Fundamentals of Semiconductor Devices

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Lecture - 09

High doping effects and incomplete ionization

Yeah welcome back. So, we had concluded the last lecture with temperature dependence of carrier density. If you recall, we also had discussed about charge neutrality so, the total positive and negative charge has to be same in a semiconductor. So, now, we shall go ahead and we shall wrap up this discussions with a few notes you know on things like high doping incomplete ionization and then will move to things like mobility and drift diffusion so that we can discuss about current transport.

Once we discuss about current transport, we can discuss devices like p n junction that makes the LEDs, photo detectors, solar cells, BJTs and so on. Things like MOSFET and different kinds of transistors, optical devices everything can be understood when we understand how electrons flow and that is carrier transport and primarily governed by drift and diffusion sort of a transport ok. And for that we need concepts like mobility and velocity. And before understanding mobility and velocity, we need the concepts of carrier concentration, how they change with temperature, you know conductivity and so on. So, that is what we are doing it. And if you recall from the beginning of this course, we had actually covered basics of band gap, carrier concentration and reciprocal space and now we are building up to the carrier statistics, how carriers are distributed and how we calculate the carrier density things like electron and hole density and so on.

So, now today we shall focus on high doping effects and incomplete ionization, and we shall thus conclude the initial section of the few lectures on carrier density, carrier recombination or carrier statistics and you know density of states and so on; so, that we can move to devices ok. So, today we shall come to the whiteboard now and will get start get started high doping effects ok.

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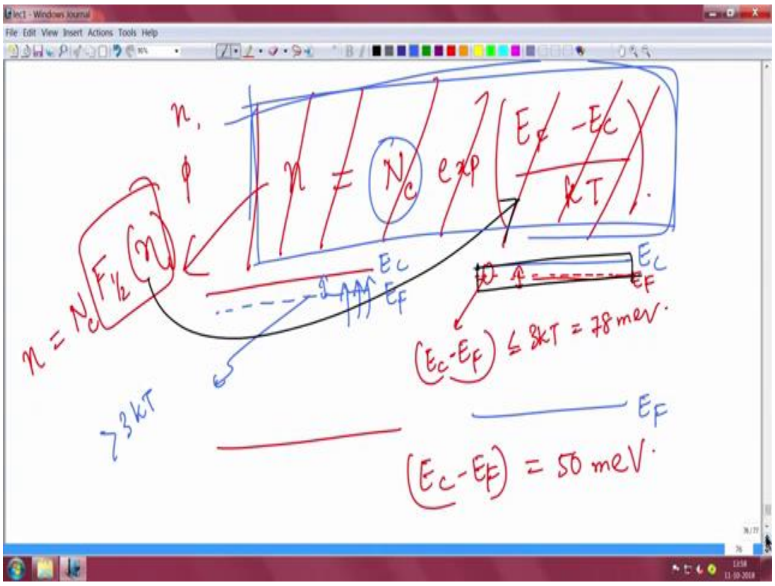

So, if you recall in the last lecture, we had in the last few lectures we have talked about you know n type and p type doping. And how do we dope them by adding impurities, we know that now. This equation is very you know very very familiar to you exponential of $(E_F - E_C)/kT$. So, you know I told you that this Maxwell Boltzamann equation is valid when your $(E_F - E_C)$, this gap is more than, at least more than 3kT right. At room temperature, this is around you know 78 meV.

So, that means, it is slightly moderate or lighter doping and this equation is valid under such circumstance and you can use this formula to calculate the electron concentration you know N_C very well. I told you that if your doping keeps increasing, then your fermi level keeps going up and up right, n-type doping and if your p-type doping increases, then your fermi level keeps coming down and down. So, if I dope it relatively higher ok; if I dope it relatively higher, such that you know your fermi level, your fermi level comes very close to conduction band. So, that this gap this $(E_C – E_F)$, this is very close. So, this is less than 3kT.

For example, at room temperature 3kT is around 78 meV, but if I dope it very high so that your fermi level comes sufficiently close to conduction band such that it is less than 78 meV. Say you know, if I say that $(E_C – E_F)$, this gap you know this gap is suppose only say 50 meV at room temperature which is less than 3kT, then this equation of Maxwell Boltzmann equation cannot be applied you know, why? Because this Maxwell Boltzmann equation came if you remember from this equation equal, n equal to N_C the fermi half dirac integral remember and this expression can be actually simplified to this particular expression, exponential only when the doping is light or when you know your (E_F - E_C), this gap is actually more than 3kT.

So, if it is more if it is higher, less than 3kT, its doping is very high.

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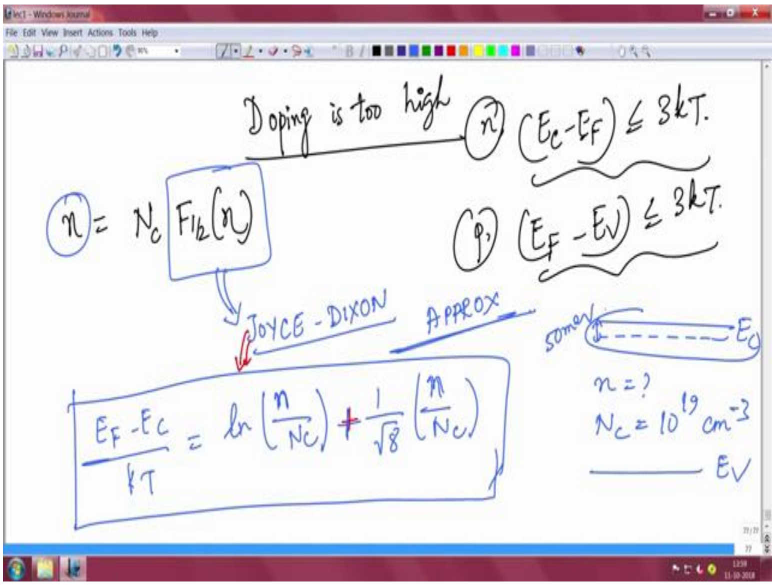

Doping is too high $\rightarrow$ ⓝ $(E_c - E_F) \leq 3kT.$

ⓟ $(E_F - E_V) \leq 3kT.$

$n = N_c \boxed{F_{1/2}(\eta)}$

JOYCE - DIXON APPROX

$$\boxed{\frac{E_F - E_C}{kT} = \ln\left(\frac{n}{N_C}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_C}\right)}$$

some

E_C

$n = ?$

$N_C = 10^{19}\ cm^{-3}$

E_V

So, if the doping I am talking about n type, same will apply for p-type also, if doping is too high right. If doping is too high which means say for n type what it means is that your sorry your E_F, your (E_C - E_F) this gap has become less than 3kT right. That is called high doping effect. And for p-type, it means that your (E_F - E_V) has become less than 3kT, right. So, in this case your doping is very high, you cannot apply Maxwell Boltzmann equation to calculate n and p. What you do is you use know you that expression was there know n = N_C, this you try to sort of do an approximation of this for high doping because calculating this exact integral will need you a MATLAB or mathematica kind of software and it is little also complicated. So, can we do some simplification.

So, that is an approximation called Joyce-Dixon approximation. Joyce-Dixon approximation, and this Joyce-Dixon approximation will basically help us to calculate the carrier concentration when your doping is high. Suppose I am you know I am giving a practical problem when your conduction band and your fermi level are very close, I tell you that this distance is only 50 meV ok. What do you do now? How do you find out n? I tell you that N_C is equal to say 10^{19}. How will you find out n now?

So, if you recall, (E_F - E_C)/kT is given by ln right of n/N_C if you recall that right is given by n/N_C. That is your Maxwell Boltzmann equation actually. This you know the expression that I have written in the previous slide, sorry this equation I have written. This is the equation actually, I am writing it in a different way, same equation I am writing in a different way. So, this is the expression right. For Joyce-Dixon, you add just one more term. You know what the term is, you add one more term and that term is plus, I will use the blue color 1 by square root of 8, n by N C.

$$\frac{Ef-EC}{kT} = \ln\left(\frac{n}{N_C}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_C}\right)$$

This is your Joyce-Dixon approximation that you can apply. When a doping is high so that your (E_F - E_C) is less than 3kT. If you can you can use this approximation and you can calculate n.

For example, you see that this is a transcendental equation. It is in the form of log x plus some constant term x right. So, it is a transcendental equation. Finding n directly you will you will have to do some numerical and iterative process, but it still can give you a better idea. So, for example, if I tell you that (E_F - E_C) is equal to 50 meV, then how will you solve this equation, right? So, it is very easy to solve now.

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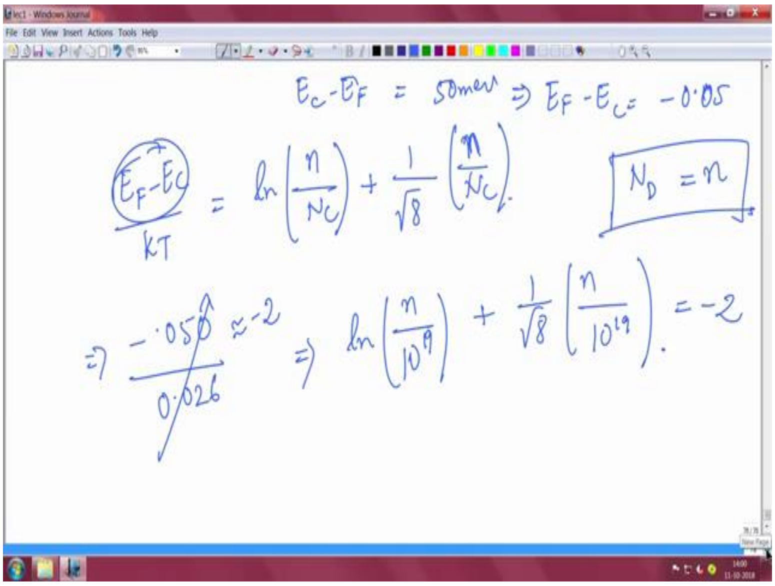

$E_c - E_F = 50\,\text{meV} \Rightarrow E_F - E_c = -0.05$

$$\frac{E_F - E_C}{kT} = \ln\left(\frac{n}{N_C}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_C}\right).$$

$$\boxed{N_D = n}$$

$$\Rightarrow \frac{-.050}{0.026} \approx -2 \Rightarrow \ln\left(\frac{n}{10^{19}}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{10^{19}}\right). = -2$$

So, I have told you that (E_C - E_F) is 50 meV which means (E_F - E_C) is -0.05, right that is 50 meV is 0.05 right. So, Joyce-Dixon approximation says that (E_F - E_C)/kT equal to ln n/N_C plus 1 by square root of n by N C, ok i.e.

$$\frac{Ef-EC}{kT} = \ln\left(\frac{n}{N_C}\right) + \frac{1}{\sqrt{8}}\left(\frac{n}{N_C}\right)$$

So, you know I assuming hundred percent ionization. So, whatever n-type doping I am giving is equal to n only I saying right. There is no p-type doping. So, you can replace this n with N_D also not an issue. Now this quantity, you know this quantity, I can write as minus 0.050 and kT at room temperature is given by 0.026. So, this is roughly around -2, not exactly, but close to -2. This is equal to log of n /N_C plus 1 by square root of n by N C. So, this N_C is supposed to be given in the question. So, suppose it is 10^{19} right sorry. So, it is given to be 10^{19}.

So, now there is only one unknown. This is equal to -2 right close to -2. So, it is only one unknown. So, you can actually do a couple of iteration or even put numerical solver and find out what is n, right, so, that is good. Same thing can be done for p-type also. So, this is the high doping effect. So, when your doping is high, right when your doping is high, you have to apply Joyce-Dixon approximation to find out the carrier concentration and another thing is if you keep doping higher and higher, then your fermi level keeps going up and up know.

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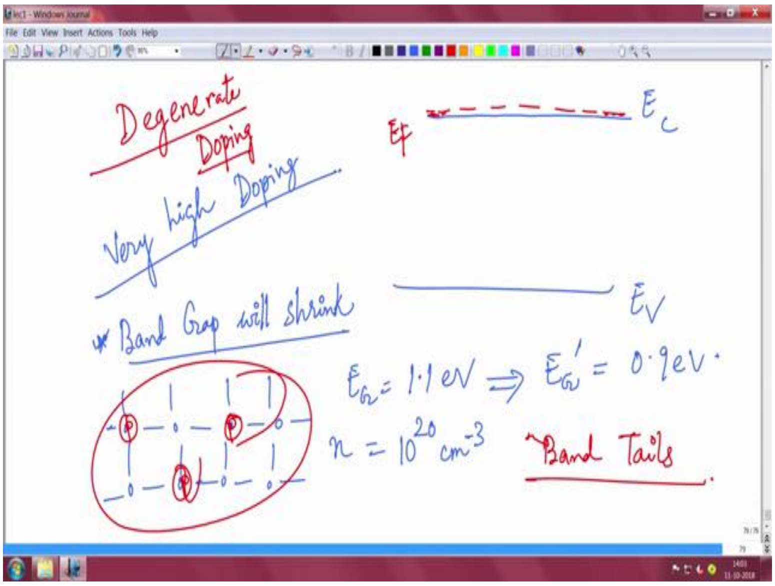

So, suppose this is your conduction band, this is your fermi level valence band. If you keep doping high and high eventually the fermi level can actually almost touch the conduction band, very high doping. In such case we call it is a almost degenerate doping ok. So, super high doping. I am talking about n-type only, but same will apply for p-type also; in the p-type fermi level will touch the valence band. So, here you see, the fermi level will almost touch the conduction band. It can even go above the conduction band you know; it can even go above the conduction, the fermi level can be above the conduction band also, if you dope it extremely high, the conduction band might actually come below the fermi level. If it is doped extremely very high if it is doped very high know very high doping, you know what will happen? Couple of things will take place actually. One very important thing is that your band gap of the material will slightly shrink or reduce.

When you dope very high, your band gap will slightly reduce. So, for example, silicon band gap is 1.1 eV. If you dope it very high; suppose you dope it n-type say 10^{20}, this band gap will now become smaller slightly. It may I do not know exactly how much it I mean I forgot the value, but it will become slightly smaller; it might become 1 electron volt or it might even become you know 0.9 eV. Do you know why? Because, with very high doping with extremely high doping you are putting too many of substituent atoms into the crystal that distort the lattice and your you know periodicity gets also distorted to some extent. The periodic potential that you have and you actually have you know this you have this periodic potential.

If you remember the atom atoms are all equally spaced apart right and when you dope it is actually like one in a million. So, you know out of one million silicon atoms only one will be replaced by say phosphorous, so, phosphorous. But, if you dope very high, then you know there will be more phosphorous for example, so, what happens is that the periodicity is sort of distorted and also that effects the band gap. So, you have things like band tails, band tails, which means the distribution actually extends below that you know band gap also.

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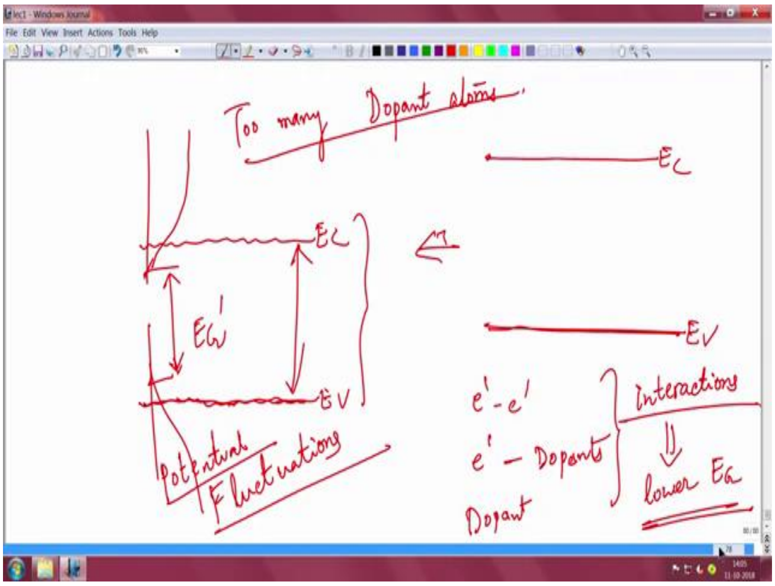

So, for example, if you have to many doping dopant atoms; if you have too many dopant atoms if you have too many dopant atoms, then because you are distorting the periodic potential or you know you are distorting the periodic arrangements of the atoms your what is what should be this E_C and E_V actually becomes little bit you know what it becomes is that, it becomes little bit jagged like it becomes slightly like this E_C E_V.

The band gap is always same at any point which is reduced slightly, but it becomes little jagged or little because this perturbation in the periodic crystal that will manifest itself as a not completely smooth conduction valence band over a position. You know, there will be periodic there will be a like a potential fluctuations ok; there will be potential fluctuations that will result in a band diagram like this.

We do not have to delete in day to day device analysis, but it just important to know that your, there is a you know there is a there is a fluctuation in the bandage and there is something called band tails. So, you know your fermi dirac distribution will extent little below the band also, will extend below the band also and so, your band gap also will shrink, right. Your new band gap E_G dash will slightly smaller than actually the true band gap of the material.

So, there is a lot of interactions between electron-electron and electron and donors or dopants right. So, too many interactions are there and all these interactions dopant-dopant interaction. This all interactions increase; this all interactions increase in magnitude when your doping is very high and all this interactions seek to lower the band gap ok. So, this is one effect you should keep in mind. You know with very high doping for example, your band gap will slightly shrink that is one thing we should keep in mind. It is very important you know implications actually later on in devices and also you should keep in mind that high doping Maxwell Boltzmann equation cannot be applied so, you have to apply Joyce-Dixon approximation. There are band edge perturbations and so on.

So, these are the things that we should you know keep in mind when they are very high doping effects. I told you conduction band you know fermi level, can actually go above the conduction band is very much possible is degenerate doping. It is extremely high doping and such high doping material you know, you know are typically not used in many devices, but there will be some devices where you have to dope very high. So, it is important to know that and then another thing we would like to discuss before we go to topics of you know mobility and carrier velocity and other things which are important for transport is called incomplete ionization.

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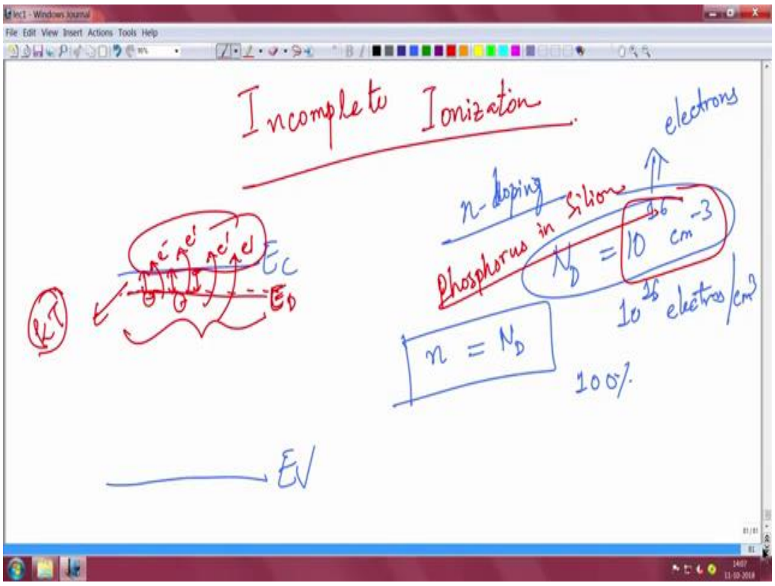

We should just be aware of this term we do not have to do the exact numericals or mathematics of this, but we should be aware of what is incomplete ionization. You know I had told probably in a very brief thing some of the last classes may be. I told you that in general, if I am only consider say n-type doping, there is no suppose p type doping ok; n-type doping. I told you that if I add 10^{16} donor atoms, say phosphorus and silicon, you know all of them are actually ionized which means all of them are giving electrons right.

So, I will get 10^{16} electrons/cm^3, right, electrons per centimeter cube and so, I said that the free electron concentration is equal to the donor concentration, right. This is assuming 100 percent ionization or 100 percent of this donors are actually ionized. And typically they are hundred percent ionized because I told you this is conduction band; this is valence band. And if I draw there is a there is a ionization energy level actually, donor energy level say this is E_D, the donor that you are putting you know like phosphorous for example, you are putting phosphorous in silicon as an n type dopant.

If you do phosphorous in silicon, then this phosphorous will introduce a donor energy level which is very close actually very very close ok. This energy difference between E_D, the donor energy level and E_C is kT or less than kT know less than kT which means at room temperature it is sufficient energy is there thermal vibration to put all the electrons up in the conduction band. All the electrons are put in conduction band and that is why if you put 100 of this, all 100 will give you electrons that is why, 10^{16} will give you 10^{16}; same thing with holes also for valence band.

Now, if for some material the doping is such that the donor ionization energy level is not so close, then what will happen? Then not all of them will get ionized right.

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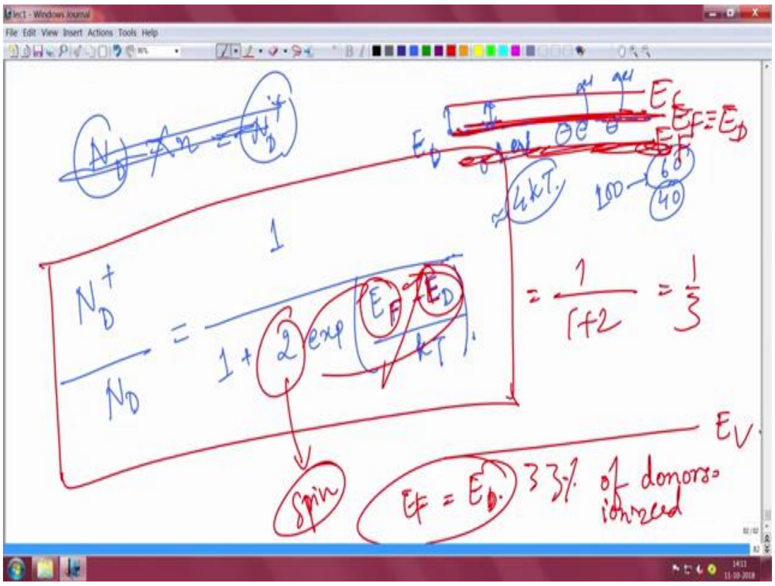

$~~N_D \approx n = N_D^+~~$

$$\frac{N_D^+}{N_D} = \frac{1}{1+2\exp\left(\frac{E_F - E_D}{kT}\right)} = \frac{1}{1+2} = \frac{1}{3}$$

$E_F = E_D$. 33% of donors ionized

So, for example, I take E_C, sorry, suppose I take E_V here and then there is a donor ionization energy level E_D which is little far away because the donor, the impurity that you are adding is such that it introduces a donor level little far away. Suppose this is you know 3kT or may be say 0.1 e V which is almost equal to 4kT almost right. 0.1 eV is little large four times the thermal energy. So, all the donor atoms that are there I mean not be able to give electrons may be some of them will give electrons some of them will not give electrons. May be if you put 100 of this phosphorous atoms or donor atoms, may be only 60 of them will give electrons rest 40 will not give electrons.

So, we see only 60 percent has ionized, 40 percent has not ionized and the donor energy level is E_D. Further away this E_D is from conduction band more and more it is possible that the ionization fraction has reduced. So, what is the fraction that has ionized? So, I told you typically you know if you assume hundred percent ionization, then N_D is equal to n which is equal to N_D^+ because all of them has ionized, all the atoms have ionized basically, right. But if it is incomplete ionization, then this is not true, right, the fraction that is ionized N_D^+/N_D typically it is 100 percent if it is fully ionized, 1 that is 1. But if it is incomplete ionize, then it will not be 1; it will be something like this, it will be something like this ok.

So, the fraction of ionized donors will depend as 1 by 1 this is a actually a fermi dirac distribution only, 1 plus 2 into exponential of (E_F - E_D) see E_F the fermi level comes into picture minus E_D, the donor level here.

$$\frac{{N_D}^+}{N_D} = \frac{1}{1 + 2exp^{\frac{Ef-ED}{kT}}}$$

The fermi level will be somewhere here know. So, fermi level minus donor level divided by kT, and this factor 2 comes because of spin, 2 states you know the spin. So, that is a factor of 2 comes there. So, for example, if I know the fermi level position and if I know the donor ionization energy level position, I know that what fraction of donors has ionized actually.

For example, if the fermi level and the donor level are the same time, at the same level, $E_F = E_D$. So, the fermi level and donor level at the same position then this will become 0, becomes 1. So, it will become 1 by 1 plus 2, 1/3. So, 33 percent of donors will be ionized if fermi level is equal to donor ionization energy level right. So, only that similarly for holes I mean this is an important thing actually not everything has to ionize similarly for holes.

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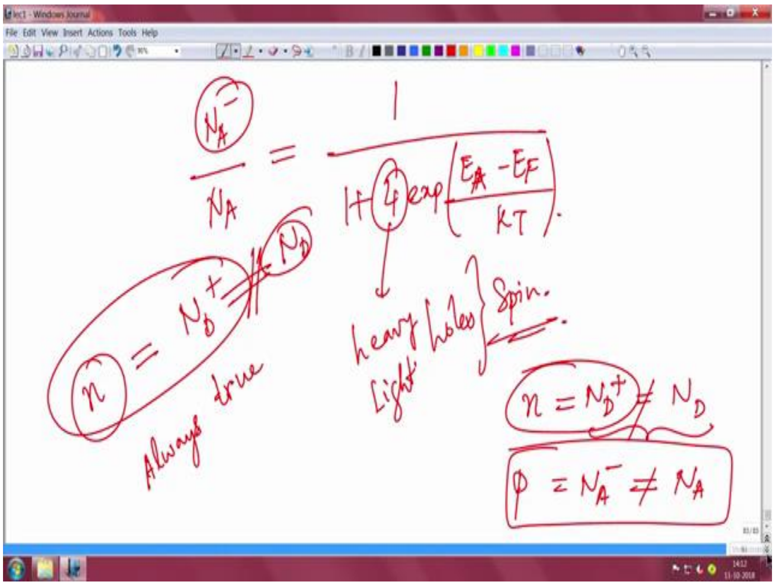

N_A^-/N_A this is the fraction of holes the not fraction of holes the fraction of acceptors that are ionized. Remember, when they ionize they give a hole or take away electron so that have become negatively charged.

This is the fraction that is ionized, that is given by 1 by 1 plus 4 exponential of (E_A - E_F) here. This is the acceptor ionization energy by kT i.e.

$$\frac{N_A{}^{-}}{N_A} = \frac{1}{1 + 4exp^{\frac{EA-Ef}{kT}}}$$

The reason this 4 comes because of heavy hole and light hole and then there is a spin so, 4 ok. You remember that for say n type doping, your electron, only if you have a electron n type doping, then the total electron, the negative concentration has to be equal total positive concentration. This will always be true for an n-type dope material ok. Even if it is incompletely ionized it is always true, these N_D^+ need not be equal to N_D if it is not 100 percent ionized.

So, n will always be equal to N_D^+, but whether all the N_D^+ comes from N^D some of them may not have come from N^D. If it is 100 percent ionized, this will be hold true. If it is not 100 percent ionized, this will now whole true. So, say 60 percent ionized or may be 50 percent, but this will always hold true same thing with holes ok. This will always be true, but this need not be true, this need not be true. So, that you know that we should keep in mind.

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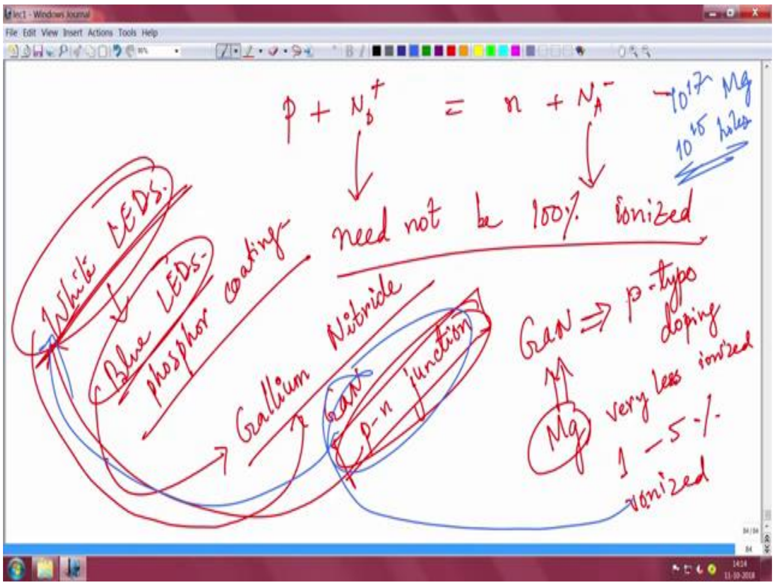

So now, if you have you know, I told you that if you have both electron and hole doping, then the total positive charge is this, total negative charge is this. This will always hold true except that this and this need not be 100 percent ionized, need not be 100 percent ionized. They can be incompletely ionized also.

So, for example, you know you have seen the white LEDs that you can buy from the market, you can buy white LEDs for 100 rupees or so. This white LEDs are actually in fact, blue LEDs and this blue LEDs, they have a phosphor coating they, have a phosphor coating to convert the blue light to white. And this blue LEDs are always made up of gallium nitride. It is a semiconductor. All the white LEDs that you see in your parking garage, on the streets, in the airports, in your houses, to stadiums all the white LEDs are actually made of gallium nitride semiconductor. And actually a gallium nitride p-n junction, a gallium nitride p-n junction will actually be the white LED that is what is happening. There is little bit more to it not telling you right now. But that is what is happening will talk about p-n junction later. Why I am telling you this, because in gallium nitride, p-type doping, for p-type doping you have to add magnesium.

The moment you add magnesium to gallium nitride, you can dope it p-type. This magnesium is very less ionized. You know what, only 1 to 5 percent is ionized as a p-type, which means if I add 10^{17} magnesium atoms, only 10^{15} holes I will get. It is so low and that has implication in the gallium nitride p-n junction which has implication in the white LED. So, these are important things that we should keep in mind. So, incomplete ionization is not something very fancy that we have to study only for our course. It is very important for practical devices like white LEDs, right. So, we should keep that in mind.

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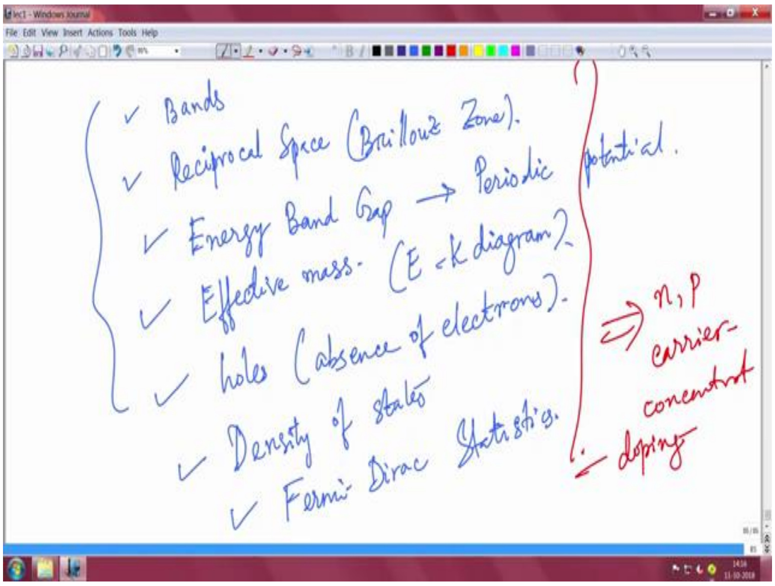

✓ Bands
✓ Reciprocal Space (Brillouiz Zone).
✓ Energy Band Gap → Periodic potential.
✓ Effective mass. (E-k diagram).
✓ holes (absence of electrons).
✓ Density of states
✓ Fermi Dirac Statistics.

⇒ n, p carrier-concentrat
∴ doping

So, now we have finished up many things see. Going from the beginning, we have defined semiconductor bands right. We have defined reciprocal space. In this course, I am just taking a stalk here ok. We have defined reciprocal space and things like Brillouine zone. Why they are important? I have told you how band energy bands are formed and energy band gap is formed right, how energy band gap is formed. If you recall the periodic potential, the periodic arrangement of the atoms, periodic potential right so, how energy band gap is formed. And I told you about things like effective mass of electrons, how the entire quantum mechanics is captured in the E-k diagram and effective mass comes from there. Electrons going from valence to conduction band leaves behind a holes.

So, I introduced the concept of hole, absence of electrons, ok. These are things we had studied initially, then we came about density of states to tell us how electrons and holes will populate the number of states that are there, also it depends on fermi distribution, fermi dirac statistics. And from there, we went to how to calculate carrier, n and p carrier concentration. Those we have discussed how to calculate carrier concentration, what is doping, right. We have done that and what are the other things we have done in the course till now.

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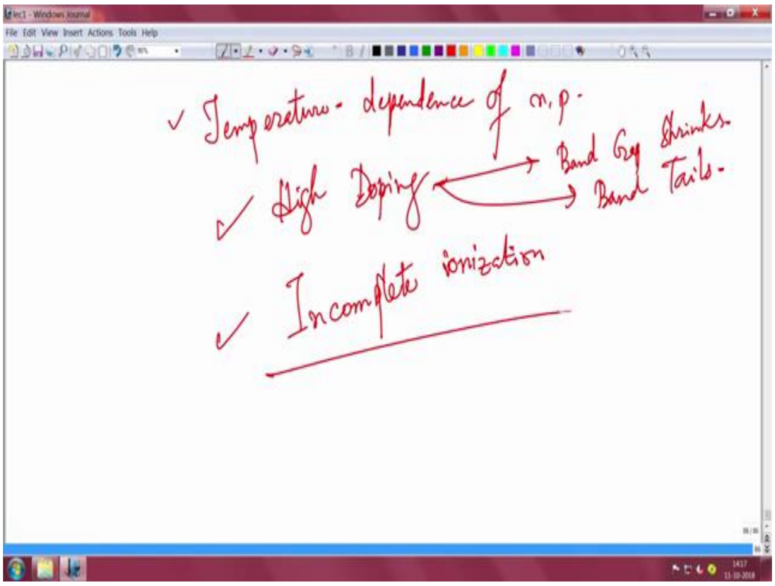

✓ Temperature dependence of n, p.

✓ High Doping → Band Gap Shrinks.
→ Band Tails.

✓ Incomplete ionization

We have done about how temperature dependence of carrier concentration, temperature dependence of carrier concentration, n and p. I have talked about high doping effects right, high doping effects, talked about incomplete ionization which is very important by the way; incomplete ionization by the way in high doping effects. We have discussed about band gap narrowing, you know band gap can shrink in high doping effect, there will be band tails, we have discussed about things and how you applied Joyce-Dixon approximation to calculate charge density and so on. These are the things that we have studied.

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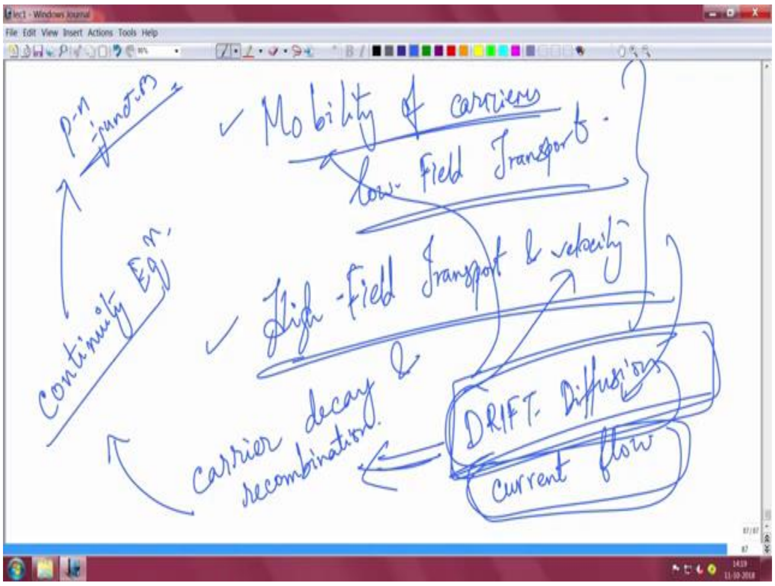

p-n junction

✓ Mobility of carriers

Low-Field Transport.

✓ High-Field Transport & velocity

DRIFT-Diffusion

current flow

carrier decay & recombination.

Continuity Eqn.

Now, what is the next point in the agenda? The next point in the agenda will be to introduce mobility, mobility of carriers ok. Mobility of carriers, low field transport, ok, low field transport and after that, will introduce high field transport, high field transport and velocity, carrier velocity. So, low field transport, high field transport will come along with that, what will come is drift and diffusion, drift and diffusion of carriers will come. So, that we can understand electron or current, you know current flow. We can discuss current flow. This transport, how electrons and holes are transporting depends on the transport drift and diffusion, they depend on velocity, they depend on mobility right, current will flow and once we discuss about current flow before that also we have to discuss about carrier recombination and decay. how carriers decay. Why they decay? We will talk about that carrier recombination and decay, and then will talk about continuity equation continuity equation.

So, this is the cycle that will complete in the next perhaps 2 or 3 lectures. We have to complete this mobility of carriers, low field transport high field transport, drift diffusion, current flow, then recombination carrier decay, continuity equation. Now after that, we are ready to go to p-n junction and devices. So, that is the agenda here. So, we will wrap up the class here today and in next class we will again look at will start from mobility of carriers. So, will end the class with the note that we have done high field transport and incomplete ionization. Today we are good we are confident and familiar with carrier statistics, how to calculate electron and hole concentration. Now we shall start to study mobility and low field transport from next class, ok. You might have heard the word electron mobility a lot ok. We will understand what the electron mobility actually means ok. So, we will end the class here.

Thank you for your time.