Fundamentals of Semiconductor Devices

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Lecture - 08
Temperature-dependence of carrier concentration

Welcome back. So, we shall continue. Now if you recall what we had learnt in the last class in semiconductor devices is that we had learnt about doping, things like fermi dirac distribution, density of states; all those things we cleared. We also had solved some very simple numerical of p and n doping together right; one is called minority, one is called majority. So, we will continue on discussion with doping, temperature dependence of your carrier concentration, things like high doping effect ok. And from there on, we will try to move to scattering mobility and other things in the subsequent lectures, ok. So, we will recall what we had last where we had last stopped, ok. So, we will go to the slide or the page where we last finished in the last lecture and I will bring that up here.

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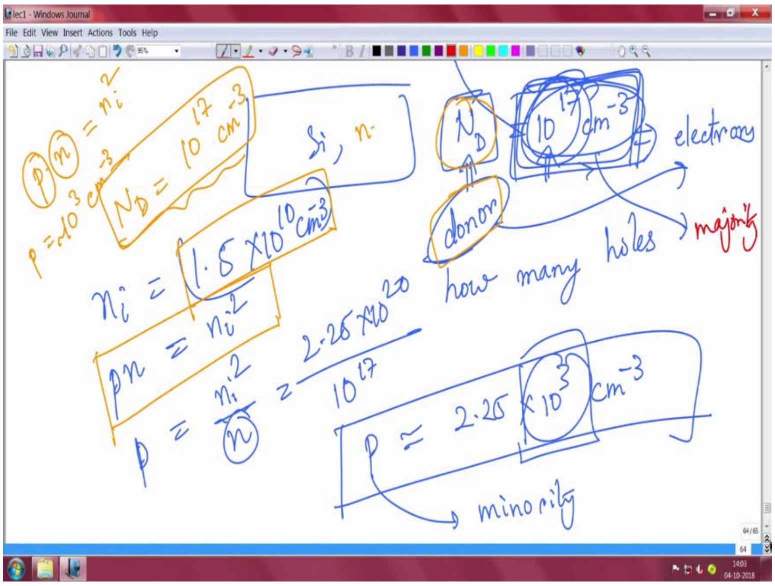

So, you can see that in the last class, I told you that you know the doping and whatever you are doping that becomes the majority. For example, if you are doping silicon n-type, then you are doping you know with phosphorous or arsenic. So, that's donor and it is a majority. Suppose you are doping $10^{17}/cm^3$ that is your say n-type doping density and it is called majority. And the minority carrier concentration can be found out by this simple formula. It is the principle of mass action which is the product of p-type and product of n-type. The total number of electrons and total number of holes has to be constant in equilibrium which is equal to the square of the intrinsic carrier concentration.

So, intrinsic carrier concentration is 1 point you know 5 into 10 to the power I told you right you know that we derived that. So, if n-type concentration is 10^{17}, then your number of holes is actually 10^{3} only, almost. So, that is called minority. So, it is a n-type semiconductor. This is a very simple way of actually doing, the finding out the electron and hole concentration.

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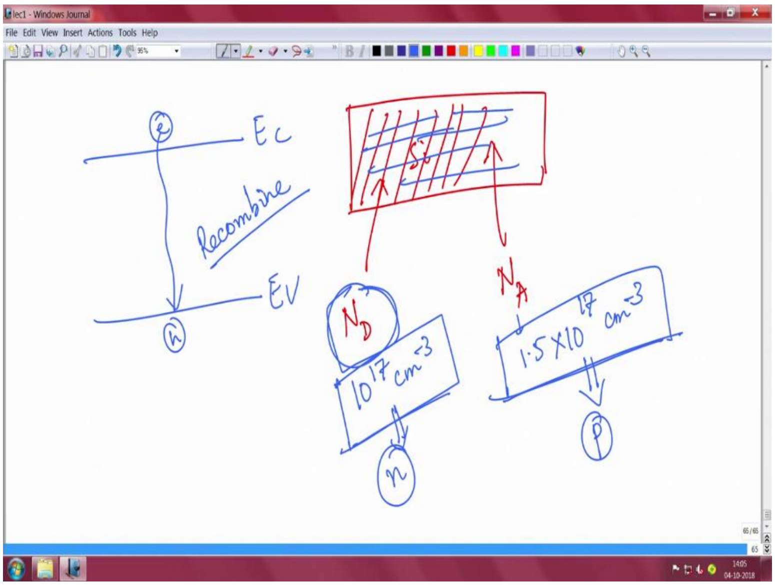

Now, another very important thing will come now here is that suppose you have a piece of semiconductor, say silicon, you are putting some n-type dopants you know N_D and also you are putting some p-type dopants in the whole thing ok. You are putting some n-type doping here, also you are putting some p-type doping in the hole. So, what will be the net concentration? Will it be electrons or will it be holes? Because you know an electron in a conduction band and a hole in the valance band; if there is an electron here, if there is a hole here, they can recombine right; they can recombine and they can release the energy, right.

So, how many electrons will be there? How many holes will be there? Suppose I have put 10^{17} doping of your n-type doping and suppose I have put you know 1.5 X $10^{17}/cm^{3}$ of boron. So, it will give you p-type doping. So, of course, I told you that if I put 10^{17} donors, then I will get equal number of electrons. So, this also will become equal to electron. If I am using so many of p-type doping or acceptor doping, then I will get this many of holes right.

So, how many of these will be electrons and how many I mean how many electrons and how many holes will eventually survive? Because many of them will recombine among each other know, between each other. So, to find that, we have to understand a very important or you have to you know invoke a very important condition which will be always valid in semiconductor physics and that is called Charge Neutrality ok.

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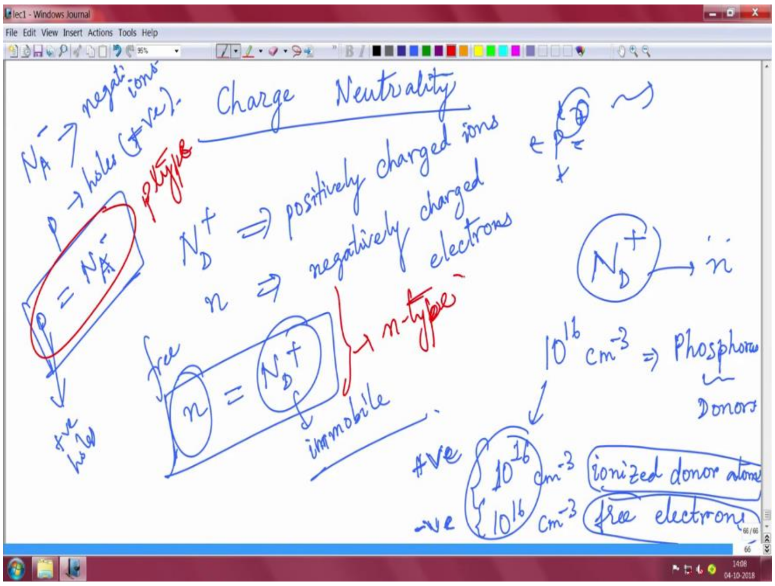

In equilibrium the total positive and negative charges have to cancel each other ok. So, the net negative and positive charge have to remain totally zero, that is called charge neutrality.

So, suppose I have a phosphorus atom I told you right, it will give it has 5 electrons and then 4 electrons it bounds and 1 electron is free that goes away right. When it donates an electron; when a donor donates an electron, it leaves behind the positively charged ionized core right. So, I told you that if we have N_D, if we have N_D number of donors, then after giving all the electrons and so n number of electrons will come which is equal to N_D. What remains behind is N_D^+ number of ionized donors which means suppose $N_D = 10^{16}$, I have added $10^{16}/cm^3$ of say phosphorous atoms I have added. So, what will happen is that there is donors right. They will donate it the electrons. So, after they all donate electrons what remains behind, what remains behind is $10^{16}/cm^3$ of ionized phosphorous atoms will remain.

I will call them ionized donor atoms right because they have given electrons. So, what remains is positively charged ion. So, this many positive charges remain because the ionized cores remain and also $10^{16}/cm^3$ of free electrons exists now; of free electrons exists now because all the atoms, phosphorus atoms have given away the electron the extra electron so, you get you know $10^{16}/cm^3$ of free electrons in the sample. These are, an electron is a negatively charged particle. So, you have 10^{16} which is the ionized charge core atoms ions and so, the total charge is 0. But electrons will be there because electrons are not recombining with donor ions there are no holes here.

So, essentially what it means is that N_D^+ number of positively charged ions will be there, positively charged ions will be there and then n number of negatively charged electrons will be there; I mean electrons are negatively charged that you all know, right, I mean that is we all know. So, negatively charged electrons will be there and under charge neutrality it is 0. So, $n = N_D^+$ ok. So, the total charge is 0, these many free electrons are there that can move and carry current. These are immobile positive ions are there, which have come because they have donated the electrons away.

Similarly, if I have a p-type material, I give N_A of you know acceptor ions, acceptor atoms then they will give the holes. So, p number of holes will be there which are positively charged, right, they are positively charged, but N_A will give holes; they will take electrons. So, this many number of negatively charged ions are there, right, at equilibrium, $p = N_A^-$, right.

So, this is positively charged holes; this is positively charged holes and this is negatively charged your ionized cores; this is negatively charged ionized cores. Now this is only for, this is only for p-type material right. This is only for p-type doping, right and this is only for n-type material, n-type material.

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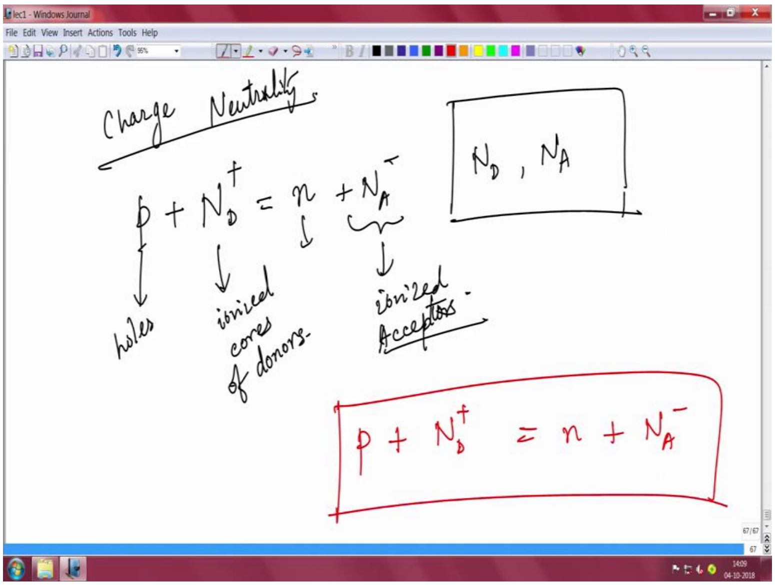

The question is what we what happens when we have both, right. Suppose, what I am trying to say is that I have given both electrons and holes know, suppose I have given N_D number of donors and N_A number of acceptors together, then what will happen? Then the charge neutrality has to written, the charge neutrality condition has to be written first. What is the charge neutrality condition? The charge neutrality condition will say the total positive charge. What is the total positive charge? One is holes. Holes will be positively charged N_D^+, sorry, N_D^+.

This comes from this is holes which is positively charged and this is your ionized cores of the acceptors, right; ionized cores of acceptors, which sorry donors sorry ionized cores of donors because they have donated the electrons. So, that remains all remains basically positively charge I told you that should be equal to the total negative charge which is $n + N_A^-$, because this comes from the ionized acceptors ok; this comes from ionized acceptors and this is free electrons of course.

So, what I can write is that I can say that, total $p + N_D^+$ should be equal to total electrons plus negatively charged ions, ok.

$p + N_D^+ = n + N_A^-$.

This is your charge neutrality condition and from this you find out everything. If it is n-type dope only, then the p-type doping is not there. So, this will not be there and this will not be there, that is true and if your electrons only p-type doping is there, then this will not be there and this will not be there right so, that basically it. So, now, we know how to do it.

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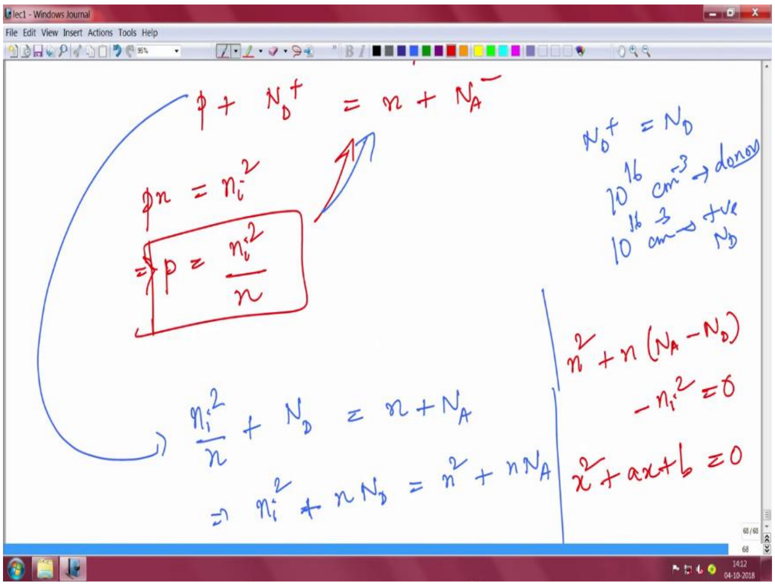

So, let us see now, I know that so I will write the equation again. $p + N_D^+$ is equal to $n + N_A^-$,

$p + N_D^+ = n + N_A^-$.

but I know that P n in equilibrium is this is all equilibrium I am talking about is equal to n_i^2 right.

So, what I know? I know that $p = n_i^2 / n$ i.e.

$$\frac{n_i^2}{n}$$

Why do not I substitute that back here ok? I will substitute that back here. So, I can write this equation now as p can I can write as $n_i^2 /(n + N_D^+) = n + N_A^-$.

$$\frac{n_i^2}{n}^{+} + N_D^+ = n + N_A^-$$

Of course, you know $N_D^+ = N_D$ only because if you have given $10^{16}/cm^3$ say of phosphorous atoms as donors, then $10^{16}/cm^3$ of you know plus positively charged ionized donors will be there.

So, instead of that N_D plus N_D minus I can just write N_D and N_A. This is the same number know, we are assuming 100 percent ionized. So, what I can write now here is, I can multiply both sides by n. So, it will become n i square plus sorry, plus n X N_D equal to n square plus n into N_A, ok.

So, what can I write here now? So, what I can write here now is that n^2 + n, N_A minus N_D plus sorry not plus minus it will be, minus n_i^2 is equal to 0 i.e.

$$n^2 + n (N_A - N_D) - n_i^2 = 0$$

So, does it look like x square plus a x plus b equal to 0 so, quadratic equation i.e.

$$x^2 + ax + b = 0$$

So, let us see how we solve the quadratic equations.

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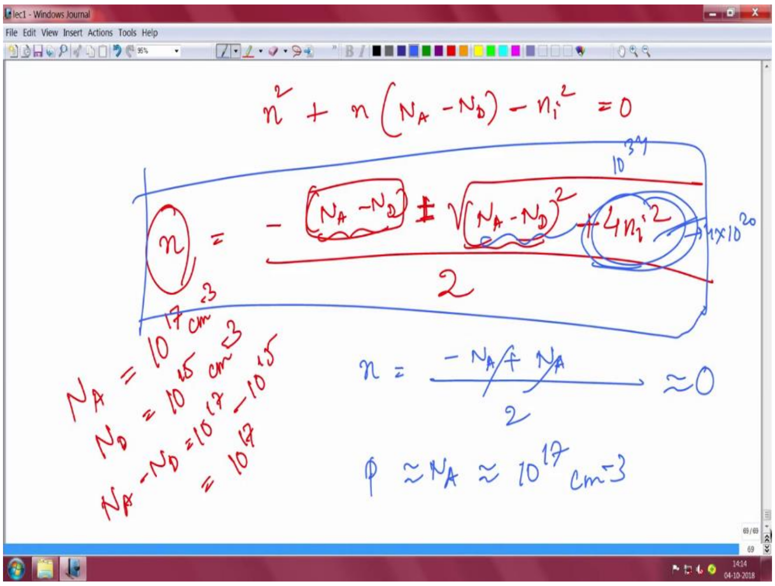

So, I have n square plus n, N_A minus N_D plus sorry not plus minus, n_i^2 is equal to 0.

$$n^2 + n (N_A - N_D) - n_i^2 = 0$$

So, n will be equal to – (N_A - N_D), depends which is the larger here, right plus minus so total number has to be positive. So, it cannot be you know the negative it cannot be negative number n cannot be negative, but the reason I am keeping plus minus is because this quantity may be negative, you know. we never know anyways. It will be equal to $(N_A - N_D)^2 + 4 n_i^2$ by 2. That is your solution. right.

$$n = \frac{-(NA - ND) \pm \sqrt{(NA - ND)^2 + 4n_i^2}}{2}$$

So, if you see the expression here (N_A - N_D) is there. If one them is very large, N_A is very large compared to N_D, then we can just you know suppose N_A for example, I am telling you a doping situation where I am doping both N and P type doping are coexisting ok. So, that is why we are doing that if both N and P type doping not exist. Only one type doping is there, then we do not care about what is because then your n = N_D only right. Only when there both types of doping this situation then this equation is valid right, I mean make sense right.

So, suppose I am doping holes 10^{17}/cm^3 P type and suppose I am doping N type only 10^{15}/cm^3, then (N_A - N_D); this equation here know this (N_A - N_D), even here for example, will be 10^{17} - 10^{15}. It is almost equal to 10^{17} only. It is like 100 minus 1 right.

So, then you can neglect N_D and then you can have a simplified expression. For example, then you will have n equal to minus N_A, right, because it will be N_A only know; there will be nothing there. So, plus N_A^- this will be much smaller. This will be 10^{15}, 10 to the power you know square of 10 to the power 20 for example, right this will be 10^{20}, 4 X 10 ^{20} approximately. And this will be equal to how much? This will be equal to 10^{34} because N_A^-. So, this is negligible. So, it will only remain N_A here by 2; this 0. So, almost electrons will be 0. So, holes will be equal to N_A which is equal to 10^{17}, that is easy right.

But if both electrons and holes are there, then it then you have to take into account these expression. And if you know your doping is low so that your this cannot be neglected, then also you have to use this equation for example, ok. So, it is important that we use that equation. In other words, this is a very universal equation which can give you the number of electrons and holes for a semiconductor where both N and P type doping are coexisting, ok. That is one thing.

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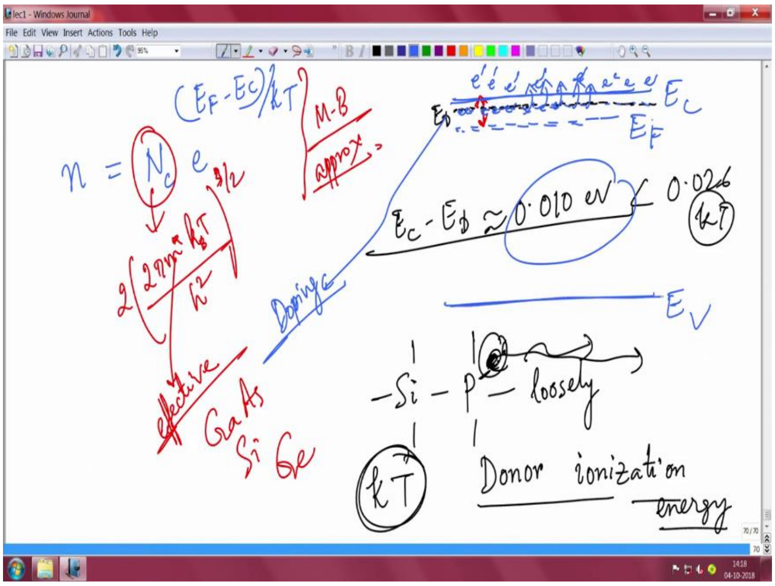

So, the next thing we will learn today is that you know I told you that suppose I have only an n-type semiconductor. Let us not make it both N type and P type. Suppose I have only N type semiconductor and the fermi level is here. You know the and you know the expression if you do not remember you should remember the total number of electrons is given by n equal to N_C exponential of $(E_F - E_C)/kT$ right.

$$n = N_C\, exp^{\frac{Ef-EC}{kT}}$$

So, $(E_F - E_C)$ is basically I keep telling you, this spacing, the negative of this spacing, $(E_F - E_C)$ is the negative of this spacing and N_C of course, depends on, you know effective mass like 2 phi m star kT. This is a Boltzmann constant by something like h^2 X 3/2. Of course, this depends on effective mass of the material and each material has a different effective mass.

For example, gallium, arsenide will have a different effective mass. Silicon will have a different effective mass, germanium will have different effective mass. So, if effective masses are different, then this N_C also keeps differing, that is one thing we should keep in mind. So, this expression comes from the you know Maxwell Boltzmann equation. If you remember, call it Maxwell Boltzmann approximation to fermi dirac and that is how we get. It takes into account both fermi dirac distribution as well as density of states.

Now, what are we going to tell you, you know here is that, suppose I take you know the donor ionization energy level is very close up here; I told you right E_D. When you at say you know if you take a silicon and I am telling you that if I put phosphorous here, then one electron is free that goes away know that is loosely bound. That is why that is free and it goes away and that loosely bound columbic energy which the electron over comes and goes away to become free is called the Donor Ionization Energy. A similar thing accepts also exists for acceptor ionization energy for P type. I am talking only of N type here.

So, donor ionization energy is basically the energy required for the electron to let loose from this phosphorus atom and go and become an N type you know, contribute to N type conductivity. So, this donor ionization energy I told you is very small, typically in the range of smaller than around thermal energy kT. So, that room temperature energy thermal energy kT is sufficient to break this into bond and make this electron free and go away, ok.

So, the donor ionization energy is typically very very small and the spacing and the energy that energy has level is positioned as E_D, very close to the conduction band its know. That it is basically (E_C - E_D) ok. If donor ionization level is there is very small is in the range of kT, it can be in the for example, kT is 0.026. So, this can be say 0.010 for example electron volt which is very small; this much smaller than 0.026 room temperature.

So, at room temperature where the thermal energy is sufficient to excite the electrons from the donor ionization energy, I mean you know, this to lose the, loosen the electron from the post atom. Actually you can imagine that this donor ionization energy is also, it is a donor level which is filled up with electrons and it is close to conduction band. It is so close to conduction band that the energy 10 meV is very small with which it is bound. So, any room temperature fluctuation of energy which exist all the time will be sufficient to excite this electrons up here. And so, on all the electrons will come here right, and that is how they populate the conduction band with lot of electrons and that is how conductivity changes and that is how doping actually changes your conductivity or conductance of the material. That is why you know semiconductor is very awesome because metals and insulators cannot do that.

So, this donor ionization energy level exists very close to the conduction band fermi level of course, can be close that or even below that for example, and then this is the valence band right. Now, we all know these things and the reason I am telling you this is again because let us zoom it very well.

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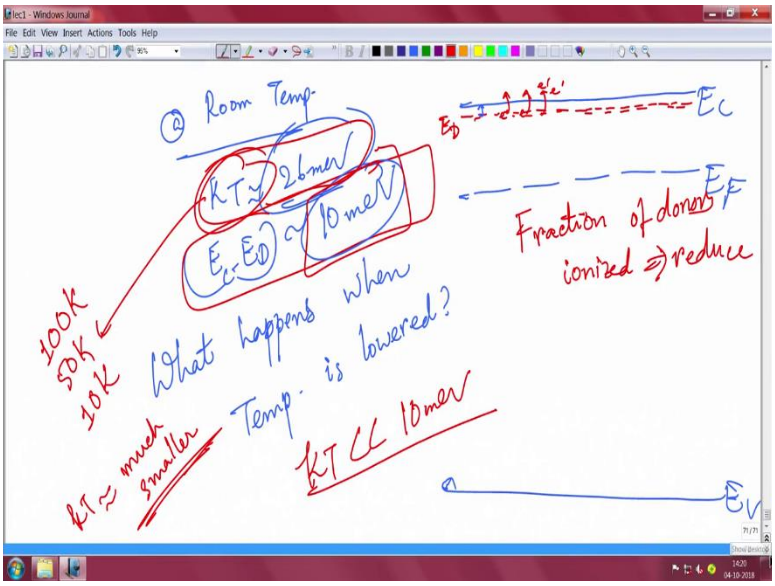

So, this is conduction band. Suppose this is valence band this is silicon only and suppose this is your fermi level. I am drawing it in a very zoomed way and suppose this is your conduction sorry this is your donor ionization energy level which is say E_D. Of course, it will be constant ok, there is no slope here. My drawing is not very good, but you know there is no slope here. So, your what is it called your donor ionization energy level is very close to conduction band, very close ok. It is close to almost you know less than kT.

Now, at room temperature at room temperature I told you that your energy, thermal energy kT is almost equal to 26 meV and this donor ionization energy I call it $(E_C - E_D)$. This donor ionization energy say 10 meV. So, room temperature energy is sufficient to actually excite the electrons from the donor level to the conduction band. That is how you get conductance, but the question is what happens. This is the important question that we want to answer now. What happens you know when for example, temperature is cooled down you know, temperature is lowered for example, or the sample is cooled down?

So, you see room temperature energy k T becomes 26 meV only at 300 kelvin. But if I go to say 100 kelvin or if I go to say 50 kelvin or 10 kelvin, then your kT becomes very small, much smaller, right, because the temperature reduce know much smaller whereas, your $(E_C - E_D)$, this quantity donor ionization energy level actually does not change it temperature much. So, it will always remain at 10 meV. So, if your kT becomes less, much less than your 10 meV, then your room then the temperature the thermal energy at low temperature will be so low that they will not be sufficient to excite the electrons from the donor level to the conduction band. What will happen then?

Then the fraction, then you know, then the fraction of donors that is ionized fraction of donors that is ionized, will also reduce, will reduce. What it means is that suppose you know, I have a conduction band I have valence band. It is N type dopes of fermi level is up here and I told you that you know there is very small you know there is a very small difference; there is this donor ionization energy level E_D.

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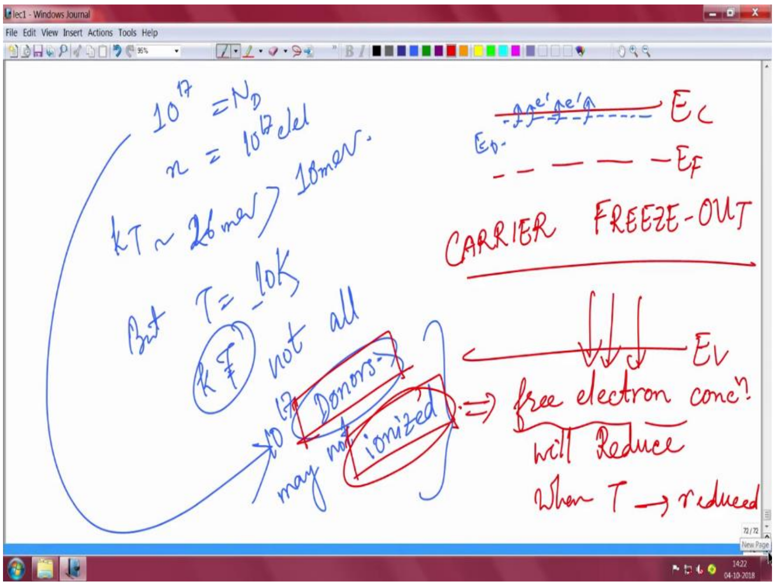

I told you that if I have 10 to the power say 17 (10^{17}) phosphorous atoms I am giving, then I am expecting to get 10^{17} electrons only, why? Because all the 10^{17} phosphorus atoms will give electrons here, right in the conduction band. Because at room temperature, kT is 26 meV is actually larger than the ionization energy say 10 meV.

But, at temperature is equal to say 10 kelvin, your kT is much lower. In that case, not all not all 10^{17} you know donors would be able to give electrons, which I call donors may not be ionized, will not be ionized. All the 10^{17} donors that I have added here will not be ionized because the temperature kT has become much smaller and that smaller energy is not sufficient to excite the carriers.

So, the fraction of donors, the fraction of donors that is ionized and that is able to give electrons will reduce with lowering temperature. What will also it will result is that your free electrons concentration will reduced, your free electron concentration will also reduce, because your number of donor atoms that are ionizing also reduces. Then why does it happen? Because your temperature is reducing say 50 kelvin, 20 kelvin, 10 kelvin. So, your kT is also decreasing.

So, it does not have energy, the thermal energy does not have energy to actually excite. So, this process the carriers reducing at lower temperature is called Carrier Freeze Out, ok. They are freezing out at lower temperature; that is what is happening. It is carrier freeze out is basically the lowering of free electron concentration of free hole I mean anything is fine; free carrier concentration is getting lowered when your temperature is reduced, right.

When your temperature is reduced than your free carrier concentration is reduced; that is called you know, it will go exponentially reduction actually it will go exponentially.

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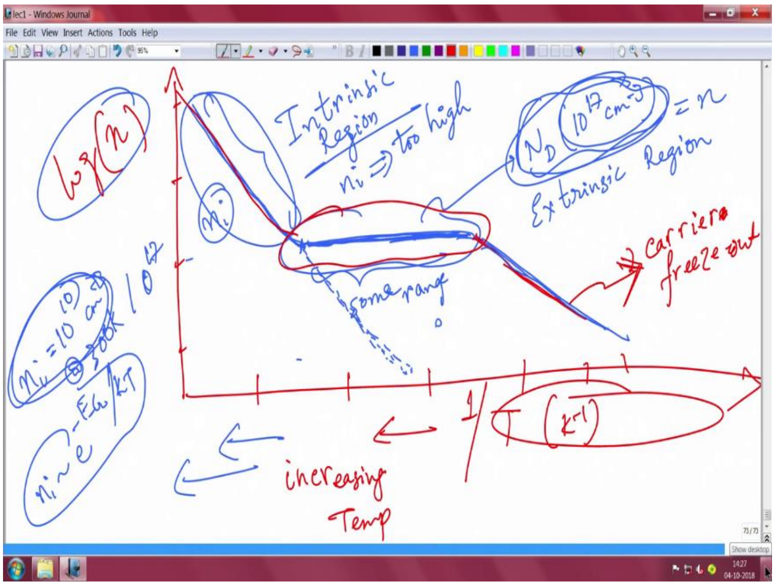

So, if I have a, the carrier concentration is suppose n, I plot on the y axis and suppose on the x axis I have temperature, 1 by temperature ok. So, it is kelvin inverse. 1 by temperature, this is very high temperature. This is lower even lower, lower, lower, lower right.

So, at very low temperature at very low temperature, what will happen? Very low temperature this region right, very low temperature, your carrier concentration will reduce as temperature reduces. So, it will decrease like this and this is by the way log of that. So, it is a log scale, it is log scale is linear. So, exponentially actually reduces.

So, what is happening is that, this is your carrier freeze out that is happening carrier freeze out. What is happening is that your carrier concentration is reducing with reducing temperature because fewer and fewer carriers are able to get ionized ok; fewer and fewer carriers are able to get ionized as you are reducing the temperature ok. As you are reducing the temperature fewer and fewer carriers are getting ionized.

As you are increasing the temperature which means you are coming this side, this is increasing temperature right, 1/T. This is increasing temperature, if you are increasing temperature, more and more fraction gets ionized eventually approaching room temperature or even lower than that, what will happen is that all 100 percent of that will be ionized. So, it will be flat. What is this flat? Because this is defined by N_D the doping that you have given.

If you have given there is no P type only N type suppose, P type can be talked about separately. Suppose you have given $10^{17}/cm^3$ of dopant to the, this is remembered number of free electrons. So, number of free electrons also will be 10^{17}, ok; number of free electrons will be also 10^{17}. That is, the number of donors which is this is equal to number of free electrons which is same 10^{17}.

So, it remains flat ok. It remains flat ok. It remains flat for a some region of temperature for some range of temperature including room temperature, it remains, some range of temperature it remains flat and this region is called the extrinsic region, because this is the region where your number of free carriers decided by the doping density N_D. So, it is flat and you want to operate the device in this regime because here you exactly know the number of free carrier which is equal to N_D, which you pre decide.

So, all conductivity, resistivity, current, voltage everything that we have to do in a device, we know exactly the carrier concentration here ok. Lower than that gradually, it drops down because the carriers are freezing out, they are not able to ionize. Although the number of doping has 10^{17}or whatever you have kept is fixed, but out of this 10^{17}, not all 10^{17} are ionizing. So, the fraction of free carrier available is reducing that is what is happening here.

And as you keep increasing temperature more and more, very high, what happens is that you have an n_i know, that intrinsic carrier concentration in silicon is 10^{10} approximately at room temperature at 300 kelvin. So, that is much lower that is much lower somewhere here right; that is much lower. But, as you increase the temperature, your n_i has a exponential dependence on E_G /kT.

So, as you keep increasing temperature, this n i will keep increasing. At sufficiently high temperature may be 5, 600 degree Celsius or so for silicon, your n_i actually it becomes so high. Actually n_i keeps coming like this from here, it will take over; it will take over here, n_i is actually gives on it keeps its very low, but with higher temperature n_i keeps increasing, increasing, increasing, increasing at some point may be 5, 600 may be, 700 may be, 400 that you can calculate, at some temperature degree Celsius. n_i will become so large that it will even more, be more than 10^{17}whatever you are doping and it will go crazy high like. This it will go exponentially like this.

This region is called intrinsic region, except that, in intrinsic region, excuse me, that carrier concentration the intrinsic carrier concentration n i has become too high. The too high much higher than N_D even. So, that here you know at very high temperature, your carrier concentration has become so high that your sample becomes almost like metallic. It has become too many it has too much too many too many electrons to many holes; n_i electron n_i holes, holes are there. This is much larger than 10^{17} almost like metallic it can carry much higher current and you lose the control of conductivity by doping which is what makes semiconductor very ideal candidate. You want the semiconductor to have conductivity that you define in this region as this one, but if it is higher temperature it will go crazy because the n_i has increased.

If you go too low also n_i, i mean n_i is not there, but free carrier will also freeze out. So, that is this is how the temperature versus log plot looks like. So, if I can draw it fresh again.

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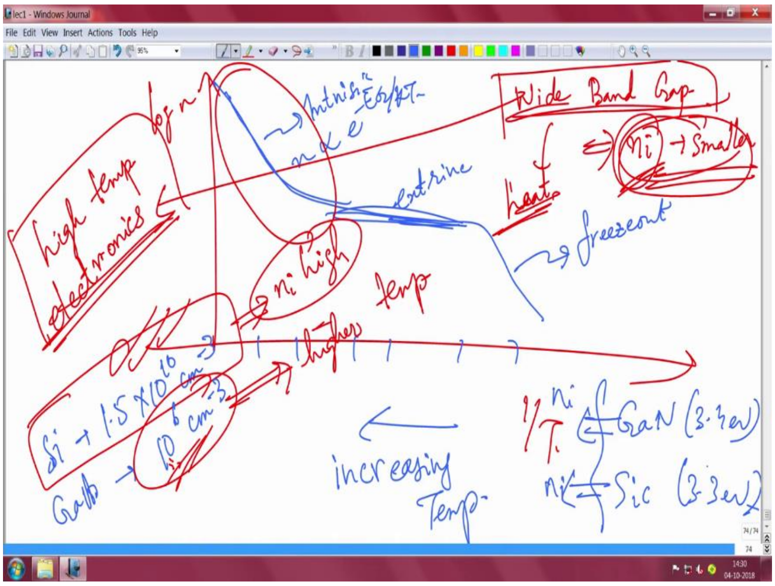

So, essentially log of n you are plotting right versus 1 by temperature. So, at very high temperature, this is 1 by temperature so, you know this is increasing temperature. So, at very high temperature it will blow up, then it will stay flat eventually it will fall down. This is called freeze out region, this is called extrinsic region where the number of electrons is almost fixed and you want to operate in this regime only, and then this is called Intrinsic Regime. Here carrier concentration will go exponentially as e to the power minus E_G/kT; that is what is happening.

So, you see silicon for example, has a band gap of 1.1. So, that carrier concentration is 1.5 X 10^{10}/cm^3. Suppose gallium, arsenide has a higher band gap; it has like 1.4 e V band gap. So, the carrier concentration is around 10^8 or 10^6, I do not remember exactly, is probably like 10^6.

So, you see silicon will basically have, it has a higher intrinsic carrier concentration. So, at higher temperature, it will blow up even faster. This will withstand little higher temperature, it can withstand till little higher temperature because the carrier concentration is low. So, you have to go to a little higher, much higher temperature than this case in order for this to blow up which means because wide band gap material. If you have wide band gap the band gap is large. I told you in one class that n i is much smaller there, right, because it goes inversely has band gap the exponent of that.

So, you can heat a wider band gap temperature to higher temperature. You can heat up a wide band gap material to a much higher temperature and expect the n_i to remain small. But for small band gap material like silicon or germanium, this n_i is already very large, I mean 10^{10}. So, if you heat up the material, then n i will become much more larger. So, wide band gap materials can be used for high temperature electronics, ok. For if you want to use high temperature electronics you want to measure some something in the fume hood of a missile or a satellite fume lot of high temperature 1000 degree Celsius or any other application industrial automotive vehicle industry where you know 5, 600 or 1000 degree Celsius processors are there.

And you want electronics to do some sensing, some measurements and some integrated circuits and so on, then high temperature electronically for that you need a material with wider band gap because wider band gap materials will have lower n_i and hence this blowing up of the n_i with temperature will be much delayed because smaller band gap material has very high n_i and so, smaller band gap material will quickly exceed this n_i quickly know, that is why you go for wider band gap material for high power electron high temperature electronics like gallium nitrite. It has a band gap of 3.4 e V, silicon carbide it also has a band gap around 3.3 e V.

So, this kind of materials are used for high temperature electronics because they can withstand much higher temperature, ok. Still be the reason is because there n_i is very small; n_i is very small. You want very small n_i otherwise it will blow up. So, that is very important thing that we have said. So, now we have considered the temperature variation of doping.

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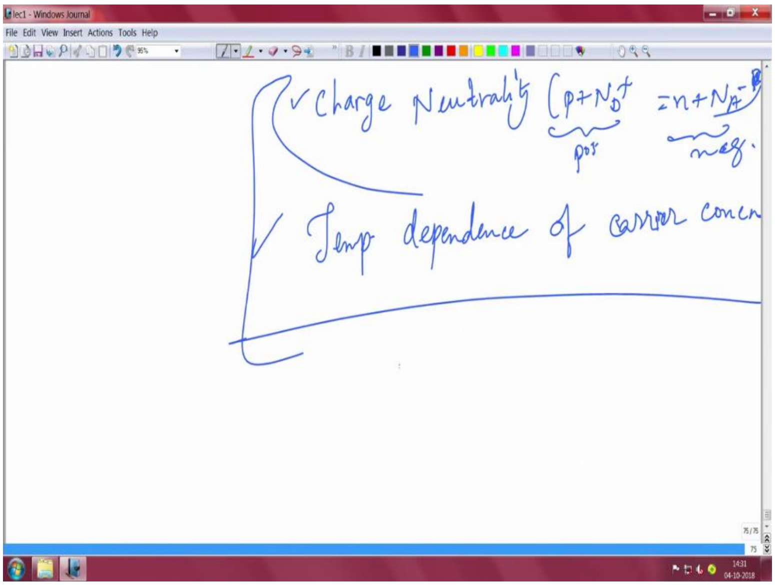

We have also talked about charge neutrality if you remember. We talked about charge neutrality in the beginning of this lecture. We talked about charge neutrality, right, which is p

$+ N_D^+ = n + N_A^-$ the total negative charge and the total positive charge has to be, you know equal right. And then another thing we have talked about is temperature dependence of carrier concentration; temperature dependence of carrier concentration, how carrier concentration changes with temperature, we talked about that also right, of carrier concentration.

So, we will end the lecture here. So, these are the two very important things. So, in the next class, we will talk about the following up, the following up the temperature dependence, what is the next thing. The next thing is actually something called incomplete ionization ok. We will quickly touch upon the facts of incomplete ionization whatever you are putting need not all get ionized even at room temperature. So, how will things look like in incomplete ionization, we will talk about incomplete ionization and then we will move forward to the basic concept of drift, conductivity, diffusion and so on. And eventually we will talk about mobility and other things may be in the subsequent classes.

So, thank you for your time. We will meet again for the next class.