Photonic Integrated Circuit
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Lecture 12
Polarization in Anisotropic Medium

Hello everyone. So, let us look at how to use the anisotropy in the material to manipulate light propagation. So, first of all we need to firm up our understanding of this optical property as well. So, in the earlier lecture we saw how the dielectric tensor and susceptibility is going to affect the material system.

So, now let us move a little bit more towards the macroscopic understanding on how the refractive index of the material is going to change and this is going to be directional. So, that means the light that putting inside the system will interact with either the x coordinate or the y coordinate or x, y plane let us say if z is your propagation direction it will interact with these different refractive indices.

So, when it interacts with this different difference in the refractive index the propagation will be affected. In isotropic medium, you do not see this effect, because when you have an electric filed let us say E_x and E_y are its components both E_x and E_y will face the same resistance or same dielectric constant then they are going to travel at same speed but now the scenario has changed. So, x and y they have different dielectric constant. So, that will result in change in your refractive index as well. So, when the refractive index changes we know the speed at which the wave is going to propagate will change. So, this is a single wave by the way.

So, this is a single electric field but then when it is moving through the system you can decompose it to x and y, the x component will move relatively faster or slower to your y component what will happen in that scenario? So, what property of light is going to change? Can we do interesting functionalities using this anisotropic propagation is what we are going to see now. So, let us understand the axis, principle axes and optical axes first.

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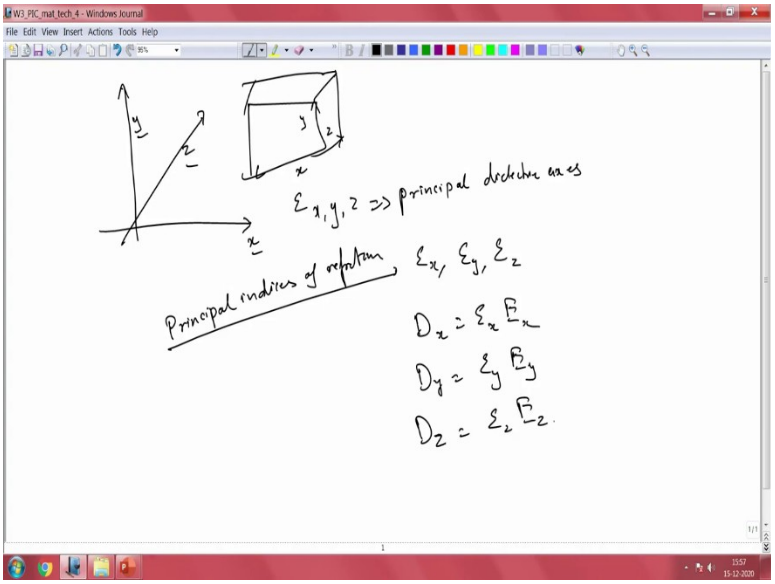

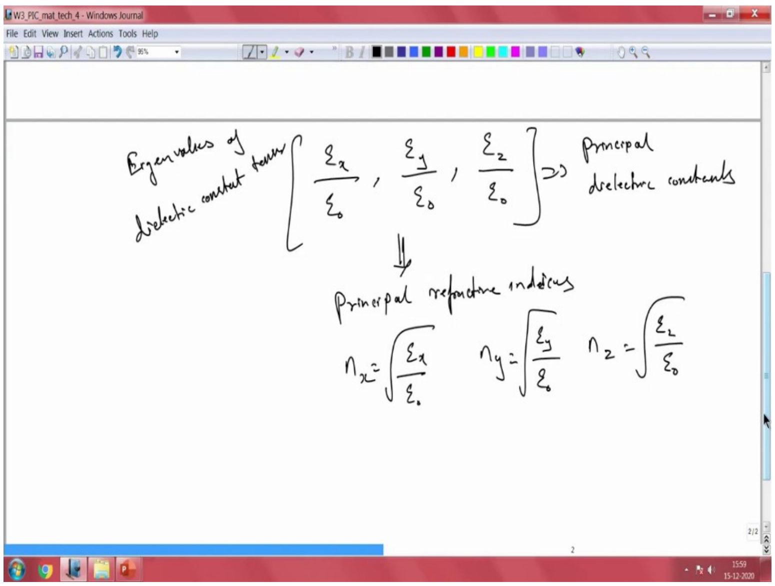

So, if you if we take a material system, we can define the coordinate system something like this so x, y and z. So, if it is isotropic when you rotate this we will not have any problems but then if it is an anisotropic material then you have to be very careful about rotation and the assignment of various axes here. So, this x, y and z that we have define, this will be a unique set of coordinates. So, it cannot be a random.

So, we need to identify a certain coordinate system. So, that you define this for a particular material. So, if you take particular material system you define what is z, what is y and what is x. So, we need to do that and when we do that then we can define these x, y and z as principle dielectric axis or simply principle axes if it is just x, y, z. So, if epsilon x, y, z is called principle dielectric axes or in general principle axes for a crystal and now once you define this let us look at the principle refraction. So, index of refraction.

So, that principle indices of refraction. So, we start with our system here. So, epsilon x, epsilon y and epsilon z. So, this is all our dielectric constants, a permittivity across the different direction. So, we can write now the displacement field simply through this. So, now we can also characterize this epsilon x, y and z as a tensor. So, we could write this as epsilon x over epsilon that is a relative permittivity and epsilon z by epsilon naught. So, these are all the Eigen values of the dielectric constant tensor.

So, this is nothing but Eigen values of the dielectric constant tensor and these are all called principle dielectric constants, this is called dielectric constants. So, the moment you define these principle dielectric constants you can clearly easily define principle refractive indices. So, that is given by n_x, n_y and n_z. So, it is nothing but epsilon naught sorry $n_x = \frac{\varepsilon_x}{\varepsilon_o}$ and here $n_y = \frac{\varepsilon_y}{\varepsilon_o}$ and here $n_z = \frac{\varepsilon_z}{\varepsilon_o}$. So, now we have three different refractive indices.

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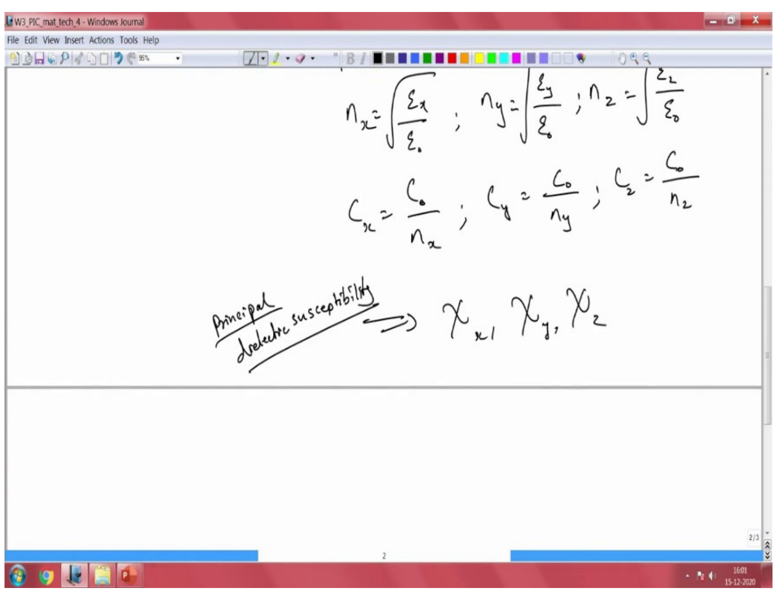

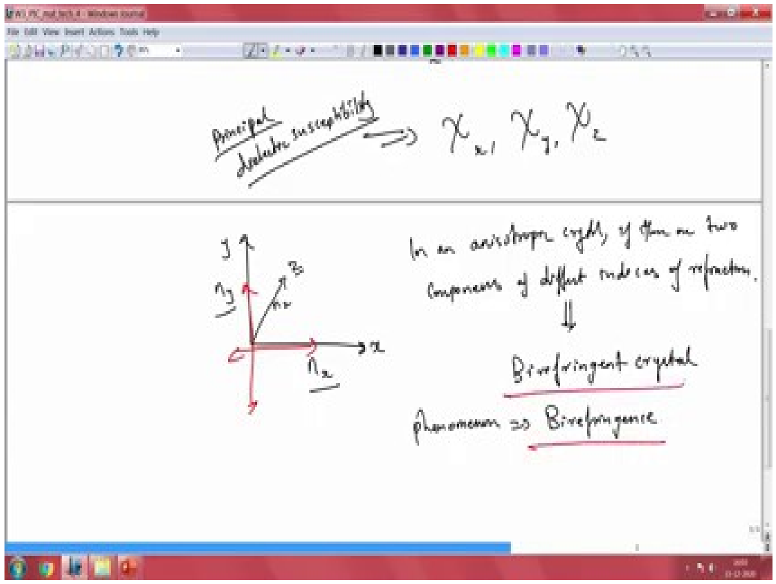

So, we all know that the speed of light is also going to change. So, now it you look at the phase velocity of light is now $c_x = \frac{c_o}{n_x}$. So, now $c_y = \frac{c_o}{n_y}$, sorry Cz is nothing but $c_z = \frac{c_o}{n_z}$. So, you can see here how change in the property as a function of orientation is affecting the refractive index and your light propagation. So, because you have change in the dielectric constant epsilon, so, is your susceptibility as well. So, your epsilon and chi are related anyways. So, now you can define the principle susceptibility.

So, now principle dielectric susceptibility you have defined in as a function of position now. So, this is all turning out to be expanding everything that we understood so far. So, if you have a system that has anisotropy then the material is a very strong interaction with the field that is propagating through to the medium. So, let us look at a very simple case I take a material that has some two coordinates here and then z. So, now this has n_x, you have n_y and this is n_z let us say. So, now for all practical purpose the direction of propagation is z.

So, the refractive index along the direction of propagation is not going to affect my propagation or the electric field now. Now, I have two refractive indexes nx and ny based on the orientation of my electric field the light is going to travel either faster or slower in this principle axes and this would result in system that has two refractive index in one medium, So, that means two refractions in one medium and these phenomena is referred to as birefringence.

So, there are two components. So, you have two refractive indices in a single medium in an and isotropic crystal if there are two components of different indices of refraction we could call this as Birefringent crystal and these phenomena is called birefringence. So, this is very important understanding from an anisotropic crystal. So, when you take an anisotropic crystal and then see there are two different refractive indices and they are going to affect the light in two different ways and you call that as birefringence these phenomena of two different refractive indices. And this particular crystal is called a birefringent crystal.

So, the moment you have this difference in the refractive index as we saw here you could come up with a way of representing this whole refractive index in a very nice simple geometric way. So, let us look at that. So, we call that as defining the index ellipsoid. So, this the index space for all practical purpose. So, let us start off with understanding that index ellipsoid starting from the relative impermeability. So, permittivity we know. So, we start with impermeability that will give us this very nice way of representing space.

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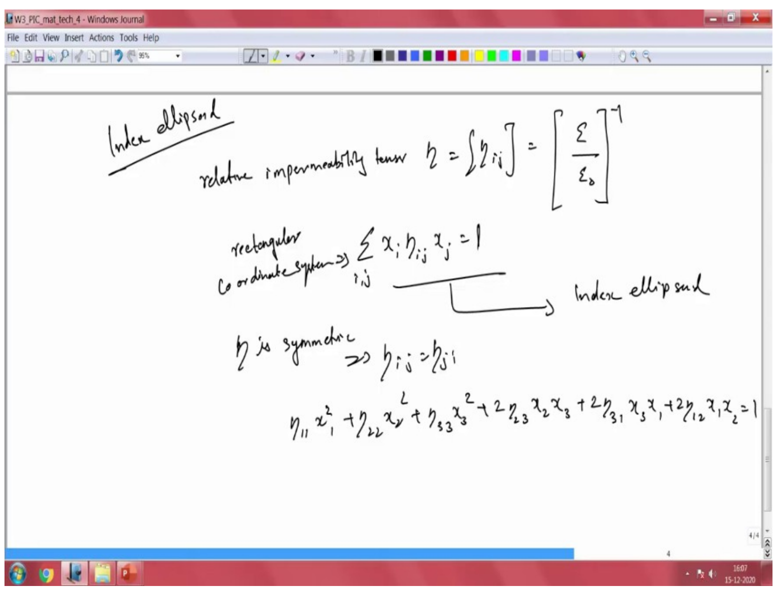

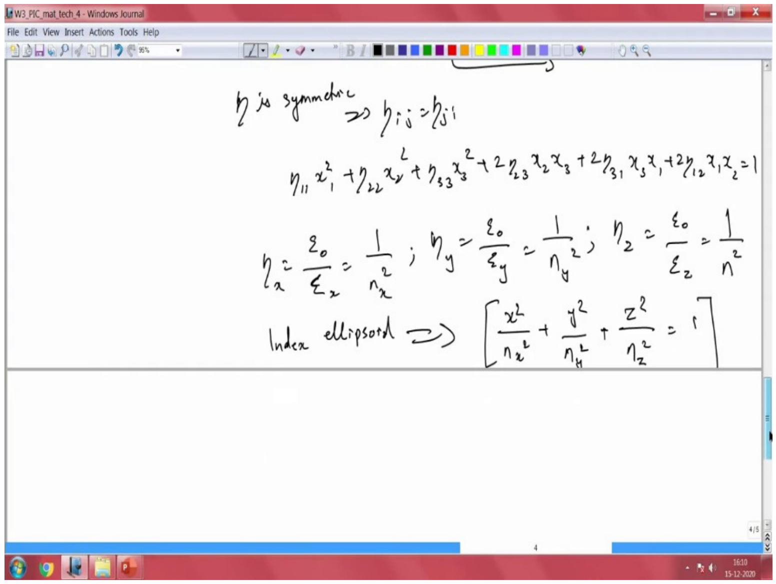

So, we call that as index ellipsoid. So, how we could understand this. So, the relative impermeability tensor $\boldsymbol{\eta}$ can be given as $\boldsymbol{\eta}_{ij}$ is nothing but $\frac{\varepsilon}{\varepsilon_o}^{-1}$. So, now since you are using the coordinate system x, y and z one can write this as $\sum_{ij} x_i \eta_{ij} x_j = 1$.

So, this is in a rectangular coordinate system. So, this is what you have and this is basically your index ellipsoid and why we call this as index ellipsoid let us just look at it little bit carefully. So, this is basically operating a column vector with your 3 by 3 matrix let us say. So, if you do that then what you end up. So, if you take the material is dielectric and non-magnetic in nature.

So, it will be $\boldsymbol{\eta}$ will be symmetric. So, this implies $\eta_{ij} = \eta_{ji}$ and the equation that we had here would become $\eta_{11} x_1^2 + \eta_{22} x_2^2 + \eta_{33} x_3^2 + 2\eta_{23} x_2 x_3 + 2\eta_{31} x_3 x_1 + 2\eta_{12} x_1 x_2 = 1$. So, this is what you would end up with. So, because your epsilon is symmetric in nature. So, your relative impermeability will also be symmetric in nature.

So, you can nicely a summarize it or deduce it into this form. So, the index ellipsoid equation that we have here is invariant with respect to rotation. So, if you change the orientation, if you rotate it at certain angle with respect to position in space then whatever equation you have can still use it. So, its invariant. So, when the coordinate system is rotated. So, now we can look at our refractive index how your permeability is affecting our index of refraction.

So, this basically this is what you have which is equal to $\eta_x = \frac{\varepsilon_o}{\varepsilon_x} = \frac{1}{n^2}$. So, similarly you can apply this to the y which is equal to $\eta_y = \frac{\varepsilon_o}{\varepsilon_y} = \frac{1}{n_y^2}$. So, this is $\eta_x = \frac{\varepsilon_o}{\varepsilon_x} = \frac{1}{n_x^2}$. So, $\eta_z = \frac{\varepsilon_o}{\varepsilon_z} = \frac{1}{n_z^2}$. So, now if the coordinate system is x, y and z then the index ellipsoid will take the form $\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1$. So, this is how you can construct an index ellipsoid or how the refractive index looks like inside the medium.

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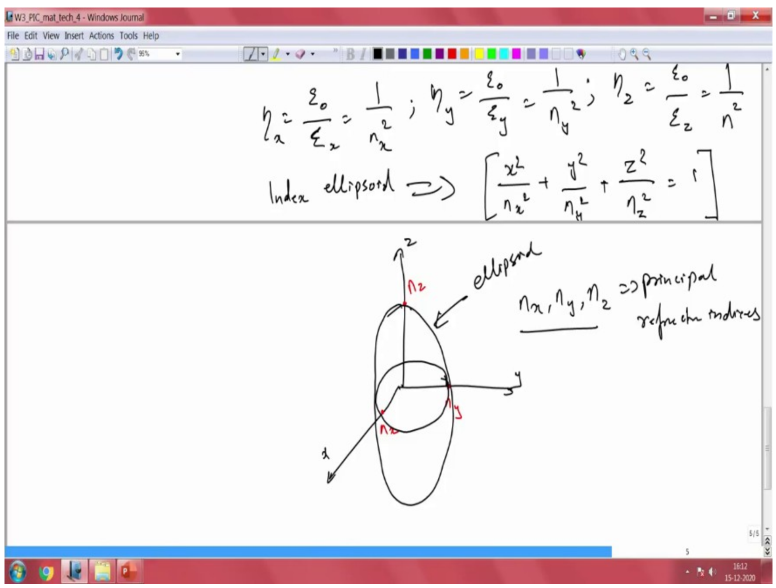

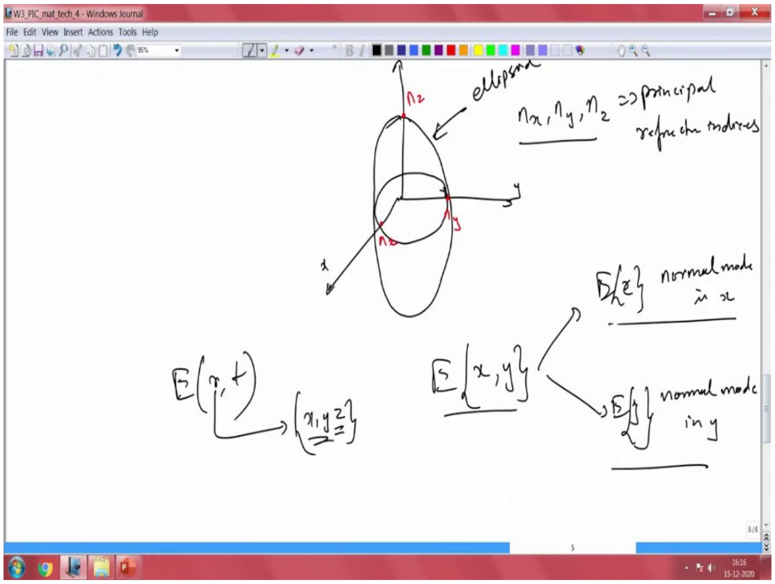

So, we started of x, y, z. So, let us construct that. So, let us say this is z and this is y and this is x. So, you have three different locations. So, you have n_z here and let us say n_y and we have n_x. So, now you could draw ellipse around this. So, when you do that you will have an ellipsoid sorry let me draw it thus and you will have a circle around this. So, this is your ellipsoid that you have and this is between x y plane where it is circular in nature. So, the n_x, n_y and n_z this is all principle refractive indices of the crystal in x, y and z coordinate system in this case.

So, you can also rotate this then you have to do some transformation in that case your principle index will change perhaps to a new value depends on the orientation that you move. So, one should keep that in mind when you are doing this transformation. So, with that understanding let us look at how the light propagation could be understood through this anisotropic medium.

So, when you take any wave. So, when you take any propagating wave. So, you represent it as a function of position and space. So, sorry position and time space and time. So, this x is nothing but your electric fields in x, y and z. So, assuming that you are looking at a snapshot and the light is propagating through along z direction, it is propagating along z then what is important is let us say the electric field will be between two components x and y because the wave is traveling like this that means it is vibrating between in x and y plane and moving in z plane.

So, you could decompose your wave into two components. So, you could decompose it into x and y and these two axes E_x and E_y that you have these fields are called normal modes. So, the vibration only in x is called normal mode in x and this is normal mode or field in y you can decompose it you can take any wave for that matter and decompose and write it into normal modes. So, the normal modes are nothing but having its field in the principle axis. So, when you have field defined in that principle axis we call that as a normal mode.

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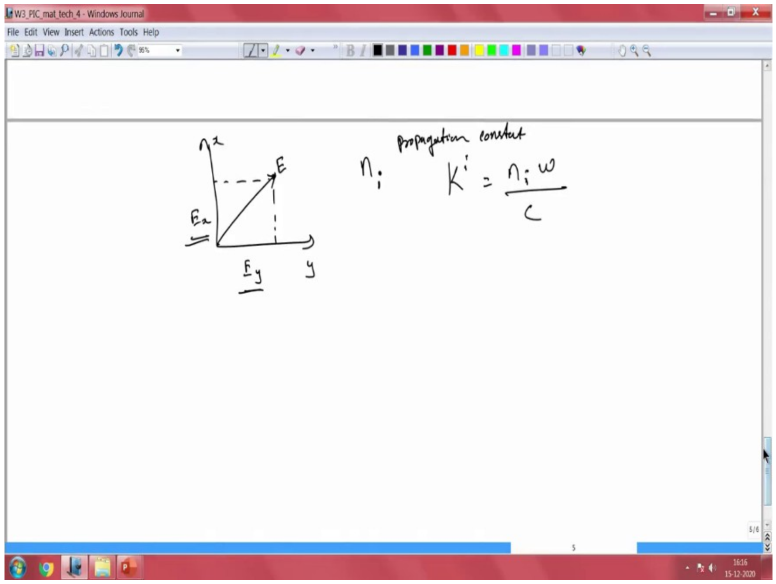

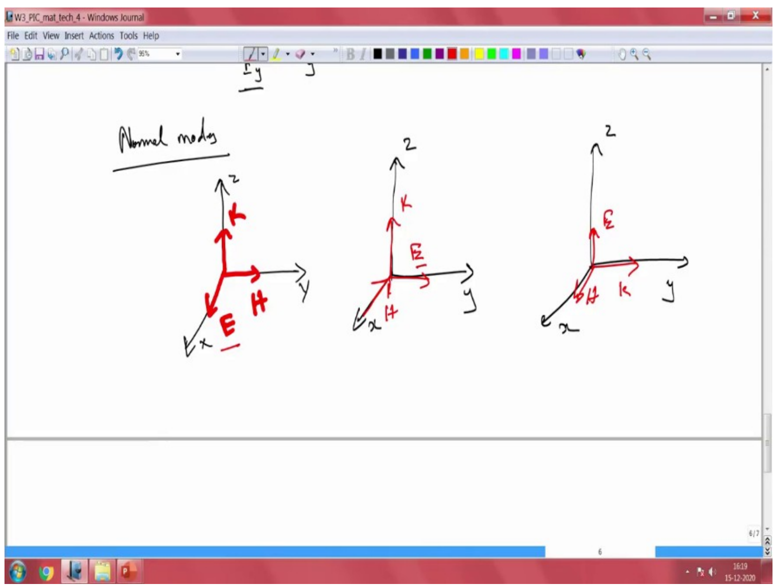

So, if you take a space and you have a electric field E then you could decompose this into E_x sorry this is E_y and this is E_x. So, we will shortly see it but this is how you can do it and this E_x and E_y are called normal modes. So, let us look at how they would propagate. So, we all know that light propagates with a propagation constant. So, propagation constant is what we use to characterize wave propagation.

So, if n_i is a refractive index along a certain direction then your propagation constant along a certain direction i is given as $k = \frac{n_i \omega}{c}$. So, this is your propagation constant and this is actually defining the polarization of light. So, what we are primarily discussing here is what is the polarization of this light. What is polarization? It is nothing but the oscillation of your electric field in a certain plane or a certain axis here. So, this is what determining your polarization but not the direction of the wave that is moving.

And that is the reason why z really does not matter because the light is propagating in z direction but its electric field is in x and y direction. So, x and y are important in defining the polarization not z. So, let us look at what are these normal modes. So, what are these normal modes. So, normal modes are defined based on our principle axes. So, let us look at that. So, you could have light propagating along z direction let me take a thick marker here. So, you could have light propagating along here and the electric field will be in this and then H is here. And now let us look at another scenario and then one more scenario let me put it, so that I do not have to change again.

So, here it is still light still propagates through this K, that is your z direction but now the field is different now. So, you have electric field in y direction, here we had it in x and here we have it in y direction. And there is another scenario where light is propagating y you have E here and then H in this direction. So, these things are possible you could have little bit more variance as well but based how, in which direction it propagates positive y, negative y and so on but these are all the fundamental modes or normal modes that are propagating along. So, where their velocity is determined by the propagation direction and the plane in which they have their x and y field.

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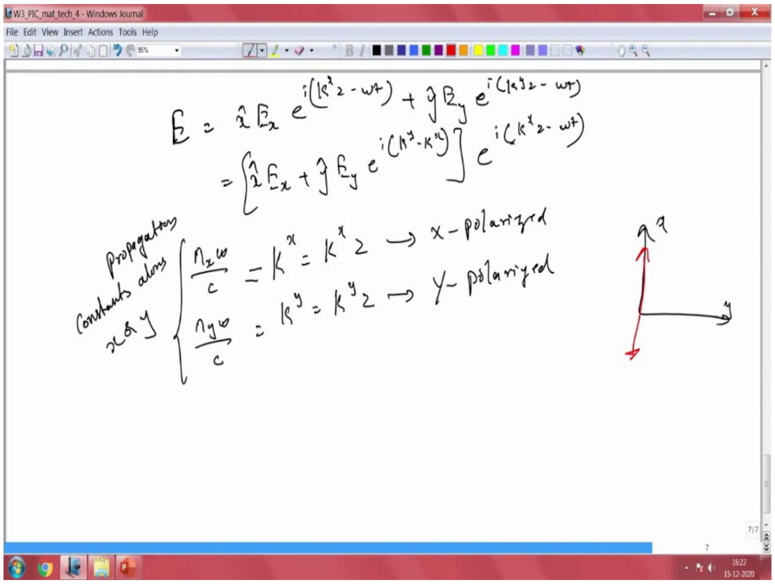

So, let us look at how we could write this wave that is propagating along x direction and y direction in the x, y plane rather. So, let us look at that. So, you can write the light that is propagating along z direction as $E=\hat{x}\,E_x e^{i(k^x z-\omega t)}+\hat{y}\,E_y e^{i(k^y z-\omega t)}$, you can do a little bit of movement here and eventually you can get something $E=[x¿E_x+\hat{y}\,E_y e^{i(k^y-k^x)}]e^{i(k^x z-\omega t)}¿$ amplitude part and then you have your phase part.

So, now the wave is propagating along x and y direction. So, this is something we saw here. So, when you are having a wave that has a field both in x and y it is going in moving in x, y plane then it will have x and y components. So, now the x and y components are defined here and they are moving with two propagation constants the wave vector along x and along y. So, this is given as this wave propagation z and your $K^x = K^x z$, $K^y = K^y z$.

So, this will be x polarized and this is y polarized. So, what is actual K y and K x is nothing but $K^x=\frac{n_x\omega}{c}$, $K^y=\frac{n_y\omega}{c}$. So, this is how propagation constant along x and y. So, this is nothing but propagation constants along x and y. So, now you could see when there is difference in the magnitude of x and y then there will be a relative difference between x and y in the movement.

So, if the light is just oscillating only in x. So, there is a electric field that has only x component, y component is 0; if y component is completely 0 we call that as a linearly polarized light along the principle axis. So, no problem here we do not have to worry about anisotropy at all because light is that particular electric field is not seeing the y direction. So, it is only moving in the vertical axis here and the same thing applies if it is in the y direction because the wave is only vibrating along y axis.

So, whatever refractive index you have it is uniform for the wave for y and then the x. So, these two waves are called linearly polarized light, but now things become interesting when you put the wave at a certain angle. So, now the oscillation with respect to x is at an angle it is not in x-axis. So, relative to x it has a certain angle. So, now it will induce a y component as well. So, you have both x and y. So, this is where things get interesting because your propagation speed or propagation constant is going to be different and they are going to travel at different speed. So, let us look at what angle these things actually make a difference.

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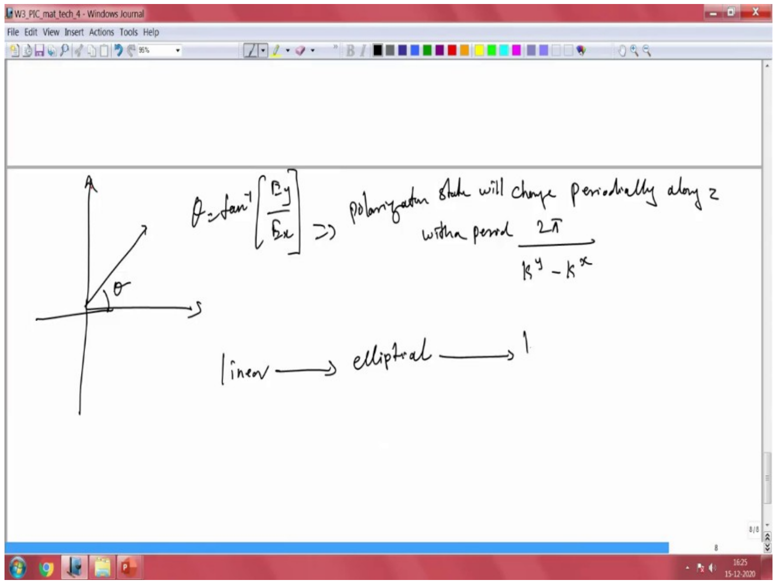

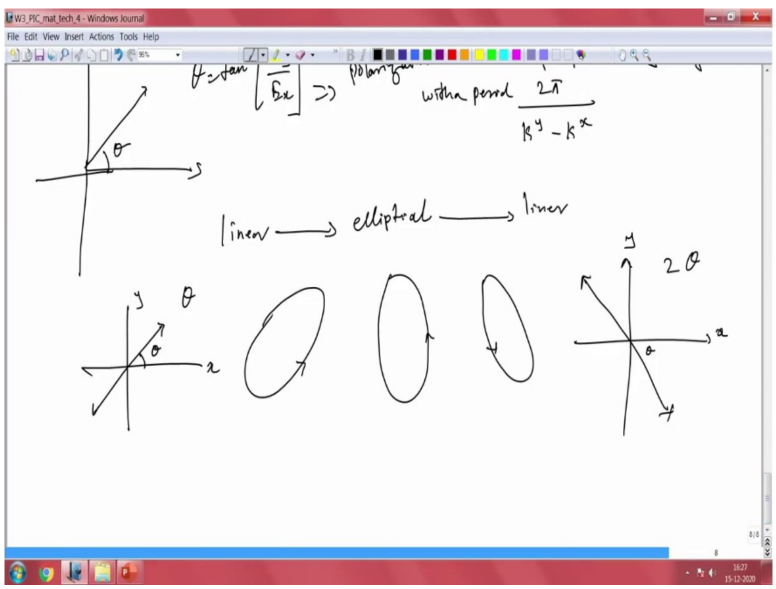

So, if we take a very simple field and I put it at a certain angle if I put it at a certain angle theta what will happen let us say what is this angle. So, $\theta = \tan^{-1}\frac{E_y}{E_x}$. So, this is from the polarization you must have studied in your basics. So, this is the polarization and it is a polarization state, the polarization of this would vary because it is going through this medium. So, polarization state will change periodically along z and that periodicity with a period $\frac{2\pi}{K^y - K^x}$. So, this is an important understanding from an anisotropic medium that is affecting the polarization of light.

So, when it is moving through a medium it is going to change the polarization of light that is going through. So, just to give you an example if you put a light that is linearly polarized, it will generate an elliptically polarized, an elliptical light and then if you even allow it to propagate through the medium it will become linear again. So, there is a periodicity associated with this. So, this will keep on repeating.

So, let us just quickly look at how this will happen. So, if you have x and y we have certain theta here. So, as it propagates it is going to create a light that is oscillating in this direction and then rotate it this way and it will become left, right and left polarized and now you have, you will come back to a completely different angle altogether.

So, what this would do is it will go from a linearly polarized to another linearly polarized but then it would have rotated by from theta to 2 theta. So, you can do this you change or flip the plane of your linearly polarized light when you go through this anisotropic medium. So, in half period your first half it goes from linear to another linear. So, that is the angle is now from theta to t to 2 theta.

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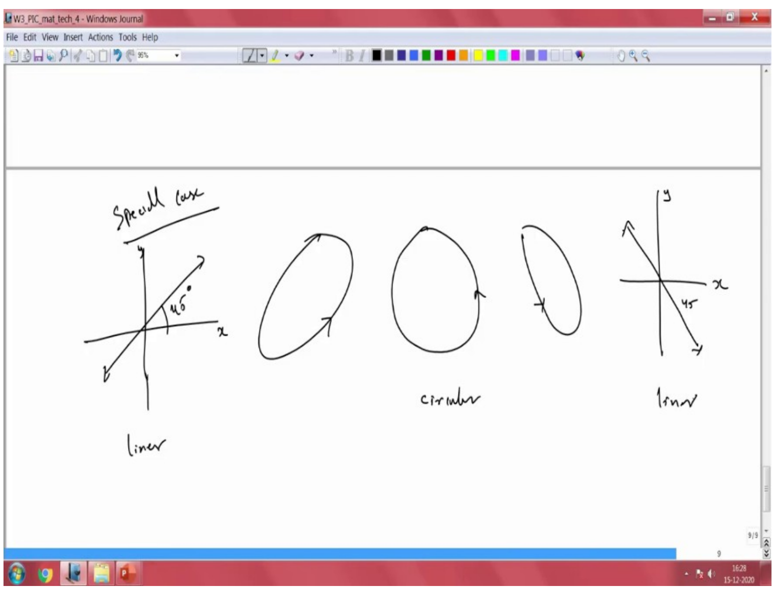

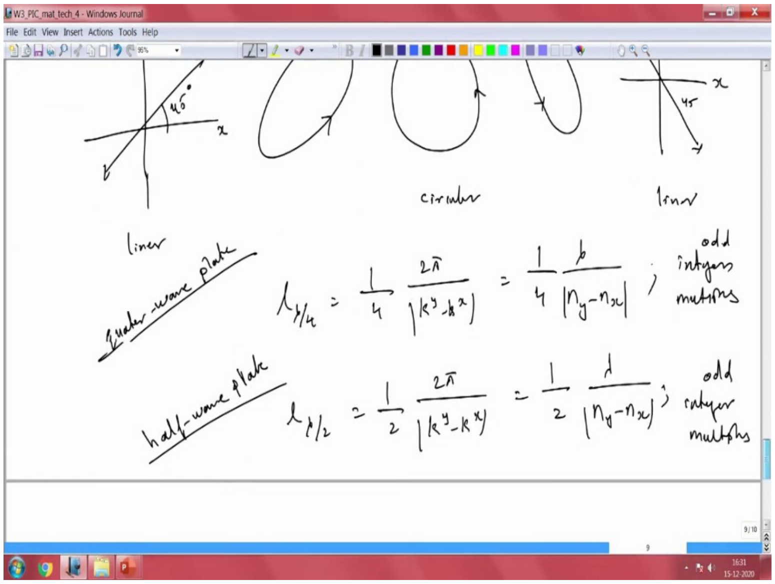

So, there is a special case where you have the theta at 45 degrees. So, when you have a light at 45 degrees this is still linearly polarized this gives you an interesting it will create similar to what we saw earlier but then the quarter wave it will create a circularly polarized and then you go back to 45 again but in the opposite direction.

So, you go from linear to sorry this is linear circular to linear. So, this is an another way of creating a different type of polarization that you want based on the anisotropy that you choose. So, let us look at what should be that length of propagation of this. So, that is given by very simple wave plate equation here. So, let us look at that. So, for a quarter wave plate or quarter wave change that is what you go from linear to circular or linear to elliptical.

So, that is $l_{\lambda/4} = \frac{1}{4}\frac{2\pi}{¿K^y - K^x \vee ¿ = \frac{1}{4}\frac{\lambda}{¿n_y - n_x \vee ¿¿}¿}$ and now for half wave that is

$l_{\lambda/2} = \frac{1}{2}\frac{2\pi}{¿K^y - K^x \vee ¿ = \frac{1}{2}\frac{\lambda}{¿n_y - n_x \vee ¿¿}¿}$. So, this is how you can find whether you need a quarter wave a conversion or half wave conversion.

So, integer multiple of this would also give the same thing. So, it could be integer multiples should give the same if I have to prefer it should be odd integer multiples and same thing here odd integer multiple should give you this rotation that you have in a periodic fashion. So, with that we could do whatever polarization you want and also we can change the phase of light that is going through it. So, that is the phase retardation. So, how can you change the phase, let us look at how phase will be affected by this.

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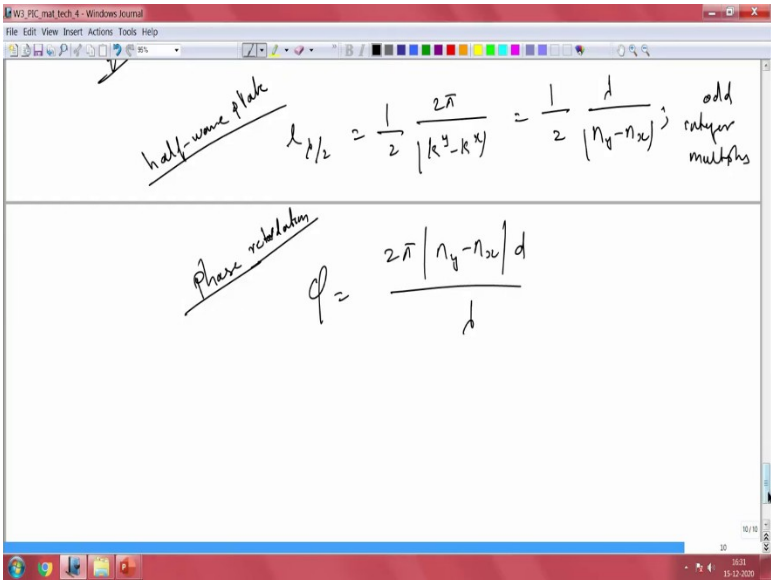

So, $\varphi = \frac{2\pi \vee n_y - n_x \vee d}{\lambda}$. So, this will give you the difference in phase between when you have an anisotropic medium. So, this is called phase retardation. So, all depends on as you can see the refractive index difference because of the anisotropy you see all these interesting phenomena that you see coming from the crystals that can do polarization rotation phase change and any arbitrary polarization that you want based on interaction of the wave with the crystal.

So, this gives us an additional tool from the material system that we can exploit to do a lot of interesting manipulation of waves that are propagating through this anisotropic medium. So, with this we have understood all the phenomena’s that one can exploit using an anisotropic medium. Thank you very much.