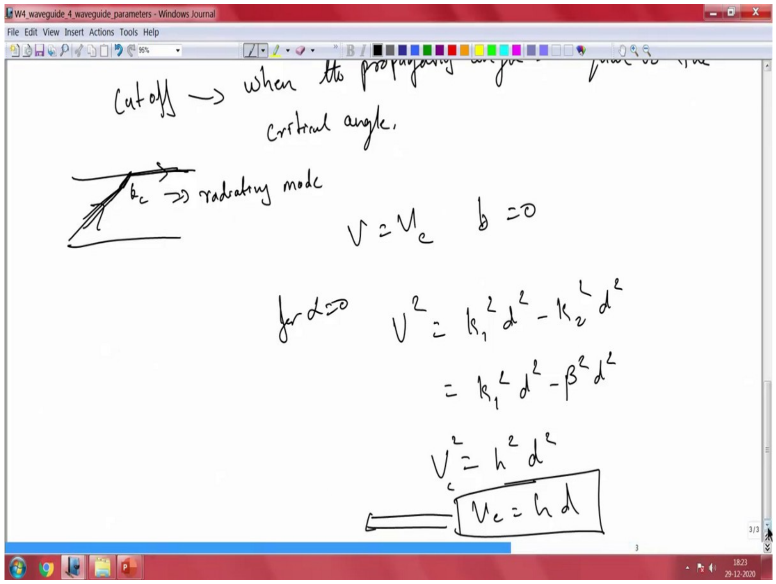

So, cutoff occurs when the propagating angle is equal to let us say the critical angle. You remember the critical angle. At that critical angle the wave, I mean there is no propagation yet that is where it is all going to start. So, they are only evanescent waves and this condition is called the cutoff condition. So, only above this you will have any propagating modes. And the guided mode, this is all solution but they are all unguided, they are radiating modes.

So, at this particular angle theta c you will have radiating mode only that means it is not actually propagating through it, it is going to be loss so you need to have angle that are greater than theta c. So, that is very important. So, the condition for cutoff. So, the condition for cut off here is your v should be equal to v c, any number beyond that. So, for this particular guided mode is the value of v at this point where b is equal to 0, so where b will be equal to 0.

So, let us see for alpha equals to 0, let us say how the v number is going to look like, based on the condition that we have and that is given by

$$V^2 = k_1^2 d^2 - k_2^2 d^2 = k_1^2 d^2 - \beta^2 d^2$$
$$V_c^{\ 2} = h^2 d^2$$
$$V_c = hd$$

So, this is our cutoff condition. So, what should be my width in order to, or the dimension in order to get any light propagate through this system. So, you can substitute this in our eigenvalue equation. So, you remember the eigenvalue equation that we saw here, so normalized equation here.

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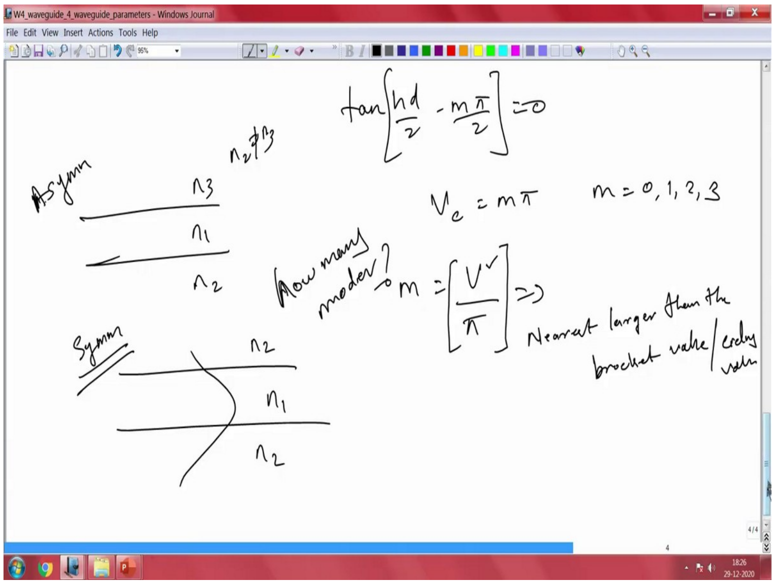

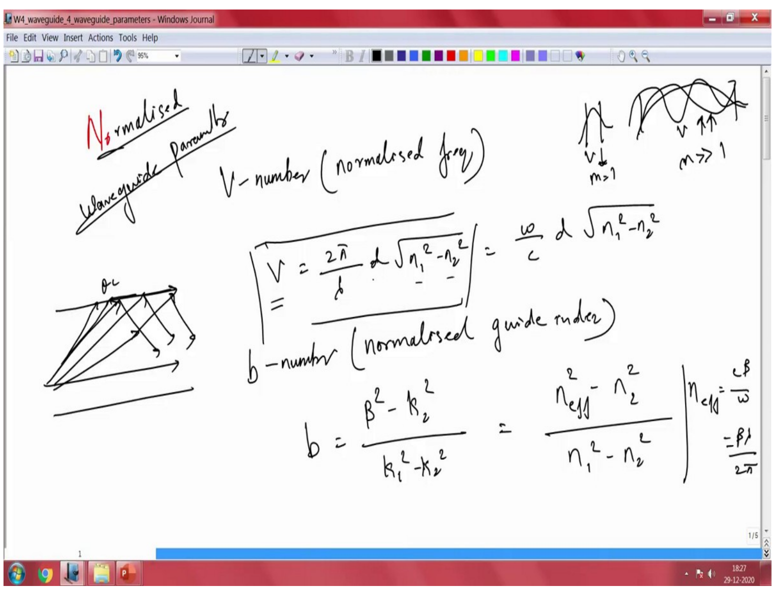

$$tan\left(\frac{hd}{2}-\frac{m\pi}{2}\right)=0$$

$$V_c = m\pi \quad ; \quad m = 0,1,2,3$$

$$m = [\frac{V}{\pi}]$$
$$V < \pi$$

So, your condition here, the number of modes or a symmetric waveguide is the same cutoff condition here, V_c becomes $m\pi$, so this m is nothing but your mode number here. So, we can easily find how many modes are propagating but just doing the reverse. So, we can do m equals v over Pi so this will tell you how many modes.

So, if you ask the question how many modes, if I know the v number, so v number can be easily found out if you know these parameters. So, we can easily find out by refractive index, the dimensions and so on. Using this v number we can easily figure out whether there is any propagating mode there or not.

So, this one important thing is, it comes with a square bracket that means nearest integer larger than the bracket value so it is a ceiling. So, you do nearest larger than the bracket value or in other words ceiling value, instead of floor we do the ceiling. So, if you do that then you can easily find the number of modes here.

An interesting fact to note here is there are two types of waveguides you can have. One is symmetric and the other one is asymmetric. And in asymmetric we have something like this where n1, n2 is not equal to n3. So, in a symmetric waveguide you will have at least one TE or TM mode. There will be at least one solution so there will be a solution always there in a symmetric waveguide. So, in a symmetric waveguide there will be always a solution. Please remember that, it is very important. But in asymmetric waveguide that is not the case.

So, now you can easily understand the implication of having a large waveguide structure here, for example, we can take this v number here. If I make this D larger, so I go from very narrow structure to a very broad structure so you will increase the v number. So, what increase in v number represent? So, v number is going to be small for here, v number is going to be very large here. So, what is the implication of having this? Your number of modes are going to be large.

Here you will have probably only one mode but in this case you will have multiple modes inside the system. So, you will have m, much much greater than 1. So, here m will be 1. So, by just using the dimension and refractive index we should be able to figure out how many waves or modes would propagate through this system. So, that is fine but what is the condition to have a single mode?

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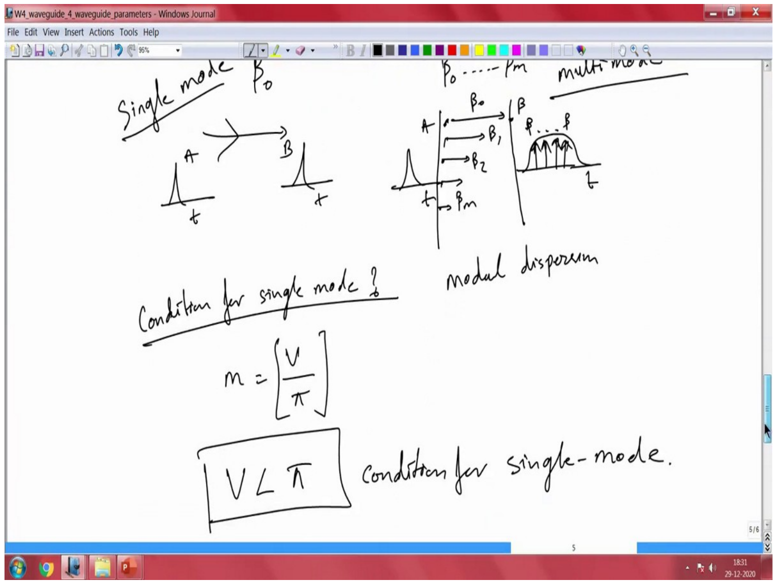

We talk about it, right, so single mode and multimode. So, this becomes a very important aspect in a guided system. The reason for this is you have only one beta and you have multiple betas. So, when you put energy into this system so this particular mode will travel from point a to b with one single propagation constant so they will reach point b with a given propagation constant or speed here but then when you take a multimode system when you go from point a to b all these betas are going to travel at different speeds and because of this the signal that you put in will not reach point b at a given time.

So, we discussed about dispersion, the pulse dispersion when it propagates through optical fiber there will be broadening because of chromatic dispersion because there is an intrinsic speed difference because of refractive index difference but in this case different modes are traveling at different speed so this is called mode, modal dispersion. Different modes are going to take this signal and distribute among themselves and they will all arrive at different time.

So, if I put a signal like this, I will get a signal like this as a function of time. The reason for this is betas are coming at different points of time but in this case when I put a signal at time t I will get the same here. So, this is one of the reasons why we try to design systems that work with single mode. We do not want to have multiple modes because handling different modes is always difficult because of modal dispersion that you have.

So, what is the condition for single mode? What is the condition here? Let us go back and look at this. So, m equals V over Pi is what we wanted. So, m should be one here. There should be only single mode here. There should be only one so that means your $V < \pi$ so this is the condition for single mode. So, the rest of the things are all higher order modes. So, when you are designing or thinking of designing any waveguide so you will look at this v number. That is the first thing you do.

You can take the refractive index of core and cladding and you know the wavelength that you are operating and based on that you choose the right dimension, so you choose the right dimension of your waveguide so that you achieve the single mode operation. So, this is how we, one can achieve single mode operation and avoid the effect of dispersion when it comes to light propagation through these waveguides.

So, with that we have come to the end of these waveguide basics. I hope you have now a comprehensive understanding of what a simple waveguide should look like and how to understand light propagation through this waveguide and also force this design to deliver the kind of modal properties that you are expecting from the material system that you have. With that we would like to close this waveguide part. Thank you very much for listening.