Photonic Integrated Circuit
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Lecture No. 16
Waveguide Mode 2

Hello, welcome back to another lecture on waveguides. So, let us quickly jump into the topic that we left in the last lecture on understanding the waveguides and how one can define different regions and also how the electric field could be propagated through this waveguide. So, particularly we are interested in slab waveguides, so we will continue doing that, let us first look at whatever we just summarized in the last lecture.

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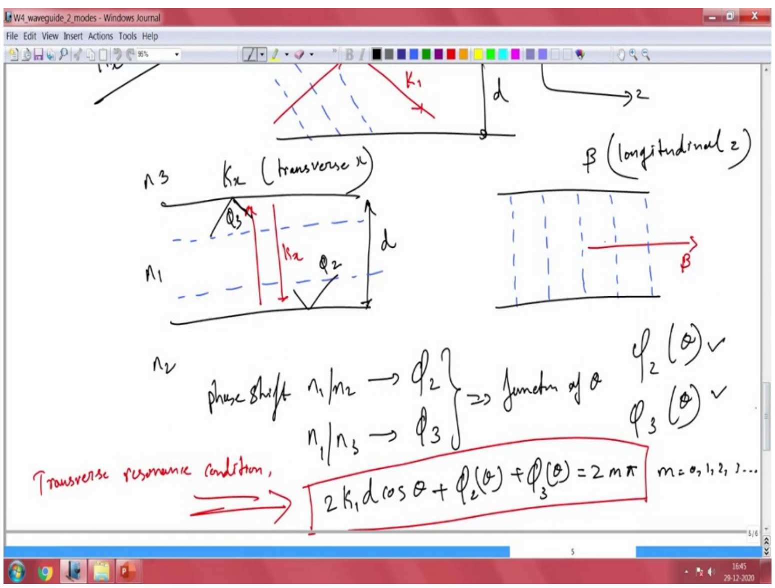

So, we concluded here, so where we had the resonance condition, the transverse resonant condition and that depends on the face that you accumulate when light is bouncing between the two interfaces here, so that is the face here $2k_1 d\cos\theta + \varphi_2(\theta) + \varphi_3(\theta)$ are the phase that is accumulated when you have reflection from the two interfaces that we have. So, we know from our you know basic understanding of reflection that this phase shift is polarization dependent or the electric field orientation dependent.

So, let us continue a little bit more into this cold reflection problem. So, let us look at the orientation of electric field and how you can visualize the flow of a field through this medium.

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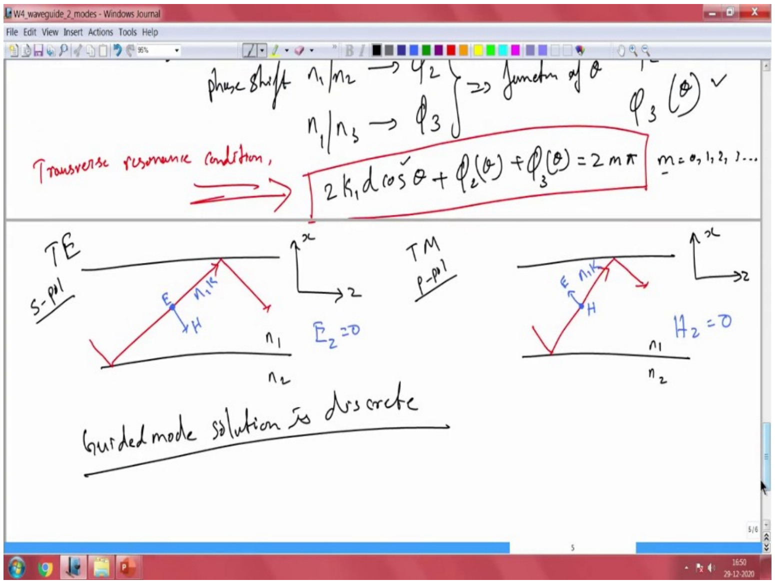

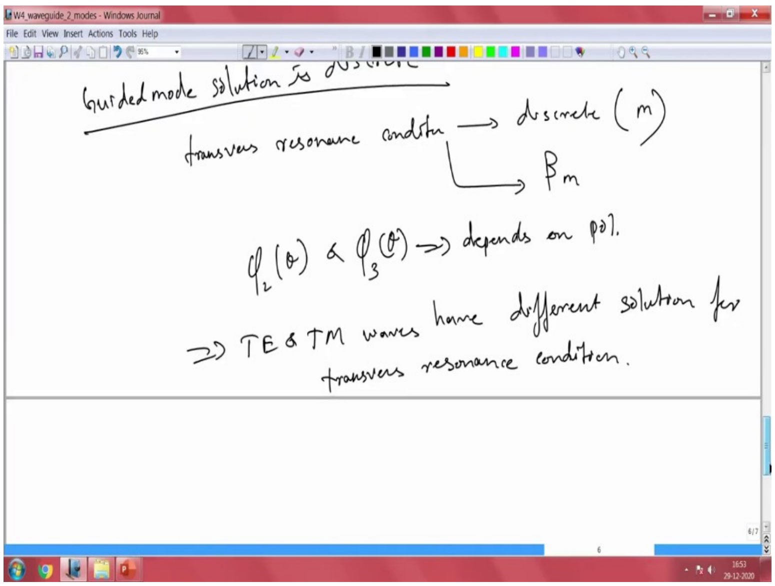

So, let us look at the transfer's electric polarization. So, that is TE polarization, so again z is the direction of propagation, so we have we this so this is n_1 and there is clad n_2. So, now the light is going to propagate through this medium like as we already saw earlier. So, now we need to mark the electric field, so as you all know since this is TE your electric field is going to be parallel to y axis and your H is here and E is here and $n_1 k$ is in this direction.

So, the electric field is pointing in y direction corresponding to the perpendicular or rather what we call the S polarization, so S polarized light here. And the field here, what you call the TE field this is entirely in xy plane, so there is something that you should keep in mind. So, there is no Ez component. So, the field lice in xy and that is the reason why $E_z$will be equal to 0. So, this is propagating around z direction.

So, let us look at TM polarized light or P polarised light, so this is what we have in this case the same geometry we take light is propagating through this and now your H field is now in along y and now our E field is perpendicular to the propagation direction and this is what we have as E and $n_1 k$, so let me write it a little bit bigger $n_1 k$ and here this is $n_1 k$. So, in this case you are magnetic field points in the y direction and for this particular polarization your H is at will be equal to 0 here.

So, this is the difference between the two polarization that we have and how they propagate through the system. And from here you can easily note that your phase shift is not going to be same for both TE and TM and because of this reason, the solution to TE and TM waves are going to be different as well. So, that is something that we should keep in mind that the solution for any slab or any waveguide structure will be different for TE and TM.

In addition to the transverse resonant condition, which is again a discrete solution, so that is something that I want to point out here, the guided mode, so the guided mode solution is discrete so it is discrete so you cannot have arbitrary M value, so it will only take integral numbers, so the mode number is 0, 1, 2 and so on, you cannot have 0.1, 0.2 as M. So, it has to be integral or integer numbers on integral numbers only.

So, if that is the case, so, that means your transverse resonance condition is discrete, because M values are discrete and this results in discrete propagation constant, so it is the propagation constant has to be discreet. So, although your theta C the critical angle both here in the interface between n_1 and n_2 and n_1 and n_3 does not depend on the polarization, but the phase shift depends, the φ_2, φ_3, so $\varphi_2(\theta)$ here and $\varphi_3(\theta)$ as a function of theta this depends on polarization.

So, this is something that you should keep in mind, so it is the solutions are discrete from our resonance condition, but then you are the phase shift that you get from the two interfaces is a function of polarization. So, there are now you refining your guided mode solution here, it is not just discrete, but even if they are discrete, there is one more factor to it, that is a polarization factor. So, this the implication here is so TE and TM modes or waves have different solution for transverse resonance condition.

So, this is a very important understanding that one should have that the polarization different polarization of this waves create different solutions and they are discrete as well. Another important thing is for a given polarization let us say whether it is TE or TM polarization the angle, there resonant condition that you have for a small beta would corresponds to small Theta, so we let us go back now probably I will draw a new one that is better.

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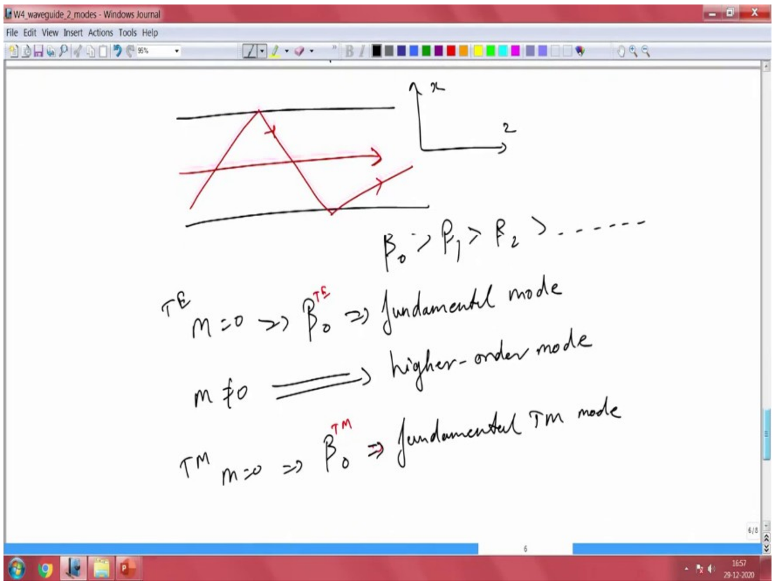

So, let us take the same slab, the waves could propagate without any reflection at all, it is through the axis, so there is no reflection at all and then you have another possibility where you have reflection. So, the wave that is going through with the smallest angle here the theta, so this will have the smallest beta. So, you will for a larger value of M, so beta will be increasing as a function of M. So, as the M increases the beta decreases in this case.

So, $\beta_o > \beta_1 > \beta_2$ and so on. So, this is and this is again something that one could easily calculate but this is what we have from our understanding here, so you can calculate it from the angle of propagation her, so there is no phase shift and the quantity here will be higher, so the phase shift will help you to do that. So, you could have β_o, so m = 0 which will corresponds to β_o and this is the first mode we call it.

So, we call this is the first solution or the lowest solution we call this as fundamental mode and when $m \neq 0$, anything higher we call this as higher order mode, so this is something that one should keep in mind because m is giving you all the integers that requires you to index your modes and the fundamental mode here is given as m_o or $\beta_o$in this case and then for $m>0$ the modes are called higher order modes. So, these are all solutions by the way.

So, this could be just for TM and for TE as well, so this is for a given polarization, you can have this, so you could have fundamental TE and fundamental TM. So, if I if one should be even more specific you can put β_o^{TE} and you could also make this for TM $m=0$ the same thing applies here, but we call this TM, but then this is again fundamental TM mode, so you could call it fundamental TM or fundamental TE. So, this is something that you can you can revise and then see how one can relate the discrete modes to the solutions with respect to polarization.

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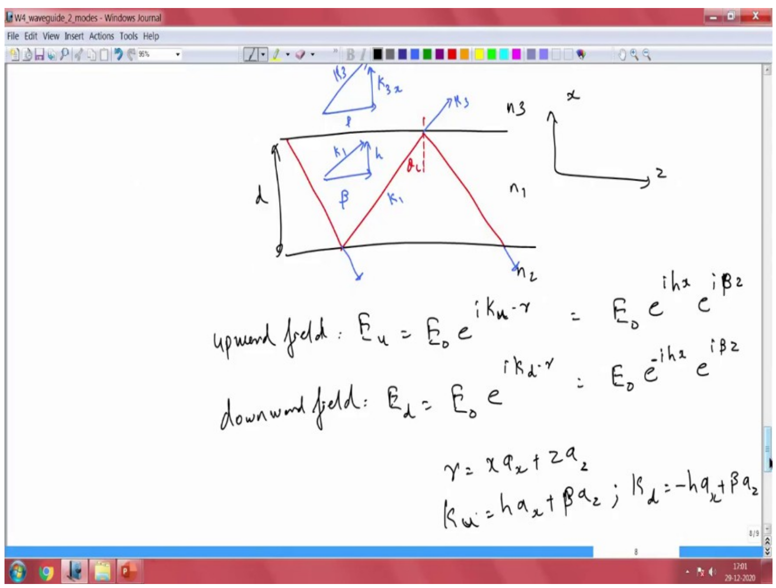

So, now let us look at some plane wave representation of this planar waveguide it is an interesting way to look at the propagation here, which will give us even better understanding of how light would propagates through this system. So, let us have this as d n_1 n_2 and n_3, so the light would propagate through this, like this system and we have theta can put θ_1 and have light that is radiating out, could be let us pick a different colour which will be good. So, light is leaking out this way it could refract, it also reflect here.

So, now we have light that is reflecting and light that is refracting, so let us put all these k vectors now. So, this will be our k_1 and let us say this is our k_3 here and we can easily draw our k vector triangle, so the k vector triangle inside will be beta here and this will be in this case h and you could have k_1, so that is one way of the layer I think this might be a little confusing let me do it this way, so this is h and this is k_1 and outside again we have beta this is k_3 and this is k_{3x} let us say.

So, there is upward field and we have downward field, so whatever you have in the clad and whatever you have inside, so the upward field is given by so upward field let us say E_u is given by $E_u = E_o e^{i(k_u . r)}$ and then so downward, downward field here is $E_d = E_o e^{i(k_d . r)}$. So, what is this k_u and k_d is?

It is nothing but $E_u = E_o e^{i(k_u . r)} = E_o e^{ihx} e^{i(\beta z)}$ and $E_d = E_o e^{i(k_d . r)} = E_o e^{-ihx} e^{i(\beta z)}$ $r = x a_z + a_z$ and $k_u = h a_x + \beta a_z$ and $k_d = -h a_x + \beta a_z$. So, this is how one could represent a plane wave representation for this planar waveguide structure. It is useful when you are doing a very simple expansion and propagation here.

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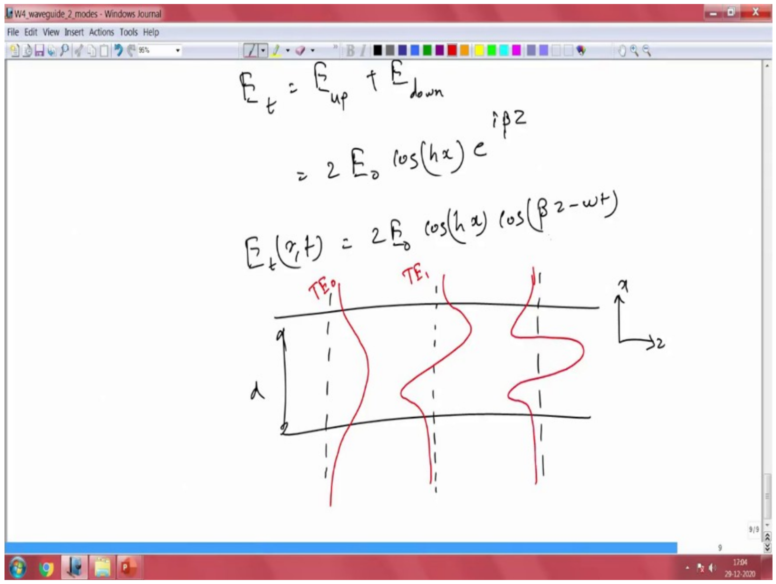

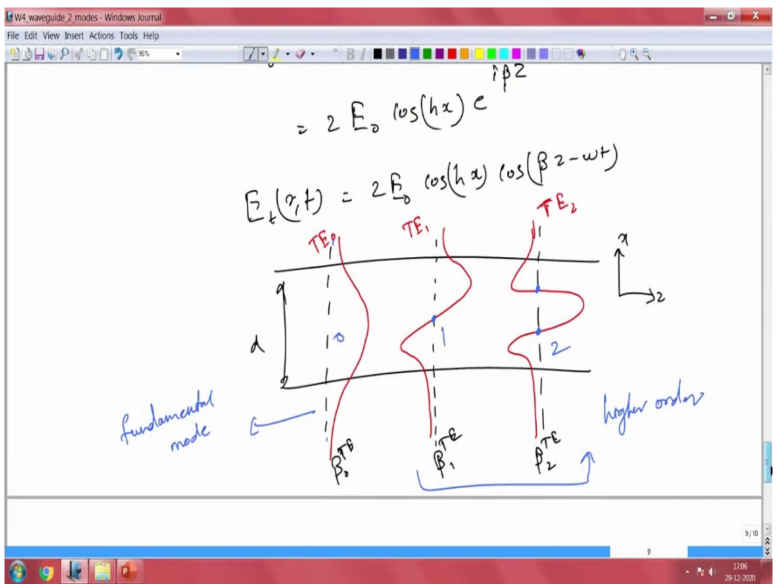

So, the total field so E total is nothing but $E_t = E_u + E_d$, so if you add whatever we had we will end up with $E_t = 2E_o e^{i(\beta z)} \cos hx$ so this is what you get. So, now we are getting close to defining how this field is going to look like, so from our earlier understanding of how the plane wave is would could be used in order to explain this propagation and now we have a total field with respect to the position, that is a total field is we will be $E_t = 2E_o \cos hx \cos(\beta z - \omega t)$.

So, this is how your field is going to look like. So, this is nothing but the real instantaneous field that you have here. So, if we if one want to see how this field would look like, so you have a certain d here, so for a very simple waveguide at a slab waveguide like this we can draw our modes now, so how the field would look like so as a function of position here? So, that should look something like this.

So, this is what the fundamental solution is going to look like we call this as TE_0. mode this is the fundamental mode. And now how the other solutions are going to look like it is basically expansion of what we just saw let me put it, so this is TE_1 this is the first order mode and then it keeps on moving. So, now you will have a node like this so this is TE_2, so this is how your mode is going to look like as you can see this is all discrete solution.

So, this is how your mode solutions are look like, so because m can only assume the integral values so this will be discrete, so that means theta's are also discrete satisfying the resonant condition. So, again this will propagate with the propagation constant of $\beta_0$and this will propagate with β_1 and this will at propagate β_2, important thing to note is this is all TE waveguide. So, this is all TE mode we could also do same TM mode as well.

So, the one way of also representing this mode is through the 0 crossing that you have. So, we call the 0 because there is no crossing across the 0 mark here, so there the field is going to decay till the end but there is no field crossing within the structure that you have, so it is symmetric we are drawing is not accurate here, but you can see here there is one crossing here, so we call this as 1. So, there is no crossing this is 0. And here you say there is two crossings and here the mode number 2. So, 1, 2 this is all higher-order so these two are higher order and this is what we call fundamental mode.

So, next let us look at the interface here, so we were able to plot this through this function, but if I go on top so we have this interface between n_1 and n_3 and n_1 and n_2, we have studied in our basic electromagnetics, so when the wave is encountering a surface then there should be a boundary condition. So, the boundary condition we have studied is the continuity in the energy, the electric field should be continuous, so the electric field vectors on the both the sides should be parallel with each other.

So, that continuity equation should be there, so unless you have that continuity energy would not be able to propagate through this system. So, here again we have two interface, so the middle you have wave propagating and we have two interface one at a top and one at the bottom. So, now the question is if this wave has to propagate through this particular structure, then there should be a continued to continuous energy distributed in all the three layers, so n_1 n_2 and n_3 so there should be a continuous energy equation between these, so the boundary condition should be continuous.

So, let us look at what is this boundary condition, in other words what is the phase matching. So, different waves can travel at different speeds we know, the waves in $n_1$will travel at a different speed compared to n_2 , because the refractive index is different. Once the diffractive index is different the propagation constant is also going to be different. Well now the question is the energy is distributed in these 3 layers, so how is it possible that the wave will propagate through the system without disintegrating without losing their light? So, let us look at that through this phase matching condition. So, let us say closely look at the interface.

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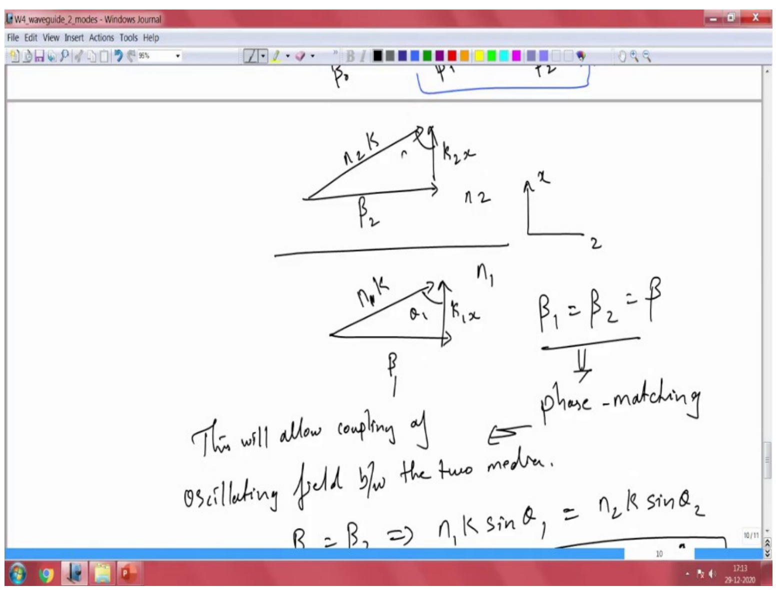

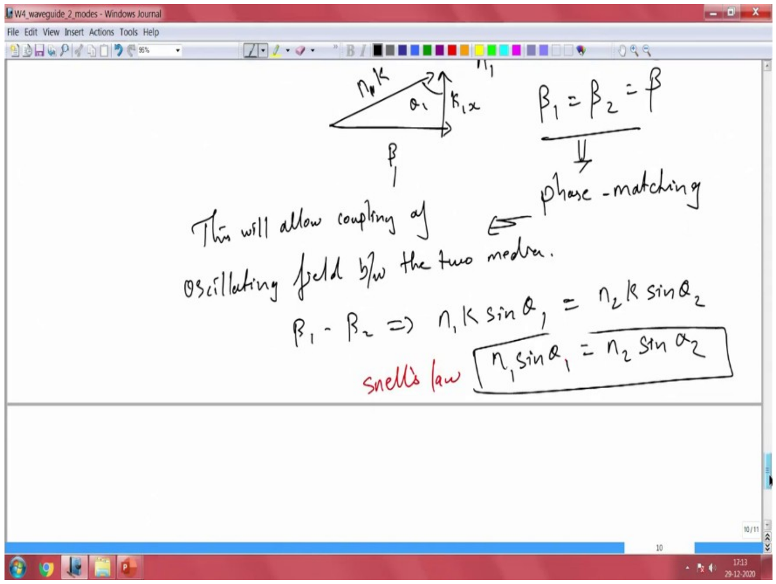

So, let us take the interface just single interface, $n_1$and n_2, let us put our k space k vector triangle here, so beta is here, let me make it a little bigger, so this is beta and here also we will have beta, so to make it specific we will put β_2 here and β_1, let me pull this little down, so this isn_1, so we have β_2 and β_1.

The next thing is we have the vertical vector that is k_{1x} and here k_{2x} and then we have the propagation constant here $n_1 k$ and here $n_2 k$. So, this is this is rather straightforward, so we can easily understand why we have done this. So, in the absence of one or the other it is going to remain the same. So, now the phase change when it propagates through $n_1$or n_2 is going to be different, so how do we make sure that the light actually continuous to flow?

And this is where we need to make sure the propagation constant in β_1, the propagation constant in $n_1$and $n_2$are equal, that means $\beta_1=\beta_2$. So, from both on n_1 side and n_2 side the waves should propagate with same velocity. So, $\beta_1=\beta_2=\beta$, this condition is known as phase matching at this interface.

So, this phase matching, so what is the purpose of this phase matching? This will allow coupling, coupling of oscillation field, between the two media. So, we want to make sure that the light is confined inside one medium in this case n_1. So, we need to make sure that the propagation constant in n_1 which is core and the one at the cladding here that is β_2, so these 2 should be equal only then we can propagate to light here.

So, quickly looking at this β_1 and β_2 let us equate this. So, what is β_1? So, β_1 should be equal to β_2 this is what we said. So, what is β_1? β_1 is nothing but $n_1 k \sin\theta_1$, so $n_1 k \sin\theta_1$, is our β_1,. What is β_2? $n_2 k \sin\theta_2$, is β_2. So, now this is really excellent, if you look at this equation so you can take out k, so $n_1 \sin\theta_1 = n_2 \sin\theta_2$, what is this? This is nothing but our Snell's law. So, Snell's law is back here.

So, in a completely different way of looking at this thing, so we did not use geometric optics that is what we normally use when we are understanding Snell's law for reflection and refraction, in this case we did not do that we actually used wave propagation and propagation vector is here, so even with that understanding we are able to, I think one thing I missed here is this this is θ_1 and this is θ_2 let me just complete that is and that is why this θ_1 and θ is coming from.

So, it is quite amazing that you can arrive at this fundamental concept from whichever direction you come from, so that is reassuring that whatever you have learned as basic can be arrived but even more complex base, but giving you very simple solutions. So, we have achieved this phase matching let us say right and what will happen to this condition if the waves are not at the angle that you want?

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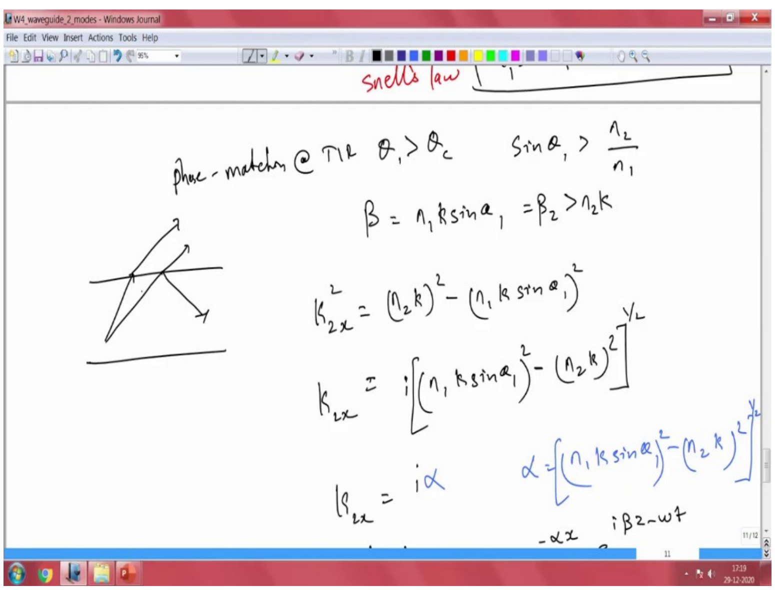

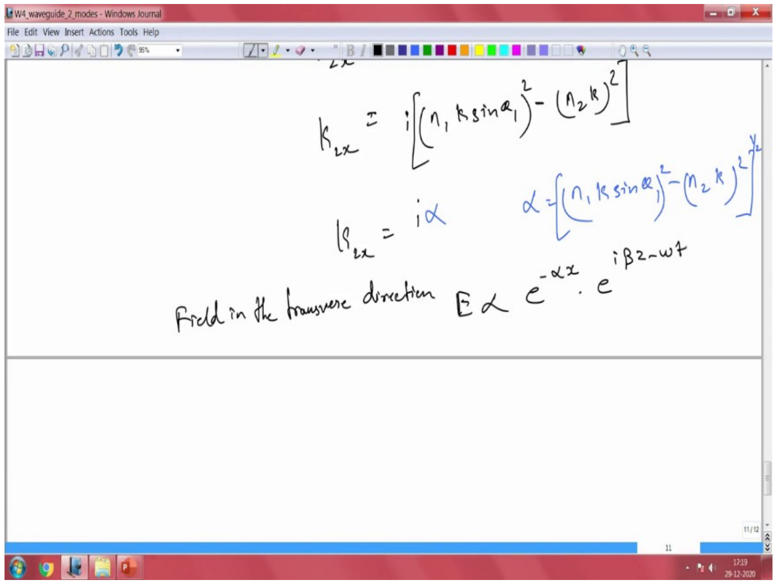

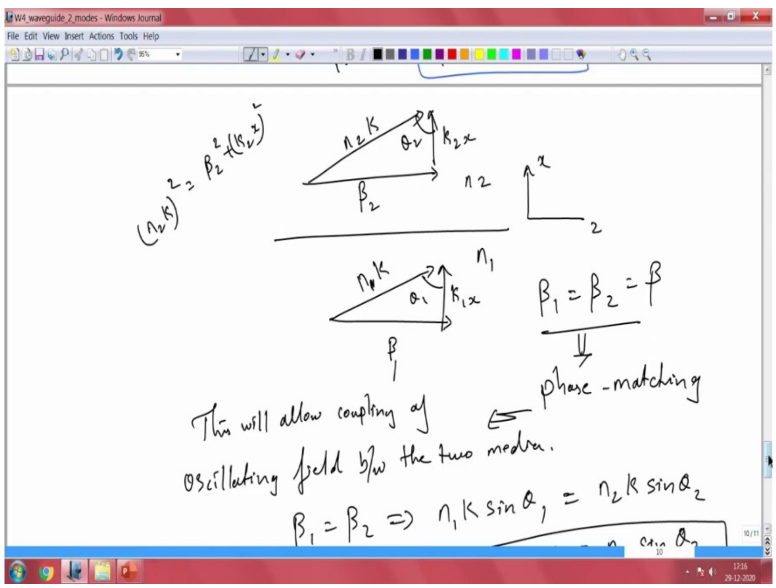

So, there is a critical angle that you are looking for, so that is your total internal reflection angle, let us say your phase matching at a total internal reflection angle, let us say some angle which is greater than the critical angle. In this case you are $\sin\theta_1 > \frac{n_2}{n_1}$, in this case you are beta is nothing but $n_1 k \sin\theta_1 = \beta_2 > n_2 k$.

So, let us look at how that the vector looks like in our simple k space, so there $k_{2x}$is what we are looking at. So, that is what we already saw, so $k_{2x} = (n_2 k)^2 - (n_1 k \sin\theta_1)^2$. So, this is beta, so this is $n_2 k$ I did not do much here you can go back and have a look at this, so this basically this is what you are finding k_{2x}, this is nothing but $(n_2 k_2)^2 - \beta^2$.

So, this is something that you can do, this is just simple Pythagoras Theorem, where you say $\beta^2 + {k_{2x}}^2 = (n_2 k)^2$. If you want me to write we can write it here, so be $(\beta_2)^2 + {k_{2x}}^2 = (n_2 k)^2$ the whole square. So, this is simple Pythagoras, so we are taking that and modifying it a little bit, moving it the side, this is what you get.

So, now the question here is you know your $\beta_2 > n_2 k$, would not be equal to the propagation constant you have here. So, this is going to be higher, so your $k_{2x} = (n_2 k)^2 - (n_1 k \sin\theta_1)^2$ will be higher or we can do some rearrangement here that means $i\sqrt{(n_1 k \sin\theta_1)^2 - (n_2 k)^2}$, I do not know why I am saying this i, so if you do this, i within roots now and this is k_{2x} .

So, now you can see that this k_{2x} the constant along x direction is imaginary now, so because this is imaginary you are going to have I can represent this whole constant let us say as alpha, so here alpha is nothing but $\alpha = \sqrt{(n_1 k \sin\theta_1)^2 - (n_2 k)^2}$, so this is what we have as alpha. So, one can represent this $k_{2x} as\, i\alpha$. So, now how is the field going to look like along the x-axis here?

So, you want to know how the field is going to look going beyond and that is given by the field, the field in the transverse direction, so now the field in the transverse direction well look something $E \propto e^{-\alpha x} e^{-i\beta z - \omega t}$. So, now you can see here the propagation that you have when $\theta > \theta_c$, so what we mean by that is this is the critical angle, so anything below the critical angle you are fine, but if your angle is too steep, what will happen to that particular field.

So, that field the angle beyond which you are going to lose that light and that that decay, how fast it would decay is something that is given through this simple relation. So, this is from our simple you know k vector representation and if you just simply do reorganization here because of the magnitude of $n_1 k \sin\theta_1 \wedge n_2 k$. So, because of that magnitude you will eventually end up with an imaginary k, the propagation vector will be imaginary. So, what that means is it is going to be attenuated. So, this alpha that we talked about is nothing but the attenuation and this attenuation is what we call evanescent field.

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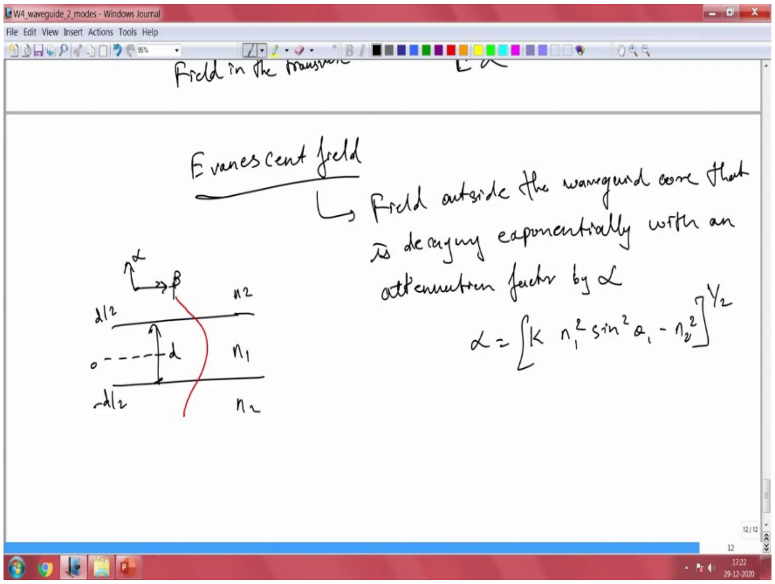

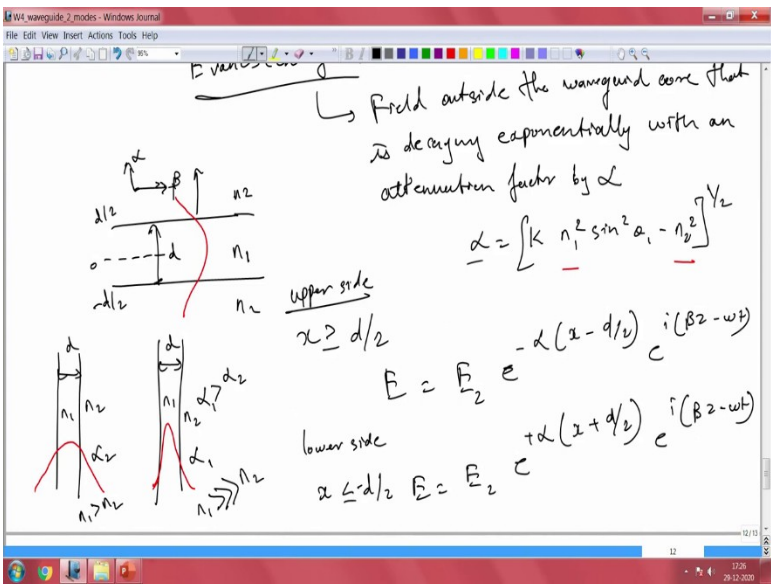

So, this field that is dying outside is called evanescent field. So, evanescent field is field outside the waveguide core that is decaying exponentially with an attenuation coefficient, coefficient are attenuation factor given by alpha $\alpha = \sqrt{(n_1 k \sin\theta_1)^2 - (n_2 k)^2}$.

So, this is actually the attenuation coefficient, so if you really want to know how quickly the attenuation happens again we can do it here, so let us say n_1 and n_2 both of both the directions and this is 0 and we know this is d and this is $\frac{d}{2}$ and $\frac{-d}{2}$ let us say the field we saw that it is going to be like this. So, here we have beta and here we have alpha, so beta is the propagation constant that keeps the wave going through, alpha is the attenuation factor, so how quickly the wave is going to decay.

So, let us look at how this will happen, so when $x \geq \frac{d}{2}$ and the upper side your field is nothing but $E = E_2 e^{-\alpha\left(x - \frac{d}{2}\right)} e^{i(\beta z - \omega t)}$. Similarly, for lower side where we have $x \leq -\frac{d}{2}$ your $E = E_2 e^{\alpha\left(x + \frac{d}{2}\right)} e^{i(\beta z - \omega t)}$.

So, this is how you will have d electric field decay when it is outside the core of this waveguide. So, this is how you can confine the light. So, we talk about wave guides can be designed to confine light and propagate light this is how we achieve that confinement. So, in n_1 you have solution that is oscillating, but then when it goes outside in n_2 it will decay and at what rate it decay? Is this alpha.

So, you can look at the constituents of this alpha, so it strongly depends on the refractive index of the core medium and the cladding medium. So, if you want your wave to be highly confined, so let me draw that, so there are two waveguides, let us say, they are all of same thickness then one waveguide you have a field like this and for another wave guide you have a field like this. So, remember that your dimension is identical, but then one is having this and one is having that.

So, now the question to ask is the decay alpha here, so let me call this as $\alpha_1 > \alpha_2$, so which one is greater? So, which one is decaying fast? Of course alpha, so $\alpha_1 > \alpha_2$. So, how did we achieve this? So, this is achieved by at the refractive index contrast, so n_1 and n_2 we can have refractive index n_1 and n_2 here, so when the refractive index n_1 is much much greater than n_2 you get very high confinement that means you have very strong decay. So, the light is not allowed to go out, so it is pushed inside but in this case and n_1 is just greater than n_2, it just meets the criteria.

So, based on the refractive index contrast one can actually decay the light the evanescent field that you have and in some cases you may want to have this evanescent field stretch longer, we will look at this in our application, in some application we do want this evanescent field to stretch out and we can engineer this decay rate or evanescent field by looking at the refractive index contrast. So, let us look at another factor we define, so we are talking about refractive index n_1 and n_2 and so on. With the waveguide got its own refractive index we call it as waveguide effective index.

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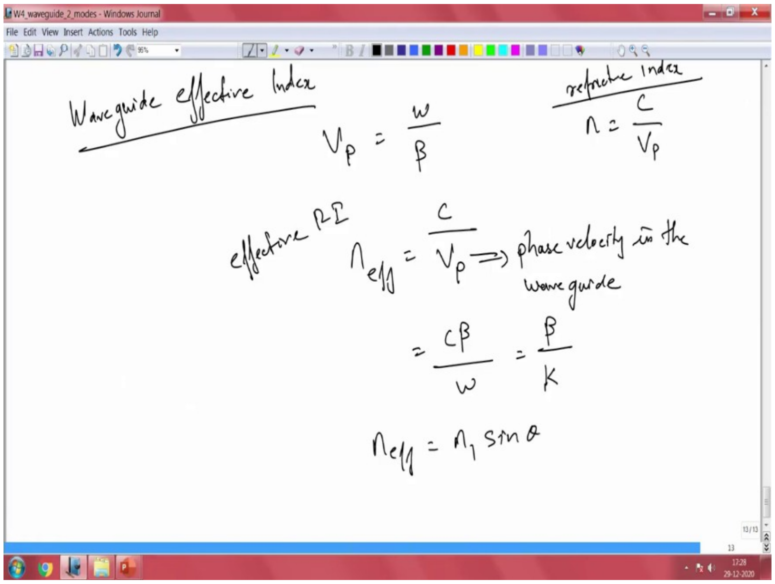

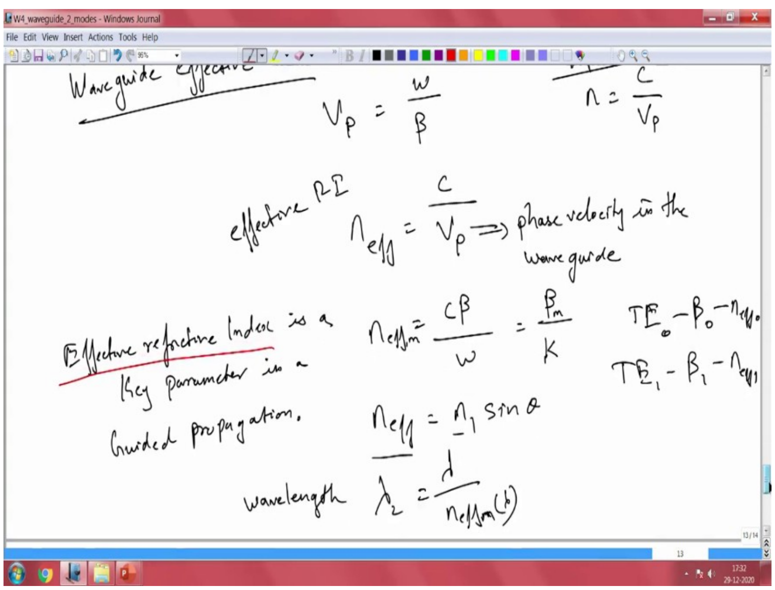

So, let us start from our understanding phase velocity, we know the waveguide phase velocity, so phase velocity is given by $\frac{\omega}{\beta}$, so this is the phase velocity. So, now if we say the refractive index, so what is refractive index? So, refractive index is nothing but $\frac{c}{v}$, so we V_p in this case. So, this is refractive index, so this is refractive index. But now what is effective refractive index?

So, effective refractive index is nothing, but n effective is nothing but $\frac{c}{V_p}$. So, here V_p is the phase velocity in the waveguide, so in other words, let us try to expand this because this looks too trivial that there is something to this, so this phase velocity is given by this, so your V_p is nothing but $\frac{c\omega}{\beta}$, so beta is the one that dictates this in other words it says $\frac{\beta}{k}$ and you can also write this in a more simplified form $n_1 \sin\theta$. So, this is the effective refractive index.

So, effective refractive index is a key parameter in a guided propagation and refractive index that we talked about n_1 n_2 those are all refractive index in an iron guided session, so n effective is actually applicable to propagating mediums or guided medium, so when you talk about any kind of waveguide that it be slab or channel you would actually ask for effective index of that medium.

Since this is beta we also saw that this beta is actually β_m depends on the mode number and if that is the case n effective should also be a function of the mode. So, a TE mode TE_0 will have β_0 and it will have n_{eff0} and TE_1 mode will have $\beta_1$and then n_{eff1}, so each mode will have its own effective refractive index here. And it will the wavelength inside this waveguide is also determined by this n_{eff}.

So, now the wavelength that we measure along z direction, so now the wavelength is now lambda here along z is given by lambda over n effective, so now with this effective refractive index the wavelength would also change and now this is also a function of mode that is m here, but again we know that your lambda and you are refractive index, so the lambda is also function of whether it is free space or in medium here, but this n effective is related to n and it is and that is one of the reason why this ineffective is also a function of lambda.

So, all this is related to the dispersion that we already discussed quite in detail earlier on. So, effective refractive index is something key to our understanding which we use to capture a lot of things here, so we are with effective index you are capturing the material property the n the refractive index or the bulk refractive index of the medium, we are capturing the lambda, the wavelength of light through k, so it combines the physical properties and also properties of light that is propagating through this medium.

So, effective refractive index is an important parameter that we use in quantifying light propagation in wave guides. So, with that we have come to end of this waveguide understanding so we know how the light is propagating through and how we can understand the confinement of light and how the interface is and the conditions are affecting the light propagation.

So, in the following lecture we will look at the actual field profile, so we will look at the field equation and then we have we have drawn the Gaussian profile and also the first order mode and second order mode, but we will actually look at the Maxwell equation and the wave equation to look at how this mode shapes evolve. So, thank you very much for listening.