Photonic Integrated Circuit
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Lecture 15
Waveguide Modes 1

Hello, everyone. So, let us look at modes in these waveguides that we discussed earlier. So, waveguide structures are two-dimensional when you look at the cross-section, and they are three-dimensional as you progress through. But most of our discussions are going to be two-dimensional. So, it makes life easy in understanding the other direction as just extrusion.

But then we need to understand how our electric fields propagate through this two-dimensional structure that we are currently looking at. So, we characterize the energy flow through these structures using something called modes. So, these are all nothing but solutions to Maxwell equation. So, you take the structure and solve it with your Maxwell equation, then you will arrive at these solutions. So these solutions are nothing but modes. So we will look at this wave equation little later.

But let us look at how the electric field is going to appear, and then how these fields are going to propagate through our basic understanding even from Ray theory. So, we can take some help from how rays are propagating and let us implement that and understand how light can propagate through this structure using very simplistic model, but, you know, right one. So simplistic model should not be too simple that you leave off the details. Here you are going to have simple discussion but then accurate as well. So, let us look at these waveguide modes.

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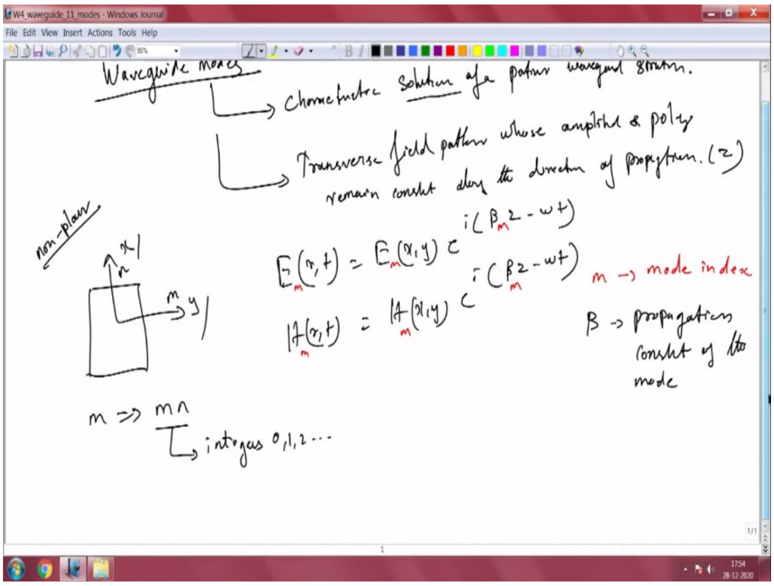

So, what are waveguide modes? These are all nothing but characteristics of a particular waveguide. So this is a characteristic solution of a particular waveguide structure. And this wave guide modes is a transverse field pattern. So, it is nothing but transverse field pattern, electric or magnetic field whose amplitude and polarization, this is very important, polarization remain constant along the direction of propagation. So, let us say along the Z.

So now you can actually write very simple electric and magnetic field of a very simple mode, let us say E(r,t) r is a function of position. One can write it as $E(r,t)=E(x,y)e^{i(\beta_x z-\omega t)}$. So, this is all from our earlier discussion. This is nothing new that we are introducing here. So, let us just keep it $E(r,t)=E(x,y)e^{i(\beta z-\omega t)}$ $H(r,t)=H(x,y)e^{i(\beta z-\omega t)}$. So this is how, you know, the electric and magnetic field of any mode could be written. you can add a number to this. So, there I said these are all solutions.

So, there are characteristic solutions. So, you could have more than one solution. So, if there are more than one solution, we can also have what is called a mode index. So you could define, let us say, m here. So we can put some m, and this m is nothing but mode index. And β_m is nothing but your propagation constant. This is propagation constant of the mode. So, there is nothing new. So, this is something that we recap on this. So, for a two-dimensional waveguide. So you will have transverse optical confinement. And there are two degrees of freedom we have, over the x and y plane.

So, your mode number V con sist of then two parameters. So, one in x and one in y. So the way to understand this is you have x and y. So you could have two degrees of freedom is what you have. So, then the mode index that we defined as m here. There will be two, so one along x and one along y. So then your m is not just a single index. So it will be double. So you have to put mn, let us say.

So the ideal way of doing this is, you know, your m would become m and n. Let us say m is along y direction and n is along x direction. So these are all number of solutions you could have along x and number of solutions you have along y. So this is this is something that we would do it and this m and n are nothing but these are all integers. So integral, so these are all integers like 0, 1, 2, and so on. These are all discrete numbers. There is you know.

So planar waveguide for example. So they do not have this y. So this is a non-planar. So let us, if you look at the planar. So in the y direction, it is rather, it is infinite. So you have complete infinite width, let us say. So there is nothing there. So you do not have, your mode field does not depend on the y-coordinate if it is a planar. So, that is something that you should also remember here. So that is something to remember.

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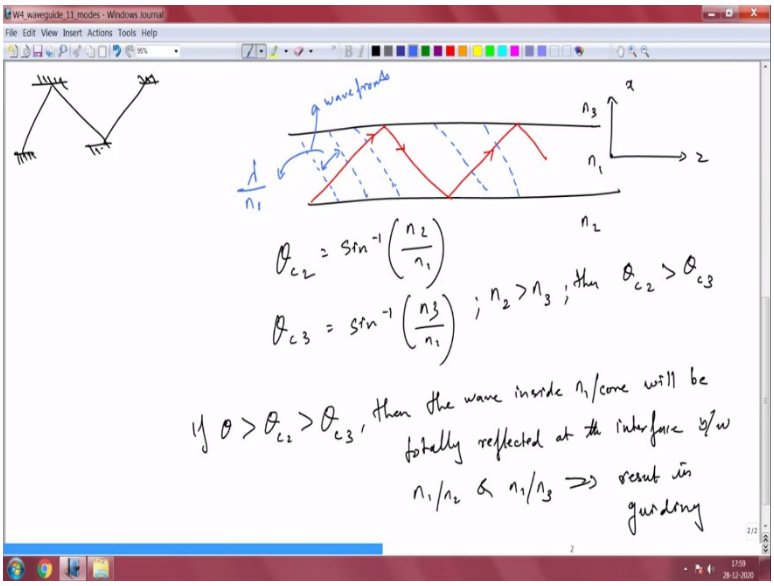

Let us move on to looking at the general idea of this wave propagation. So we have a very simple a step index waveguide, let us say. A thin slab, let us start from the slab. How you could propagate through this? So you have propagation along z, this is x and this is y. So light is propagating along z. And refractive index here is n_1 and there is n_2 and for general case, let us put it as n_3. So this n_3 could be equal to n_2. So that is a special case.

Let us keep that n_2 and n_3 are two different. So now I want to propagate light through this. This is something even in the introduction class I actually told you guys, how to make light propagate. It can be also done through this kind of mirrors. So I can put mirrors and mirror will reflect light. So we want a similar kind of scenario here. So what we need? The light should bounce back and forth. It should not go out.

So I want lights to go up and reflect back and reflect back again, and so on. So I want the light to propagate in this fashion continuously. So how do we achieve this? So we need to achieve this by looking at the critical angle, you remember total internal reflection? So you need to have total internal reflection here and total internal reflection in this surface. So you want total internal reflection between n_1 and n_3, and then you need to have total internal reflection between n_1 and n_2.

So let us look at how to achieve that. So for that we need to know the critical angle. So there are two critical angles here. So critical angle with respect to n_1 and n_2 that means $\sin^{-1}\frac{n_2}{n_1}$ will give you critical angle with respect to n_2. The critical angle with respect to n_1 is given by $\sin^{-1}\frac{n_3}{n_1}$. So now you know because $n_2 > n_1$ let us say. So let us take this as the condition. Then your θ_{c2} will be greater than theta θ_{c3}. So this is something that.

So how do we make sure that we achieve the critical angle here? If $\theta > \theta_{c2} > \theta_{c3}$, then the wave inside n_1 or core will be totally reflected at the interface between n_1, n_2, and n_1, n_3. And this result in guiding. So this is how we can start guiding right now. So I hope this is this is clear. The other thing to notice is, you know, this is a ray that is propagating. you have wavefronts following this.

So there are wavefronts following this. So that is something that you should keep in mind, which we will use it later. So these are all nothing but wavefronts and this wavefront, distance between two wavefronts is nothing but your Lambda. So this is nothing but $\frac{\lambda}{n_1}$. You have, you should not forget about the medium you are propagating through. So $\frac{\lambda}{n_1}$ is the wavelength inside the medium n_1.

So, the light is going to get reflected back and forth, these two interfaces. So the guided mode can exist, so you only when a transverse resonance is achieved. So that is what we mean by reflecting back and forth. So reflecting back and forth between the surface is nothing but creating this resonance between the two layers, the top, and the bottom layer. So let us, you know, reassure ourselves of what we mean by the guided mode.

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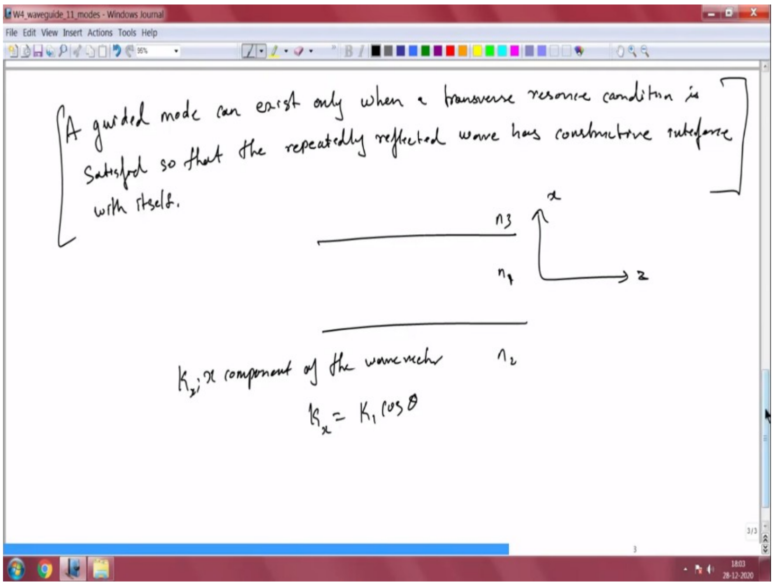

A guided mode a guided mode can only, can exist only when a transverse resonance condition is satisfied, so that the repeatedly reflected wave has constructive interference with itself. So this is a important definition of a guided mode. So the modes are nothing but solutions. But then, what is a guided mode? So the guided mode can only exist when a transverse resonance condition is satisfied. That means the light is bouncing back and forth. So between the two layers here repeatedly and has a constructive interference with itself.

So light has to exist as it propagates through, so it should not die down. The only way that one can assure that is by having an constructive interference because you see here the light is going back and forth. So your wavefronts are also moving. So you should have constructive interference between these wavefronts in order to keep the energy flowing forward. Otherwise, you will have leakage and you will have loss of light.

So there will not be any guiding anymore. So let us look at, wave vectors that are that one can define as it propagates through. Let us let us let us make a new one here. So you took a thin slab and this propagation direction. So we had n_1, n_3, and n_2. So the vertical component here. so the k_x component, so you can decompose this into k_x and k_y component. Here k_x. That is your x component of the wave vector. So $k_x = k_1 \cos\theta$.

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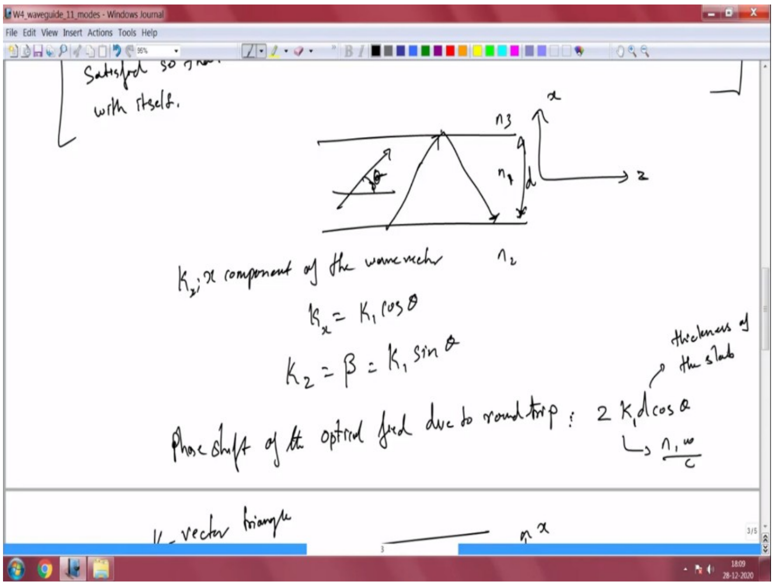

So now the, at an incident angle theta here. So we are talking about what angle it is going to make so that angle here is given. So now what is the component along z direction. So k_z, or we call that as propagation constant $k_z = \beta = k_1 \sin\theta$. So now the wave is bouncing back and forth. So it is going this way. So then there will be a phase shift in this optical wave due to this round trip.

So when we travel from one position A to position B, we all know that light will accumulate a certain phase and this phase depends on the refractive index of the medium, the wavelength to, and then the distance between this. So what is that. Let us look at the phase shift. The phase shift of the optical field that is, this is due to round trip between the two interfaces. So that is nothing but $kd\cos\theta$. So this is $k_1 d\cos\theta$. So this is this is very simple phase shift relation that we have seen earlier. So now let us expand our understanding of this propagation by using a K-vector triangle, which is which is not very difficult to reach out. But let us let us try to do a K-vector.

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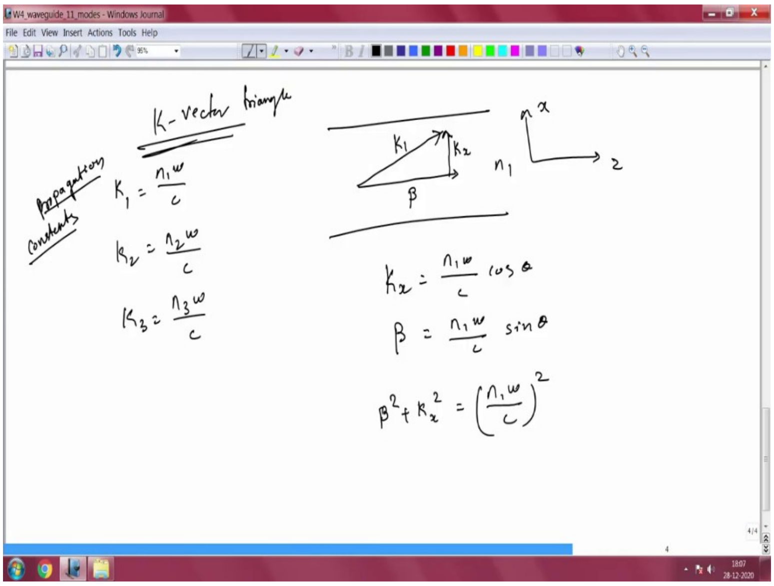

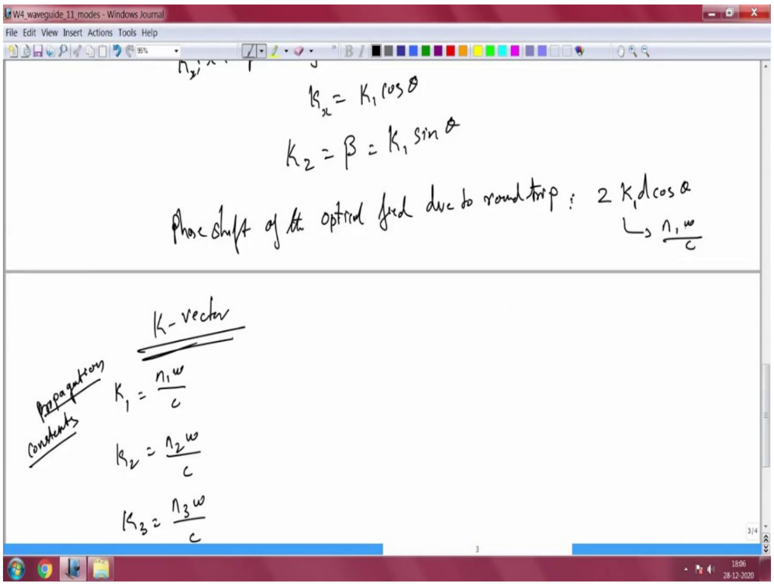

So before doing that let us write all our, you know, propagation constants k_1. $k_1=\frac{n_1\omega}{c}$, $k_2=\frac{n_2\omega}{c}$, and $k_3=\frac{n_3\omega}{c}$. So this is all propagation constants of n_1 and n_2 and n_3 layers. So this $k_1=\frac{n_1\omega}{c}$. So that is something that you can take it. So let us now look at the components here. So, we have, again let us draw this simple propagation here.

So this is n_1. So light is propagating along this direction with a propagation constant beta. And it has propagation constant k_x, the vertical component of the propagation constant k_x, and this is represented by your k_1. So this is where this is where light is propagating. So now so the $k_x = \frac{n_1 \omega}{c} \cos\theta$ which is something we already saw, that is $k_1 \cos\theta$. And $\beta = \frac{n_1 \omega}{c} \sin\theta$.

So now we can easily write using Pythagoras what is the dimension of this will be. So this is very simple K-vector triangle, you can call it K-vector triangle if you want, how the orthogonal components of your propagation constant beta and k_x are related. So this will be very useful further on when you are designing any wave guide, what should be the magnitude of k_x. And what is the magnitude of beta based on k_1 that you have in the propagation. So let us decompose this into their components and dig a little bit more and understand what this k_x and beta look like.

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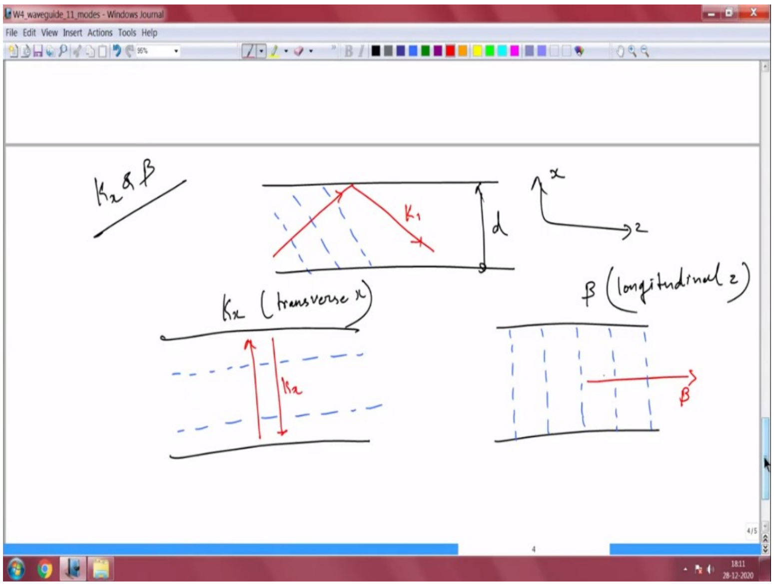

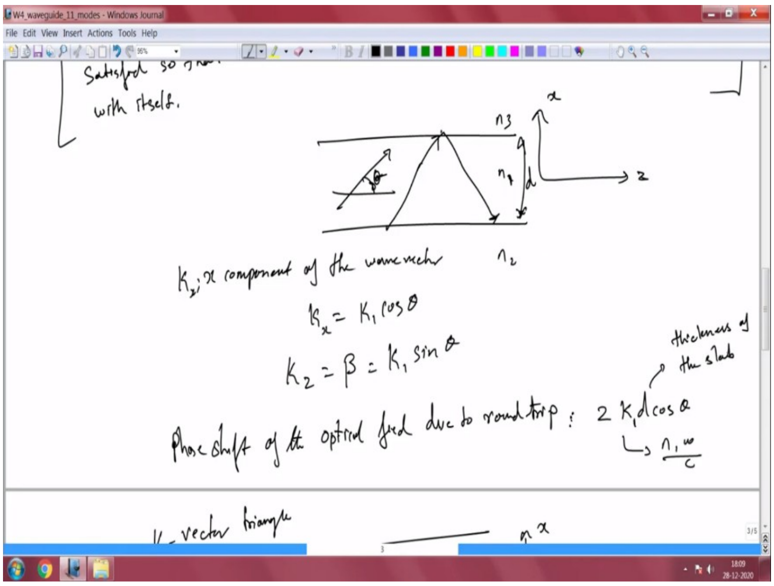

So let us say $k_x$and beta, let us look at that. So the wave was propagating along z like this, let me take the other color because, we, that is what we have done earlier. So this is basically k_1. So this is k_1 and the wavefronts we saw, like this and it had a certain distance. The thickness is d, perhaps if it was not clear. So this is nothing but d. So d is the thickness of the slab. So now they are moving. So now we have decomposed this into k_x and beta. So let us look at how beta and k_x look like. The first thing is is looking at k_x component.

So k_x component is basically vertical component here. So this is what k_x looks like. It is a vertical resonance, so the wave is moving up and down. Along the x direction here. So now let us look at beta. So now beta is given in this direction. So this is beta. So this is k_x and this is beta. This is beta. And now beta is defined along the propagation direction. So now our beta wavefronts are located this way. So as they propagate. So now we can clearly see that you can decompose or propagating zig-zag wave.

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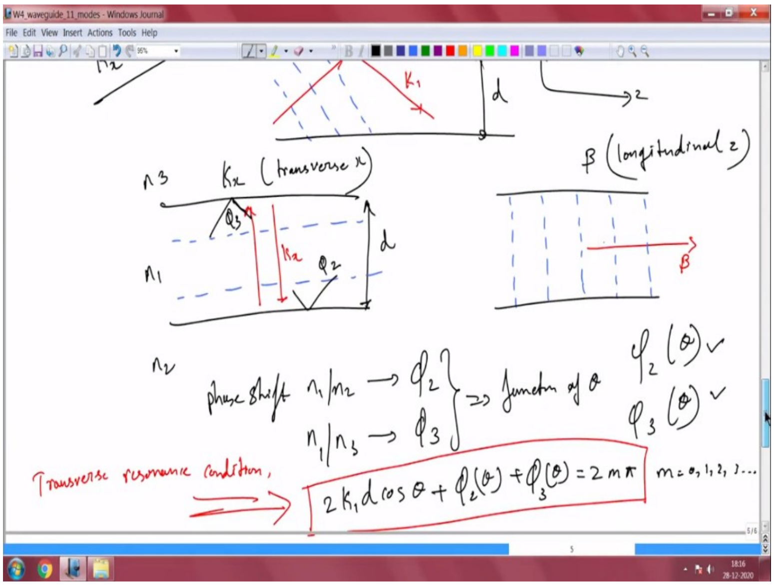

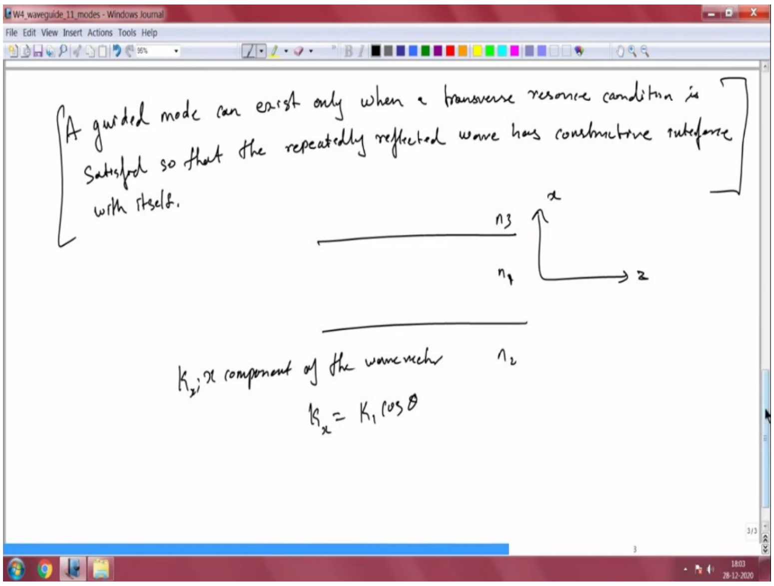

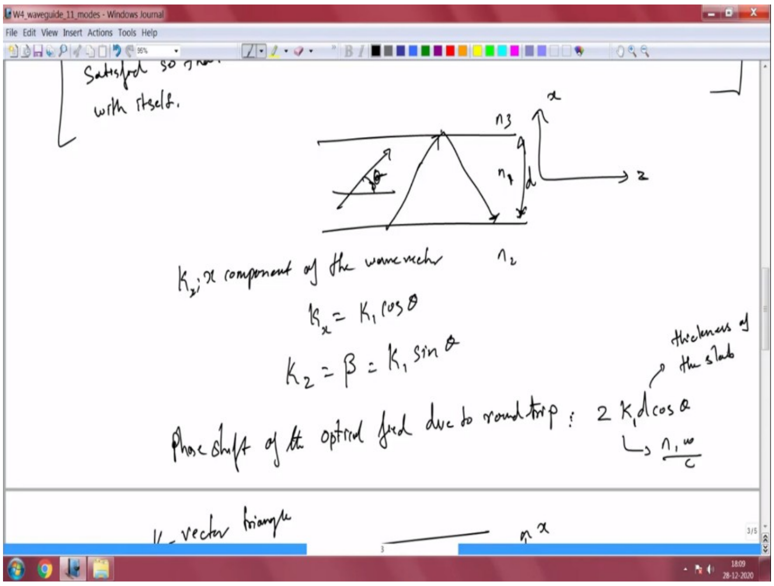

So zig-zag wave that it is propagating could be decomposed into two orthogonal components. So the longitudinal component and the transverse component. So this is the longitudinal component along z and this is the transverse component. So the transverse components are going to go up and down. So this is this is what we mean by constructive interference. So it is like a resonant cavity. They are going to move up and down, creating a constructive interference.

So I go back to the definition we had here for a guided mode. So the guided mode can exist only, it can only exist when there is a transverse resonant condition. And they should be constructively interfered. So there should be a resonance and they should constructively interfere. So this is this is the condition again we come back to this, based on our understanding so far. So how do we do this resonance condition? How do we know that a wave is in resonant condition? We can easily derive this resonant condition.

So let us look at the phase shift because whenever you reflect off the surface, you will have a certain phase shift, let us say. So you have n_1, n_2, and n_3, let us say. So you have reflection now. So there is a phase shift when you go from n_1 to n_2, and we call that phase shift φ_2, let us say. And the phase shift between n_1 and n_3, we can that φ_3, let us say. So now we can easily take this phase shift from our simple reflection understanding. So for TE wave and TM wave, or in other words S-polarization and P-polarization reflection of a surface.

So that that is something that we already studied long time ago, we understand that. So this φ_2 and φ_3, so this reflection th at we have off here and off here, so this is φ_3 and you say φ_2. So these two are a function of theta. So that means $\varphi_2(\theta)$ and then, sorry, $\varphi_2(\theta)$ and $\varphi_3(\theta)$. So what angle it comes in to create the constructive interference.

So now if we consider the phase shift, the two phase shifts we have, and then we also know because it is traveling a certain distance d. We know what is that phase difference is. So, if we put all these things together, we will get the resonance condition. So let us look at that, how we can come up with that condition.

So φ_2, φ_3 as a function of theta and what is the phase shift because of this thickness d, this is something we saw earlier. So the phase shift of the optical field due to the round trip, so that is $2k_1 d\cos\theta$. So that is something we already know. So $2k_1 d\cos\theta$ is because of the round trip, the distance that it is travel. And the other phase shift is because of the interface between 1 and 2.

The next one is between interface 1 and 3. So all these phase shift put together should be equal to $2k_1 d\cos\theta+\varphi_2(\theta)+\varphi_3(\theta)=2m\pi$ for the constructive interference. So this is your resonance condition now. So this is the transverse resonance condition. And what is m here? So m is a integer. So m will be 0, 1, 2, 3, and so on. So only integral values will go and that means there are certain, only certain values of theta can satisfy this transverse condition. So not any angle. So that is the solution that we saw earlier.

So, with that, we have come to a conclusion here or at least a summary of what we discussed so far. We started off with a with a understanding of the field, how one could have a solution and how one could define the electric field, both in x and y direction and then we looked at what is the condition. The critical angle that is required to have total internal reflection to keep the light propagating through the waveguide and we also looked at the phase shift that happens because of this round trip that you have.

And we defined the guided mode, what is the guided mode here. And that is nothing but a resonance that we get in the x direction, defined by the k_x here and we also saw what are all the phase shifts that has going to be there because of the reflection and in addition to the round trip phase shift and the interface phase shift you will get a resonant condition that is represented here.

So with this is condition we should know whether a particular injection at a certain angle is going to give us a propagating mode or not. So with this, we are having some understanding of the requirements coming from the resonance condition in a x direction while you have a clean propagation along z-direction defined by beta. So in the following lecture we will look at how we can elaborate this to electrical and magnetic field and also we will look at the wave equations for this. Thank you very much.