aPhotonic Integrated Circuit
Professor Shankar Kumar Selvaraja
Centre for Nano Science and Engineering
Indian Institute of Science, Bengaluru
Lecture 11
Anisotropic medium and reciprocity

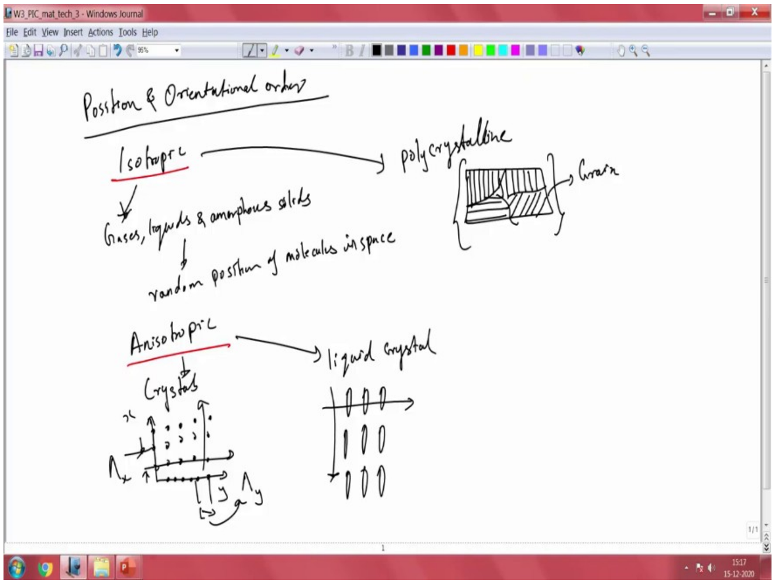
Hello all, welcome to this another lecture on photonic integrated circuit. In this particular lecture we are looking at material technology. So, we already looking at material properties and how light will interact with this material and how the property of material is changing the propagating wave. So, far we primarily handle materials that are isotropic in nature. So, when we say isotropic whichever direction you move you do not see any periodicity or you do not see any variation in the material property.

So, that means they are completely uniform in the property. So, let us take a simple refractive index as a measure. So, we can give a simple x, y, z coordinate system and in that system we can define refractive index along these different directions. So, if the refractive index is uniform in whichever direction or whichever plane you move then that material can be characterized under isotropic material, but in nature we have both isotropic material and also anisotropic material.

So, in anisotropic material the property of the material changes with respect to the direction that you take. So, for example if you move along x direction the refractive index perhaps is very different compared to y direction but along x it is uniform but compared to y it is different. So, if you have two different refractive indexes moving in two different axes then those materials are classified under anisotropic material.

So, in this lecture section we are going to look at how you can create a mathematical understanding of this anisotropic material with respect to light propagation and how the microscopic and macroscopic property of material is going to affect the light propagation. So, let us look at a very simple example of how you can develop this understanding.

(Refer Slide Time: 2:45)



So, when we say anisotropic material we are talking about position and orientational order, position and also orientational order. So, this is what characterizes isotropic material or anisotropic material. So, what is isotropic? So, isotropic material we have gases. So, we have gases, liquids and amorphous solids. So, all these material the molecules are located total randomness. So, random position of molecules in space and are relay and themselves they are isotropic or oriented in whatever direction they want to there is no uniformity or periodicity there and then you have another class that is polycrystalline.

So, in some polycrystalline material we also see this isotropic nature. So, in a polycrystalline material what you see is let us say arrangement of grains here. So, what you see here they are uniformly arranges within that particular grain, we call this as grain. So, each grain you could have highly oriented crystals but then when you put all this together as a combined unit this would act as a isotropic medium. So, that is isotropic.

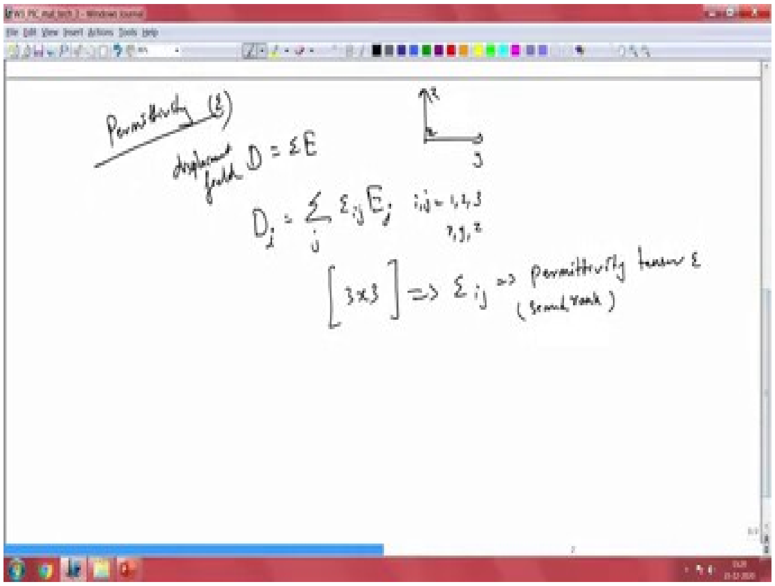
So, let us look at anisotropic. So, typically crystals show anisotropy. So, crystals show anisotropy and then we also have another type of crystals liquid crystal. So, this also shows anisotropy. So, crystals primarily the molecules are organized in space according to a regular periodic pattern and they are oriented in same direction. So, if you look at two different directions, they are highly periodic in nature and you can see they could have two different patterns here.

So, along y direction you have a periodicity Λ_y. So, this is the periodicity that you have along y, along x the periodicity could be different and because of this arrangement that are very directional then the properties are going to be also directional and this is the origin of anisotropy. Similarly, liquid crystals, so they have orientation of these molecules in a certain direction based on temperature and also based on the electric field that you apply.

So, when you move in this direction and when you also move in this direction you see difference in the density or the packing of these molecules and this is the origin of anisotropy in a liquid crystal. So, crystals in general exhibit anisotropy and they are they can be oriented in one direction or the other that one can you exploit it for interesting properties. So, let us look at how the macroscopic properties could be understood.

So, we use tensors in order to understand this matrix because now we have moved away from one dimensional understanding or uniform material system now we need to talk about all the macroscopic elements and microscopic elements in terms of orientation. So, let us look at the permittivity.

(Refer Slide Time: 08:11)



Permittivity (ε)

displacement field $D = \varepsilon E$

$D_i = \sum_j \varepsilon_{ij} E_j$ i,j = 1,2,3
x,y,z

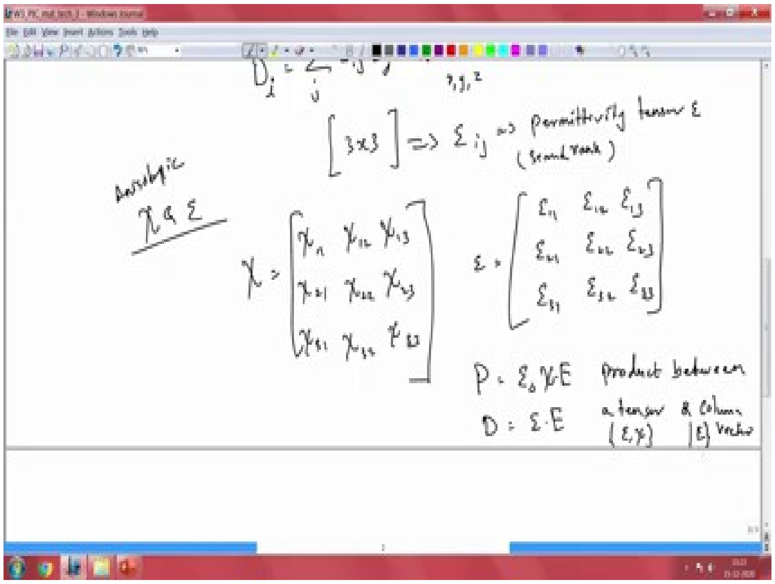
$[3\times3] \Rightarrow \varepsilon_{ij} \Rightarrow$ permittivity tensor ε
(second rank)

So, permittivity we all know it is epsilon but then we call this as a permittivity tensor when we start talking about crystals. So, in a dielectric medium each component of displacement field let us say, so D we all know could be written as $d = \varepsilon E$, this is from our earlier understanding. So, this is what we know this is nothing but displacement field. So, now let us look at a coordinate system x, y and z and now when you have an electric field applied in one of these components then the permittivity should also be a function of the position because we are talking about material it is anisotropic in nature.

So, it is not anymore just an epsilon it is not just E it is not just D. So, now everything has to comply with a coordinate system now. So, where i comma j is 1, 2, 3. So, representing x, y and z coordinate system. So, this will eventually result in a 3 by 3 matrix. So, it will be a 3 by 3 matrix for your dielectric constant here or permittivity in this case.

So, this would constitute to what you call epsilon i, j which is nothing but electric permittivity tensor could be captured as epsilon and this tensor is of rank two, second rank matrix. So, the moment we define I know the permittivity tensor then the rest of the things follows. So, we also know our susceptibility which is also related to our permittivity will also be a tensor and tensor of rank two it is also going to be 3 by 3.

(Refer Slide Time: 10:58)



$D_i = \sum_j$ x, y, z

$[3\times3] \Rightarrow \varepsilon_{ij}$ as permittivity tensor ε (second rank)

Anisotropic $\underline{\chi \,\&\, \varepsilon}$

$$\chi = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{bmatrix} \qquad \varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

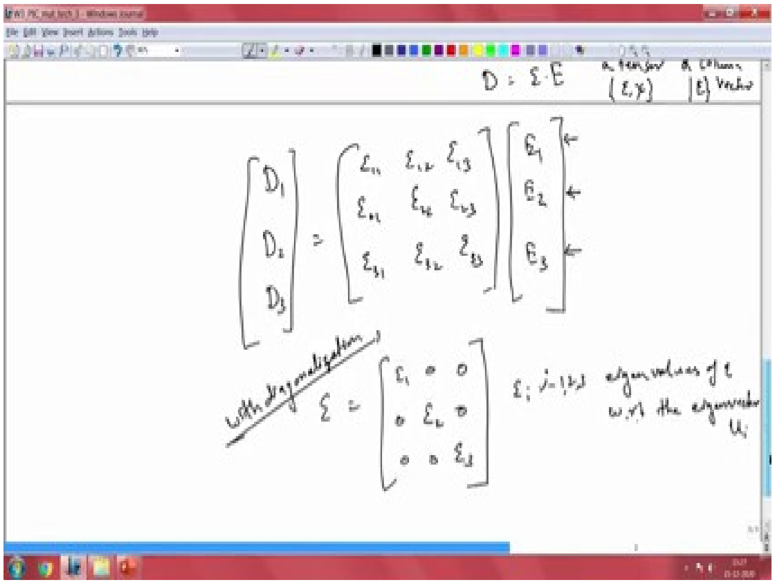
$P = \varepsilon_0 \chi E$ product between a tensor (ε, χ) & column $|E|$ vector

$D = \varepsilon \cdot E$

So, now for in an anisotropic medium. So, now the susceptibility and epsilon this is, so, anisotropic susceptibility and epsilon now can be written as a matrix now and you can do it this way. So, you can fill up this 2 3, 3 3 let me fill it for completeness 2 2, 3 2 and now we have epsilon which can be written as $\varepsilon 11, \varepsilon 12, \varepsilon 13, \varepsilon 21, \varepsilon 22, \varepsilon 23, \varepsilon 31, \varepsilon 32, \varepsilon 33$. So, these two are our anisotropic susceptibility and permittivity tensors.

And now the polarization that we all understand is not anymore a single value. So, we all know this is how we wrote our polarization and this is how we write our. So, now all of this are going to be vectors. So, this will be a product between a tensor and a column vector. So, here the tensor is our chi and epsilon let me do it in the next page. So, here tensor is nothing but our epsilon and chi and your column vector is nothing but your electric field.

(Refer Slide Time: 13:05)



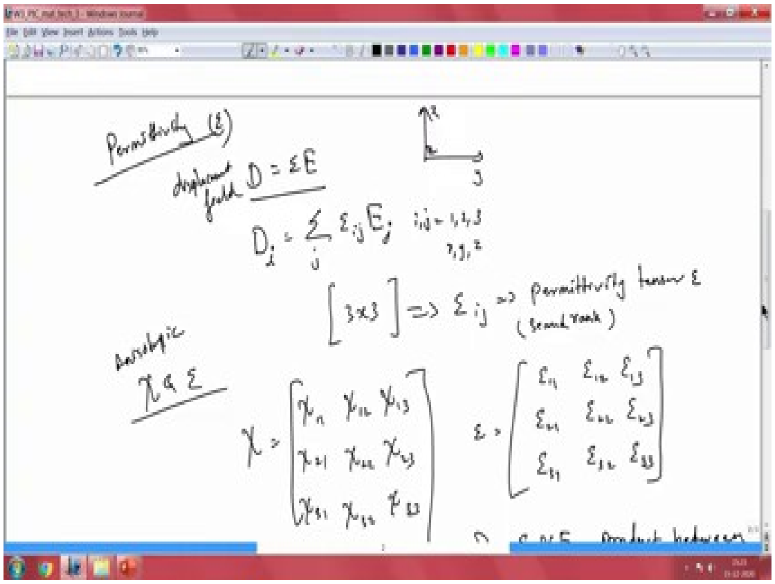
$D = \varepsilon \cdot E$ a tensor (ε, χ) & column (E) vector

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

with diagonalization

$$\varepsilon = \begin{bmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & 0 & \varepsilon_3 \end{bmatrix}$$

ε_i $i = 1,2,3$ eigenvalues of ε w.r.t the eigenvector u_i



Permittivity (ε)

displacement field $D = \varepsilon E$

$D_i = \sum_j \varepsilon_{ij} E_j$ $i,j = 1,2,3$ x,y,z

$[3\times3] \Rightarrow \varepsilon_{ij} \Rightarrow$ permittivity tensor ε (second rank)

Anisotropic χ & ε

$$\chi = \begin{bmatrix} \chi_{11} & \chi_{12} & \chi_{13} \\ \chi_{21} & \chi_{22} & \chi_{23} \\ \chi_{31} & \chi_{32} & \chi_{33} \end{bmatrix} \quad \varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{21} & \varepsilon_{22} & \varepsilon_{23} \\ \varepsilon_{31} & \varepsilon_{32} & \varepsilon_{33} \end{bmatrix}$$

product between

So, let us try to write that for example if I want to write D. So, D1, D2 and D3 this is our displacement field $\varepsilon 11, \varepsilon 12, \varepsilon 13, \varepsilon 21, \varepsilon 22, \varepsilon 23, \varepsilon 31, \varepsilon 32, \varepsilon 33$ and your column the electric field in E1, E2 and E3. So, now you can clearly see that we earlier on we started off with a very simple understanding of displacement field with respect to the electric field that we apply into the material that has a permittivity epsilon.

But when you bring in a material that is anisotropic that means your permittivity is not a function of, it is a function of position and direction and because of that you no longer got one single epsilon value. So, your epsilon value is going to change whether you are moving in x direction or y direction or z direction if you like, but for the moment let us just restrict ourselves to x and y. So, whether you go in x or y you are going to feel the difference and that difference can manifest in various ways at least in the immediate equation that we saw.

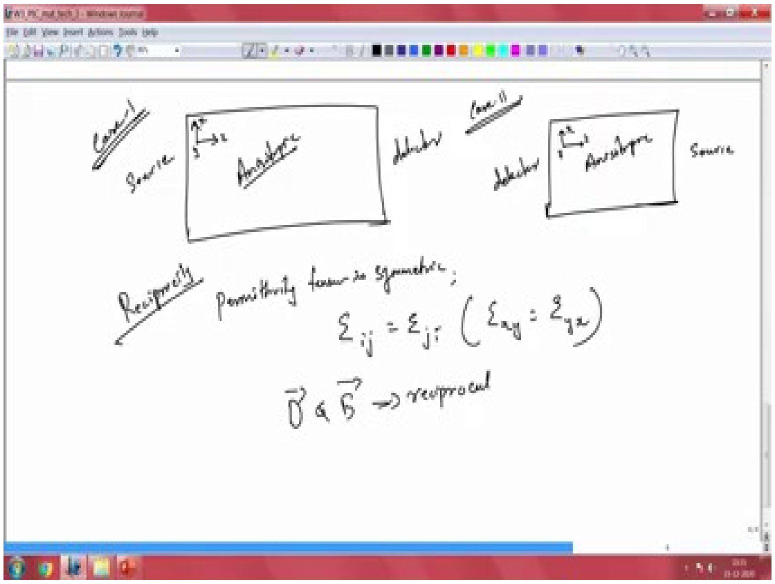
Your displacement field is now a function of a permittivity tensor operated on the electric field. So, in general the matrix the epsilon matrix that we have here is not diagonalized, but it can be diagonalized based on proper choice of your coordinate system and so on. So, if you can do that, so with diagonalization one can write epsilon as $\varepsilon_1 0 0, 0 \varepsilon_2 0, 0 0 \varepsilon_3$. So, now 1, 2, 3 this is nothing but epsilon i all these where i is 1, 2 or 3 this is nothing but the Eigen values with respect to the Eigen vector some Eigen vector U_i let us say.

So, now you can clearly diagonalize the tensor that you have this is not just for the permittivity tensor you can also do this for the susceptibility as well. So, you can use diagonalization to make it much more handleable when you are operating it. So, with this we have understood that when you have a material that is anisotropic then your displacement field is going to be affected because you have different field here x, y and z field in this case if you replace 1, 2 and 3 coordinate system with x, y and z.

So, when you do that your displacement currents are going to be different. So, one thing that to keep in mind is your E1, E2 and E3 let us say even if you consider them to be identical in their amplitude, your epsilon need not be identical because you have anisotropic material. Even when you have a material that is anisotropic your field Ex and Ey field could be different.

So, E1 and E2 need not be equal if that is the case then the displacement current that you are going to get, displacement field that you are going to get will be affected by the interacting field itself because the material enables such interaction. So, this kind of things are very useful in manipulating the properties using external electric field. So, let us look at this anisotropic material and then define how one could perceive the material reaction to a input and an output.

(Refer Slide Time: 18:15)

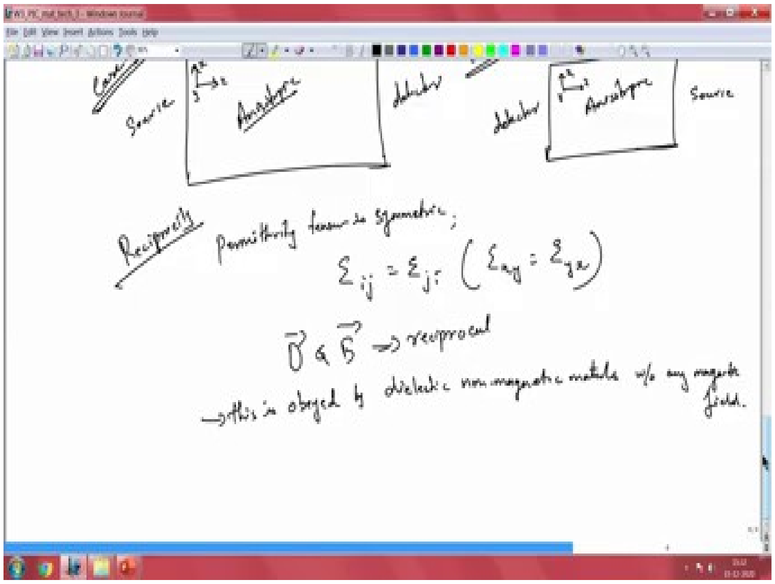


Reciprocity

Permittivity tensor is symmetric;

$$\varepsilon_{ij} = \varepsilon_{ji} \quad (\varepsilon_{xy} = \varepsilon_{yx})$$

$\vec{D}$ & $\vec{B}$ → reciprocal



Reciprocity

Permittivity tensor is symmetric;

$$\varepsilon_{ij} = \varepsilon_{ji} \quad (\varepsilon_{xy} = \varepsilon_{yx})$$

$\vec{D}$ & $\vec{B}$ → reciprocal

→ this is obeyed by dielectric non-magnetic materials w/o any magnetic field.

So, let us say I have a source and then a detector and then I have an anisotropic material. So, this is a scenario one let us say this is case 1. So, this is anisotropic you have to be very, very careful here and now what I do is in case 2, I am going to reverse the input and output. So, I put the source here. So, now this is the material remains the same at the center it is anisotropic. So, I have not changed the material in order to do the mark it, let me do it something like this, let us say this is x and this is y.

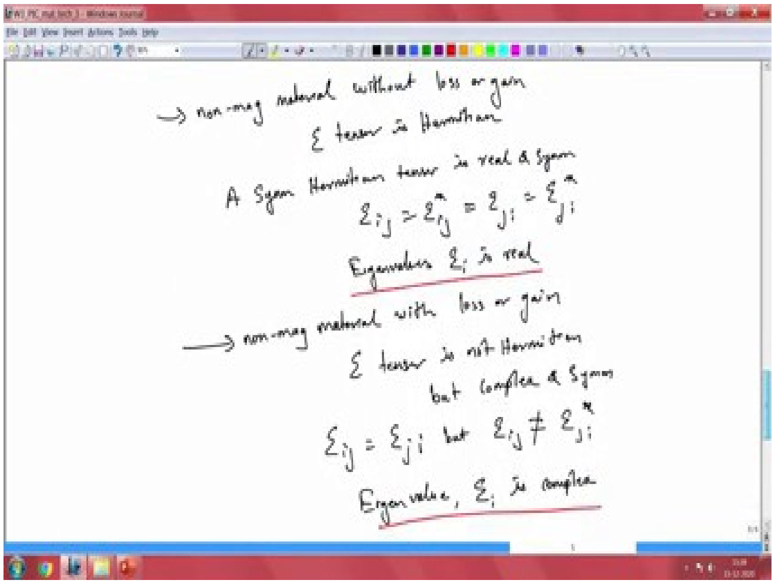
So, I am not changing the material orientation. So, I am keeping the material stationary I have on the case 1, I have source on the left side and then detector on the right side in case 2, I have source on the right side and detector on the left side. So, when you interchange the source and the detector if you do not see any effect of this flipping then what you say that it is the system is reciprocal there is a reciprocity associated with this system.

So, most of the systems without any magnetic, non-magnetic material without any magnetic field will act as a reciprocal medium. So, we need to understand whether one can induce non-reciprocity here which is really fascinating. Let us first understand this anisotropic material and the reciprocity. So, first let us understand reciprocity say. So, for most dielectric material there the permittivity tensor is symmetric. So, the permittivity tensor is symmetric. So, what does that mean? That means $\varepsilon_{ij}=\varepsilon_{ji}$ or in other terms if you are looking at the coordinate system $\varepsilon_{xy}=\varepsilon_{yx}$.

So, this means the relation between the displacement current and displacement field and electric field is reciprocal. So, this is what is important. So, you have the D the displacement field vector and the electric field vector. So, this is the relation between those two are reciprocal. So, because of this you can interchange the source and detector. So, this system obeys for dielectric medium and they are non-magnetic material. So, this is very important to understand. So, this is exhibited by dielectric, this is obeyed by dielectric non-magnetic materials without any magnetic field. So, one has to be careful.

So, without any external magnetic field, so the dielectric medium just to obeys this dielectric reciprocity principle. So, because of this reciprocity you can change the coordinate system without affecting your final result. So, we saw case 1 and case 2 when we change the source but then when you have this reciprocity, then you can also change the rotate the crystal, when you rotate the crystal everything remains the same. So, you do not have to worry about reciprocity altogether. So, let us look at how one could understand this reciprocity with a little bit more conditions let us say.

(Refer Slide Time: 23:42)



→ non-mag material without loss or gain

ε tensor is Hermitian

A Symm Hermitian tensor is real & Symm

$\varepsilon_{ij} = \varepsilon_{ij}^* = \varepsilon_{ji} = \varepsilon_{ji}^*$

Eigenvalues ε_i is real

→ non-mag material with loss or gain

ε tensor is not Hermitian

but complex & Symm

$\varepsilon_{ij} = \varepsilon_{ji}$ but $\varepsilon_{ij} \neq \varepsilon_{ji}^*$

Eigen value, ε_i is complex

So, I am going to give you a few conditions and the solutions to that. So, that we understand this reciprocity of the reciprocity in the medium a little bit more from what you know now. So, let us look at a non-magnetic material it is a non-magnetic material without loss or gain. So, that means optically they are not active. So, there is no gain and there is no loss in the medium. So, that means you can say that this epsilon tensor will be an Hermitian in nature.

So, asymmetric, so we all know epsilon tensor is a asymmetric in nature. So, asymmetric Hermitian is real that means $\varepsilon_{ij}=\varepsilon^{¿}_{ij}=\varepsilon_{ji}=\varepsilon^{¿}_{ji}$ So, this is the most important thing it is symmetric in nature, real and symmetric. So, the Eigen values for this is real. So, that is what it tells you. So, the Eigen values the ε_i. So, this is case 1. So, you have a non-magnetic material without loss or gain.

So, what this is basically a simple dielectric material, it is a transparent dielectric material where you have symmetry in all the direction that you see. So, now let us look at a non-magnetic material with either loss or gain. So, now we are making things a little bit more interesting. So, initially there was a transparency there is no activity in the inside the material and now there are two possibilities; you could either absorb the light, so that is what loss means or you can add more light to it, so, there is a gain.

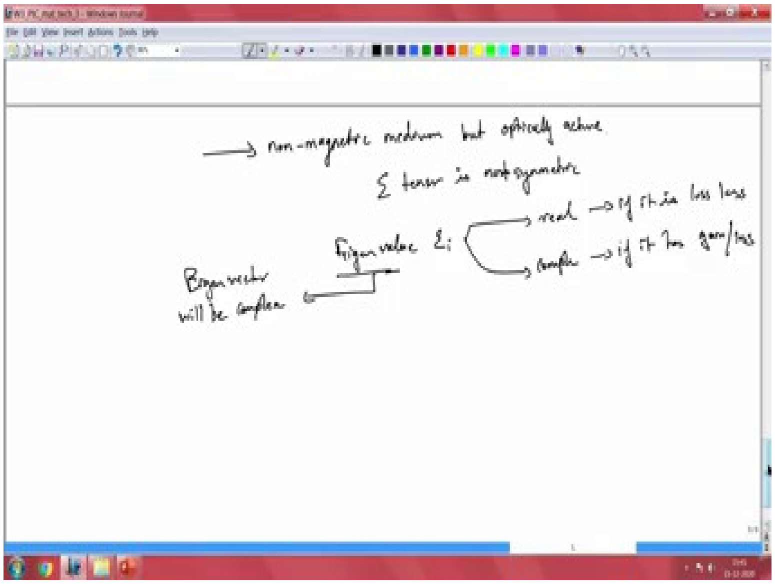
So, you could reduce the number of photons or in the intensity of the field or you can add more photons or increase in the field, so, you can do that. In that case epsilon tensor is not Hermitian in nature but complex and symmetric. So, what that means is since it is non-hermitian but symmetry you preserve the symmetry but it is not Hermitian. So, that means symmetry is given by this $\varepsilon_{ij} = \varepsilon_{ji}$.

So, that is the symmetry that you have but $\varepsilon_{ij} \neq \varepsilon^{¿}_{ji}$. So, this is the basic principle from our base mathematical understanding but the Eigen values here epsilon sorry epsilon i is complex. So, this is something reaffirming us from our earlier understanding your dielectric permittivity that you have strongly depends on the nature of the material whether it is absorbing or transparent. So, when you have a absorbing material then your permittivity has to be complex in nature because there is damping associated with this.

So, this is from our material understanding if you remember you must have studied about the resonances in the material like material dispersion modeling. So, there you had this real part and the imaginary part. So, the permittivity will be complex in nature when the material has loss. So, when the material is lossless that is where you have only real component. So, these are all important points that you may want to keep in mind.

So, there are multiple ways to reach the same conclusion. So, we started from a microscopic model to understand the nature of epsilon whether it is real or complex based on resonances in the material but now we are coming from a macroscopic model. So, from a macroscopic understanding we still come to the same conclusion.

(Refer Slide Time: 29:46)



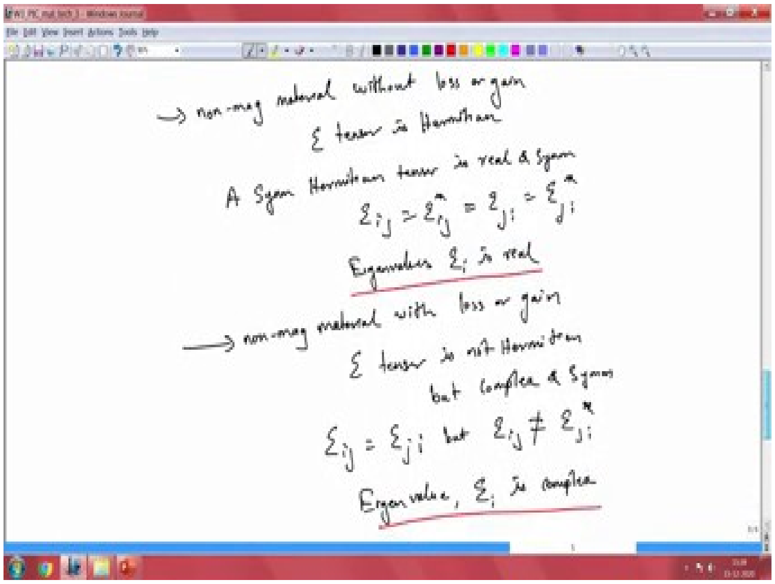
→ non-magnetic medium but optically active

ε tensor is not symmetric

Eigen value ε_i → real → if it is loss less

→ complex → if it has gain/loss

Eigen vector will be complex



→ non-mag material without loss or gain

ε tensor is Hermitian

A Sym Hermitian tensor is real & Sym

$$\varepsilon_{ij} = \varepsilon_{ij}^{*} = \varepsilon_{ji} = \varepsilon_{ji}^{*}$$

Eigenvalues ε_i is real

→ non-mag material with loss or gain

ε tensor is not Hermitian

but complex & Symm

$$\varepsilon_{ij} = \varepsilon_{ji} \text{ but } \varepsilon_{ij} \neq \varepsilon_{ji}^{*}$$

Eigenvalue, ε_i is complex

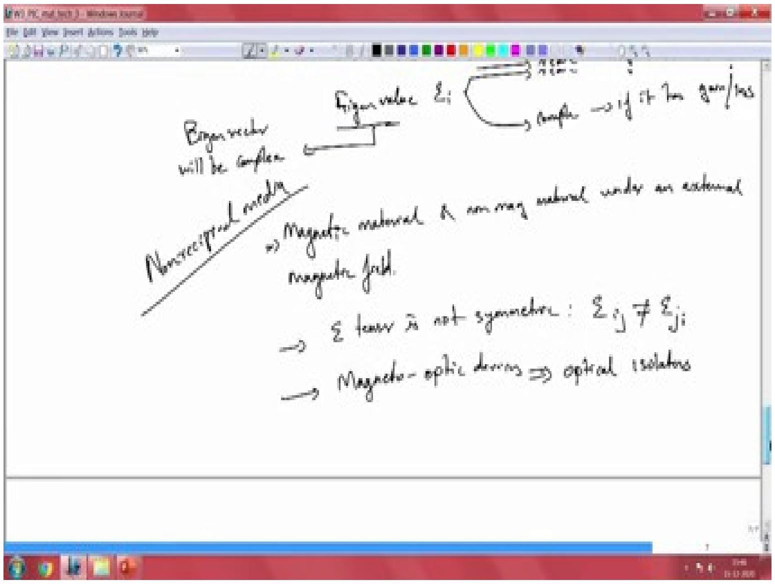
So, we could have a third condition where the material is non-magnetic, non-magnetic medium but optically active. So, this is though it is optically active this is still a reciprocal system here epsilon tensor is not symmetric but the Eigen value. So, the Eigen value could be, the Eigen value epsilon high it could be real if it is lossless and it will be complex if it, if it has gain or loss as simple as that. However, your Eigen vector, will be complex for this particular case.

So, again if the material is optically active you could have both scenarios, either real that means loss less or you could have complex that gives you certain amount of loss. So, that is about the reciprocity. So, you have a reciprocal medium here generally, we refer this reciprocity this generally given for a non-magnetic material in the absence of an external magnetic field, so that is all taken care.

The other important thing that you should understand is there are primary two scenarios where the material, a non-magnetic could have loss or gain or in the other case without loss or gain. So, when you have a material without any loss or gain then your epsilon tenor is symmetric, it is Hamiltonian, sorry it is hermitian and you could have the Eigen vectors that are all real. However, when you get a material with loss or gain then you will have a tensor which is non-hermitian but complex and symmetric in nature.

So, what happens in that case your Eigen values will be complex in nature. So, this complex part brings you the damping or amplification part for loss and gain respectively. So, the next thing to understand is the non-reciprocal medium. So, far in this case it is all reciprocal in nature because of the symmetry. So, you are able to easily take that into a count but what happens if the magnet, if the magnetic field is affecting your material.

(Refer Slide Time: 33:38)



Eigen value ε_i → complex → if it has gain/loss

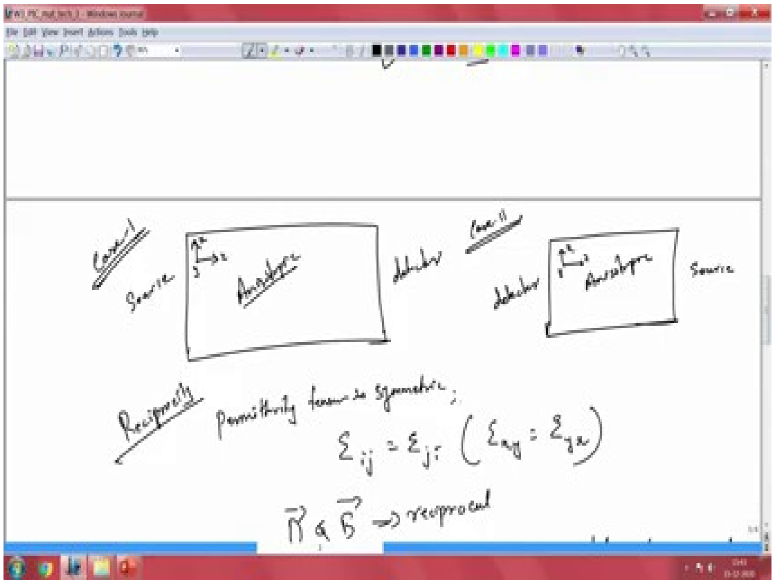
Eigen vector will be complex

Nonreciprocal media

⇒ Magnetic material & non mag material under an external magnetic field.

→ ε tensor is not symmetric : $\varepsilon_{ij} \neq \varepsilon_{ji}$

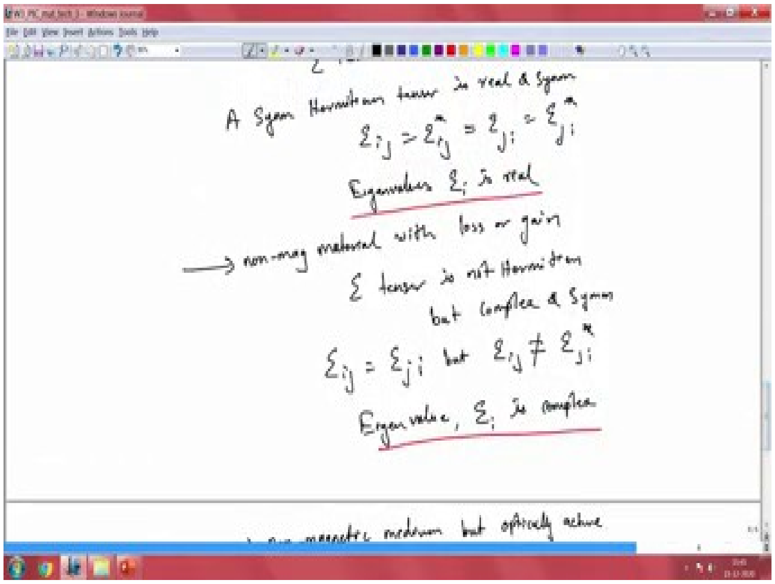
→ Magneto-optic devices ⇒ optical isolators



Reciprocity Permittivity tensor is symmetric;

$$\varepsilon_{ij} = \varepsilon_{ji} \quad (\varepsilon_{xy} = \varepsilon_{yx})$$

$\vec{\mu}$ & $\vec{\sigma}$ ⇒ reciprocal



A Symm Hermitian tensor is real & Symm

$\varepsilon_{ij} = \varepsilon_{ij}^* = \varepsilon_{ji} = \varepsilon_{ji}^*$

Eigenvalues ε_i is real

→ non-mag material with loss or gain

ε tensor is not Hermitian

but complex & Symm

$\varepsilon_{ij} = \varepsilon_{ji}$ but $\varepsilon_{ij} \neq \varepsilon_{ji}^*$

Eigenvalue, ε_i is complex

→ non-magnetic medium but optically active

So, a magnetic material when it is subjected to an external magnetic field. So, that is where your non-reciprocal media comes in. So, what do we mean by non-reciprocal, let us go back to our initial idea here, so when you change the source and the detector right side. So, source to detector detected to source side, when you do that, you will not be able to get the same output at the detector side.

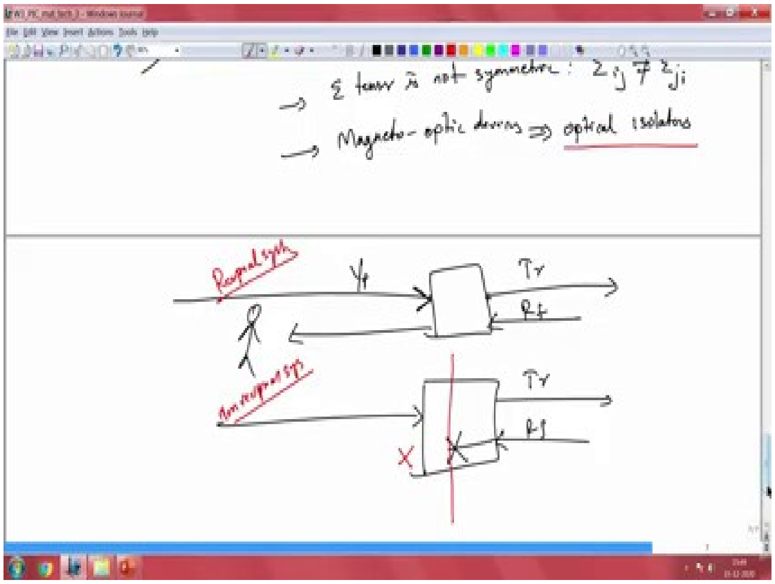
So, when you switch the detectors side, then you will not be able to see the same effect or same effect of the material on the propagating light because you have something that is being changed. So, that is a very interesting property of a system and that system is called that non-reciprocal system. So, you cannot change input and output, this primarily happens when the material is magnetic.

So, magnetic material and non-magnetic material under an external magnetic field. So, when you have an external magnetic field and this could result in a non-reciprocal system. So, in such a medium there is no symmetry exist between the source and detector when you are doing some signal exchange. So, in that case your epsilon tensor is not symmetric. So, what do we mean by that, $\varepsilon_{ij} \neq \varepsilon_{ji}$.

So, this really makes life so different from whatever we discussed here. So, you can easily see here you have symmetricity there. So, easily you can switch between the coordinates without seeing any effect same thing true even when you have loss or gain in the system however when you have a non-symmetric permittivity tensor then you will not be able to easily switch the input and output sides.

So, such materials are used for interesting application when there is a magnetic field involved. So, primarily magneto optic devices and one of the important application of this device in building optical isolators. So, what is an optical isolator? Isolator allows light to pass through only in one direction.

(Refer Slide Time: 37:22)



→ ε tensor is not symmetric : $\varepsilon_{ij} \neq \varepsilon_{ji}$

→ Magneto-optic devices ⇒ optical isolators

So, in some cases you want to block the light. So, it goes through a certain medium and you will have let us say reflection from here. So, there is a signal that is getting reflected but if it is a reciprocal system. So, let us say this is input and this is your output and there is a reflection let us say or instead of output I will call it as transmitted signal and there is a reflected signal. So, when there is a reflected signal if this is a reciprocal system, if it is a reciprocal system you will see the output here.

So, that means you are when you are sitting here observing putting some high power into this particular device, it can come and shoot you back because of the reflection this can damage the resources that you have on the input side. So, particularly when you are building lasers, with the high power lasers you want to only go through one direction, any reflection would result in destabilization of the laser and to protect whatever instrumentation you have.

But when you have a non-reciprocal system, again you have an input, there is a transmission but then when there is a reflection the reflection will be arrested, the system will not allow, so non-reciprocal system, the system will not allow any reflection altogether. So, these kind of systems are used to avoid any back reflection and that is what we do using optical isolator. So, isolators isolate the input to output there is no back propagation of any of the waves that you have put into the output side.

So, this is how one could understand the signal propagation between the two systems here reciprocal and non-reciprocal. So, which we use it to develop a lot of interesting functionalities both bulk and integrated optic cells well. So, with this we would like to summarize that anisotropic materials are got interesting features and the way to understand is through this tensor matrix and there are multiple ways to exploit this if the material can be easily diagonalized.

So, then it is pretty easy to handle anisotropy, it might you look like too much to start with but then if you can diagonalize it, it is all relatively easy to do that. Later on we will see in the lecture how we are going to exploit the effect of this anisotropy for various other functional that you are going to do. With that we would like to close this lecture and in the following lectures we would see continuation of this anisotropy but for signal processing. Thank you very much.