Fundamentals of Semiconductor Devices
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Lecture – 13
Current continuity equation

Welcome back. So, if you remember in the last class we had introduced a first we have, you know derived Einstein relation between drift and diffusion, mobility and diffusion coefficient. And then we had introduced the concept of recombination, traps, you know how electrons are captured electrons are emitted holes are captured holes are emitted. There was an eventually we came to an expression which gives the total recombination rate of electrons or holes because of this traps.

This recombination rate is very important, whether you are shining light, you are not shining light in the presence of traps, the operation of devices, like LEDs and photon detectors and so, on this is a very important things. Because this recombination leads to a current called recommendation current we will see later, or generation recommendation current, that has real impact on how the device performs. So, we should definitely try to understand this recombination very well. So, we will come to the whiteboard now, we will stopped, we will see where we stopped in the last class.

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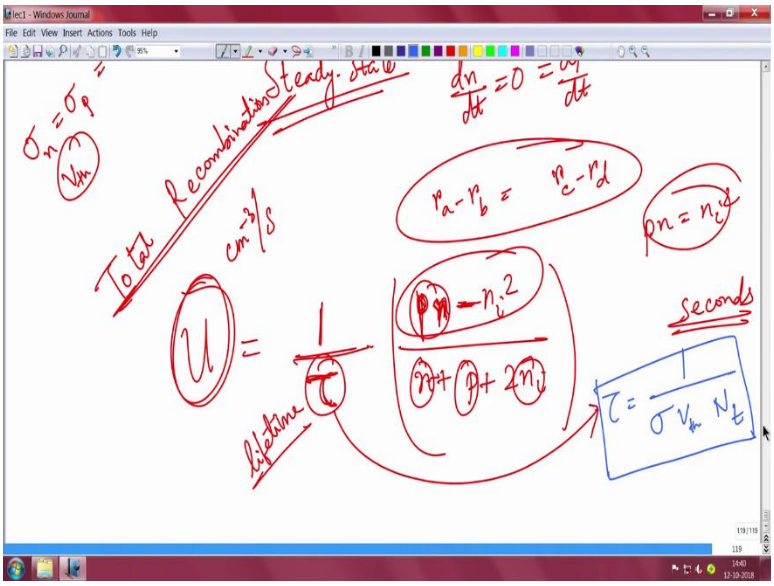

There was a typo in my last slide here; type typing mistake this actually is not you know I define τ as the life time, is actually given by 1 by sigma thermal velocity times the total trapped density, not n, the total trapped density there was the typing mistake there last time i.e.

$$\frac{1}{\sigma v_{th} N_t}$$

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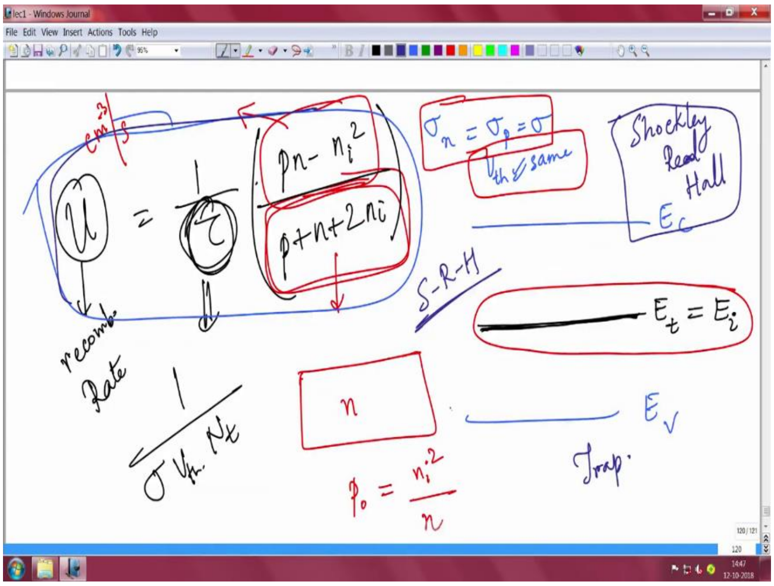

So, I told you that, let me come to a new slide here. I told you that electron cross section is assumed we hole cross section is assumed to be sigma, the thermal velocity electron and hole is assumed to be the same. And, another thing is that, if this is your conduction band, this is your valence band another thing that has been assumed actually in all the mathematics that we solved which I did not show it here, is that your trap energy level ,actually I am not drawing it well, your trap energy level E_t is assumed to be at E_i which is at mid gap ok.

If the trap energy level is at mid, the mid gap or E_i then, your you know recombination rate becomes maximum. So, that is why we estimate here and that helps in the mathematics otherwise some cos hyperbolic term comes there, any ways when you do this then you get a major recombination, this is your total recombination rate, recombination rate ok. This is equal to 1 by τ into $(pn - n_i^2)$ by $(p + n + 2n_i)$ i.e.

$$U = \frac{1}{\tau}\frac{pn - {n_i}^2}{p + n + 2n_i}$$

n_i is the intrinsic carrier concentration and τ as I told you is given by

$$\frac{1}{\sigma v_{th} N_t}$$

So, if your τ is large. You know if your τ is large, which is a large lifetime then a recombination rate is also small, it will recombine small. And if your tau is very small, say nanosecond or picosecond very small then your recombination rate is faster, it will recombine too fast ok. This sort of recombination and you remember this pn and n_i^2, they are not equal here because it is a non-equilibrium, you are shining light for example, the excess carrier that are coming out because of this traps emitting and cross sections on this is not an ideal situation here.

So, this actually is driving the recombination, this is actually giving the resistance for recombination here, ok. So, this has three assumptions: one is that cross section of electron and hole traps are same, thermal velocities are same and a third and the third assumption is that the trap energy level is supposed to be exactly at mid gap in which case this is valid ok. This is a total recombination rate, the unit is $cm^{-3}s^{-1}$ here ok. So, suppose I take a moderately you know n type doped semiconductor moderately n type doped semiconductor I am shining light generating some carriers if I am generating some carriers.

So, in equilibrium, the hole concentration will be say p_0 in equilibrium which is n_i^2/n, n_i^2/n right. So now, I am going to shine light and generate some excess carriers and I am going to understand how the recombination rate depends on that; how the recombination rate depends on that.

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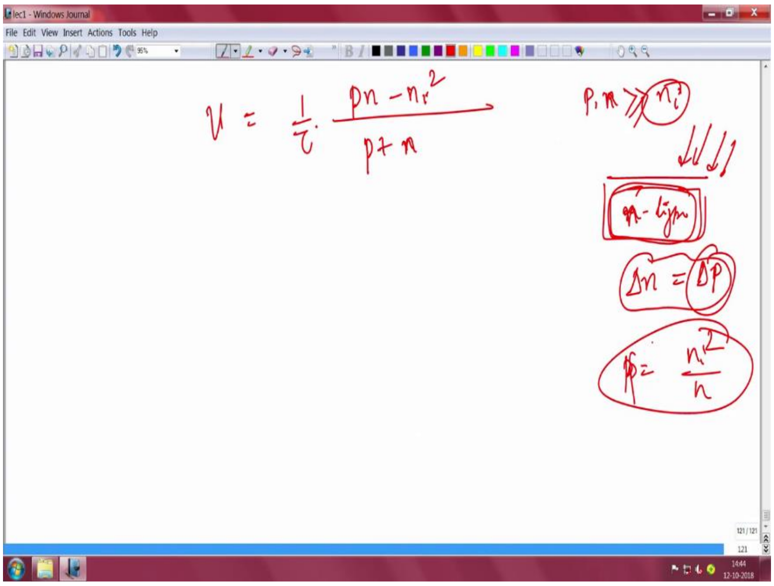

So, u look at u here, 1 by τ (pn - n_i^2) by sorry minus sorry (pn - n_i^2) by (p + n + 2n_i) i.e.

$$U = \frac{1}{\tau}\frac{pn - n_i^2}{p + n + 2n_i}$$

Now because it is a non-equilibrium a shining light generating some excess carrier they are large numbers you know this is the large numbers that are generating compared to n_i, much much larger ok. So, you can conveniently ignore n_i in the top in the bottom at least here you can ignore bottom. So, what you can do is that, you can ignore 2n_i here, you can ignore 2n_i here and then I told you that you have n, it is an n type doped semiconductor predominantly. It is an n type doped semiconductor and you are shining light for example, they are generating excess carriers, excess electrons and holes because it is an n type excess electron that are generating, see excess electrons that you generate is suppose Δn.

Because you are shining light, that will always be equal to excess holes that you are generating; but because it is n type doped that this quantity is much smaller than the background n type doping. So, you cannot it this is it can be neglected in term in comparison to n type, but because the p type is minority and that is given by n_i^2/n because p type is minority, the Δp is actually larger than p ok. So, for n type semiconductor, when you generate excess carriers the excess holes actually matter more than excess electrons because p is minority here ok. So, I can neglect the p also here, I can neglect the p here.

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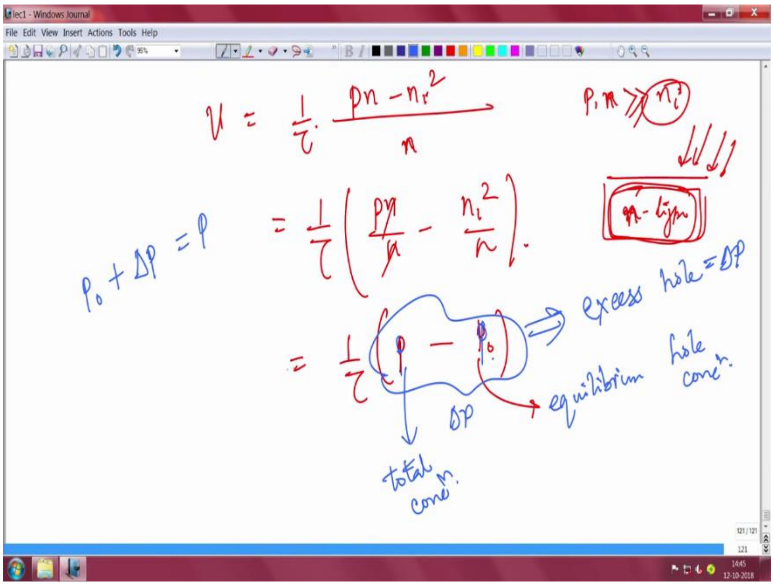

So, what I can do is that 1 by τ (pn/n) - n_i^2/n.

$$U = \frac{1}{\tau}\left(\frac{pn}{n} - \frac{{n_i}^2}{n}\right)$$

So, this is given by 1 by τ p remember n_i^2/n actually is the equilibrium hole concentration. This is your equilibrium hole concentration without any shining light, equilibrium hole concentration, and this is your total hole concentration that you are coming, total hole concentration because of shining light. The difference of total hole concentration minus the equilibrium hole concentration is called the excess carrier that you are generating, excess hole concentration that you are generating which is equal to Δp.

See you have a baseline p_0 which is the equilibrium concentration on top of that you are shining light and generating some excess carrier Δp, the total is p the total electron hole concentration. So, (p – p_0) gives you Δp. So, in other words if I write this equation again better here.

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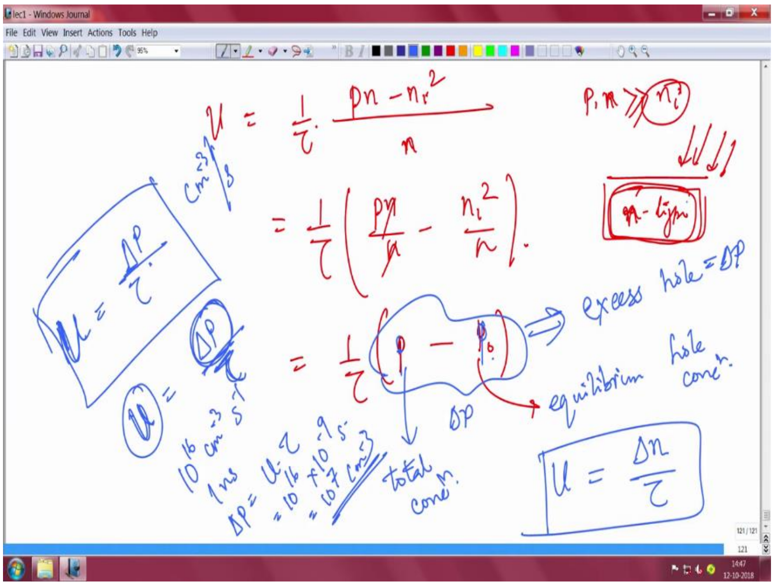

It means that u the total recombination rate is equal to 1 by τ times Δp right i.e.

$$U = \frac{1}{\tau}\Delta p$$

So, this is Δp/τ. This is a very important in a fundamental very important quantity ok. It is given by cm^3/s. It tells you that the recombination happens as delta excess carriers by τ. How many excess carriers are there and how they are decaying ok.

So, if you see, if your lifetime is very short your recombination is very fast and if you have more holes, then of course, the recombination also will be very fast. This is a very important, very very important relation if you are told that you know your recombination rate is say $10^{16}/cm^3$ and your lifetime is say 1 nanosecond, what is the excess carrier generating, the excess carrier generating is Δp is equal to u times τ which is 10^{16} times 10^{-9} second is equal to $10^7/cm^3$ of excess electron holes you are generating, ok.

So, this is a very powerful relation also for electrons it will hold true in a moderately p type of semiconductor, excess electron by that life time that you are doing. This is your total recombination rate, the rate at which electrons or holes are recombining, the rate at which electrons or holes are recombining and in case I did not tell you before this entire premise, this entire expression here that we have we have learnt here you know, the entire expression that you have learnt here, this recombination, this is this is called S-R-H recombination statistics. What does the S-R-H stand for; S-R-H stands for Shockley Read these are three people actually Read and Hall.

Shockley Read Hall statistics gives you this recombination this is a trap related recombination this is a trap related recombination. I can and I am showing you a shining light example to show how the trapped recombination can actually you know take part here. So now, we know the recombination, we know also generation when you shine light you might generate carriers, for that the light has to be absorbed by a semiconductor and you typically shine light that can be absorbed by the semiconductor. So, the light's energy has to be equal to or more than the energy of a semiconductor, if you remember that ok.

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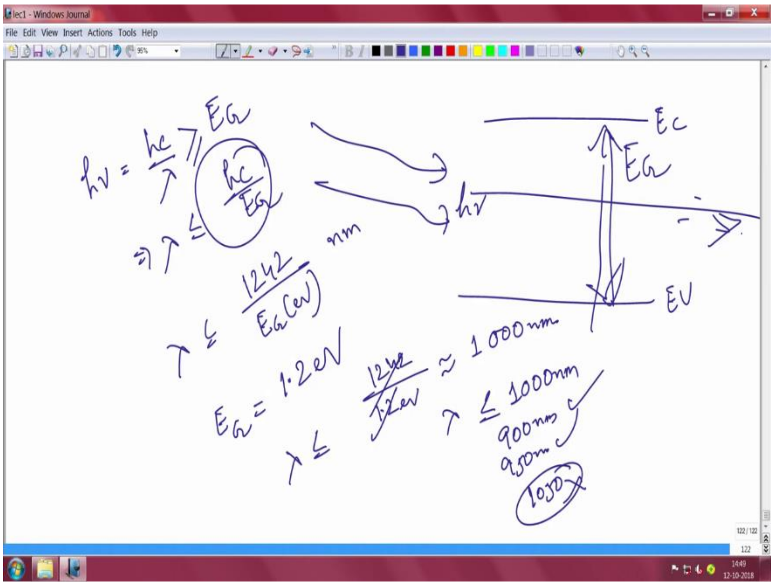

So, suppose I have a semiconductor, I have a semiconductor and the band gap is E_G. So, the light that I have to shine, the energy of the light for it to get absorbed, the energy the light is hν which is equal to hc by λ i.e.

$$h\nu = \frac{hc}{\lambda}$$

that has to be at least greater than or equal to E_G. If it is less than the band gap, then it will pass through, it will not get absorbed it has to be at least the band gap of the energy or more than the band gap of the energy of the band gap. So, from here I can say that λ has to be less than equal to hc/E_G and this is simplified as you know 1242 by E_G in electron volt will give you in nano meter.

What I mean is that λ. Suppose the band gap of a semiconductor is 1.2 electron volt, then the λ minimum λ you know is λ less than equal to 1242 by 1.2 eV will be almost equal to how much almost equal to 1000; so, 1000 nanometer. So, λ less than or equal to 1000 nanometer will be absorbed in the material. So, 900 nanometer will be absorbed, 950 nanometer will be absorbed, but 1050 will not be absorbed, because 1050 nanometer is lower in energy than this ok; so, that we should keep that in mind just in case ok.

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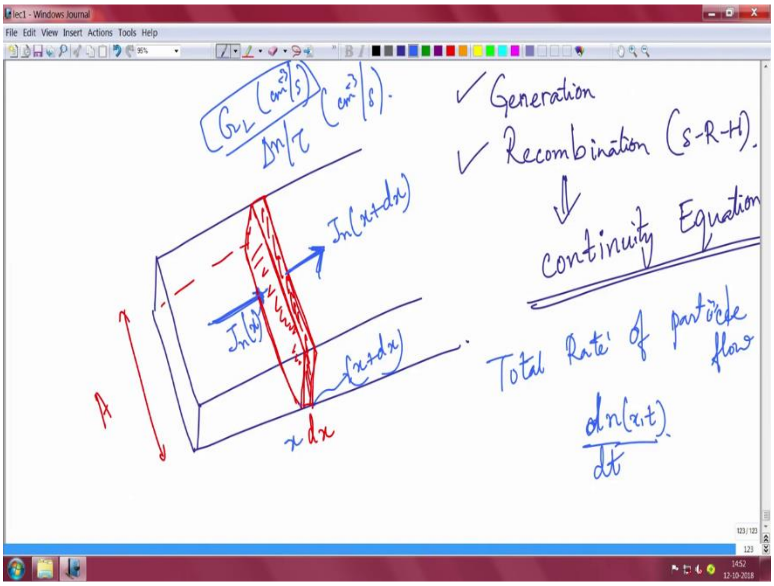

Now, we know generation when you shine light you can have generation or then you can have more thermal generation in other case. We have recombination, which is because of say traps and then it is called Shockley Hall Read combination ok. There can be also direct band to band recombination that might emit light, that is a different thing we are not that is that is during your led that we will talk about we will not talk about that right now. What is important next logically is continuity equation, continuity. This continuity equation will allow you to derive the solution for any kind of semiconductor situation. What does continuity equation actually mean, suppose I have a semiconductor and here I am going to take, a, I have a semiconductor like this and I am going to go and take a slice of this, I am going to take a slice of this ok.

I am going to go I am going to take a slice of this, which has a very small thickness, this is the thickness, ok. I am going to take a slice of this, the slice thickness is dx and the area uniform area is A, there is current that is flowing, J of n this point is x this point is x plus dx. So, the current at this point is $J_n(x)$ the current that is coming out is $J_n(x+dx)$ now you might say it will not it will violate Kirchhoff's law because, the current has to be continuous everywhere true. But, here there will be some recombination taking place in this volume, that is why the current the particle rate has to be conserved, not the current ok.

Because there will be some recombination generation of carriers that will come out here in this volume. If you see this volume well; so, some current is flowing here $J_n(x)$ some current is coming out here $J_n(x+dx)$. Within this volume, within this volume the red shaded volume that I am giving area is A ok. Within this volume, you have an generation of G_L/cm^3s^{-1}, may be optically it your shining light. Within this volume there is also recombination, the recombination is given by, you know if I am talking about electron then $\Delta n/\tau$, this is also centimetre cube per centimetre cube by second.

So, this is what at rate at which the electrons are recombining, this is the rate at which electrons are generating, electrons are flowing in there is a current, electrons are flowing out there is a current. So, the total rate of particle flow within this block, the total rate of particle flow, electron flow I mean, the total rate of particle flow, the total rate of particle flow is given by $\frac{dn}{dt}$, the rate of flow of particle, ok. Actually, it is the position of both x and t, the rate at which particles are flowing here and what will that rate be.

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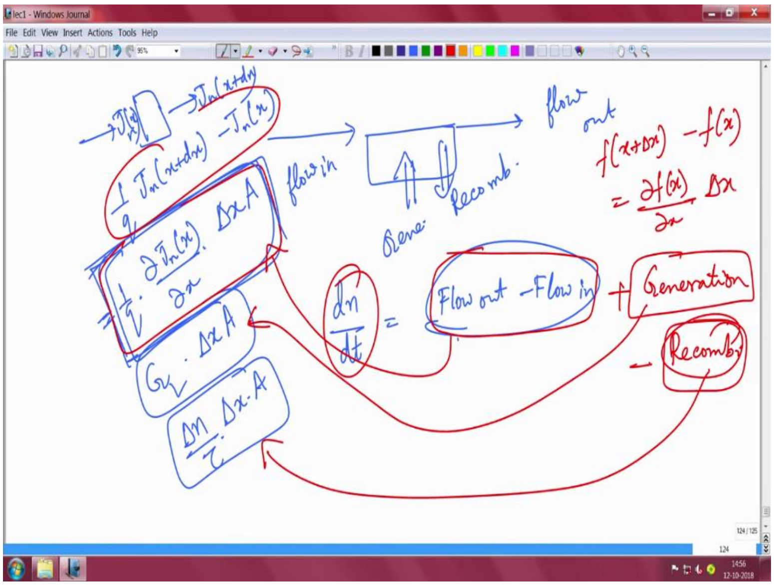

So, actually there is a current here flow in. There is a current out here flow out. There is generation here. There is recombination lost also currents particles are lost here. So, $\frac{dn}{dt}$, the rate at which this is going, is equal to is equal to what, is equal to actually the you know if your current is flowing J_n and the current that is flowing out is $J_n(x + dx)$ this is $J_n(x)$. Then the rate of particle flow is actually given by 1 by q ($J_n(x + dx)$ - $J_n(x)$), which can be written as, which can be written as what 1 by q $dJ_n(x)/dx$ into Δx i.e.

$$\frac{1}{q}\frac{dJ_n(x)}{dx}\Delta x$$

Of course, this is the current density. So, if you want to get normalized current, the total current, you have to multiply the area A you have to multiply the area A. Similarly this is the total particle flow in flow out minus flow in flow out minus flow in. This is a total current flow in and flow out and G_L is the optical generation rate. So, this also has to be multiplied by Δx times A. So, this gives you the total generation rate and delta n/τ is the recombination, the carriers are losing because of recombination times Δx into A gives you the total recombination current here the rate the recombination rate i.e.

$$\frac{1}{q}\frac{dJ_n(x)}{dx}\Delta x A$$

So, if you look at the previous slide, the total rate of particle flow is given by particle flow out minus particle flow in plus generation minus recombination, ok. So, the total rate of particle flow is given by flow out minus flow in, which is given by this quantity plus generation because your generating carrier, which is given by this minus recombination, which is given by this. You see what is that flow out minus flow in that is the generation rate plus optical generation rate or any other generation rate minus recombination rate gives you the total rate of particle flow. So, now I can rephrase this equation and write it very well ok.

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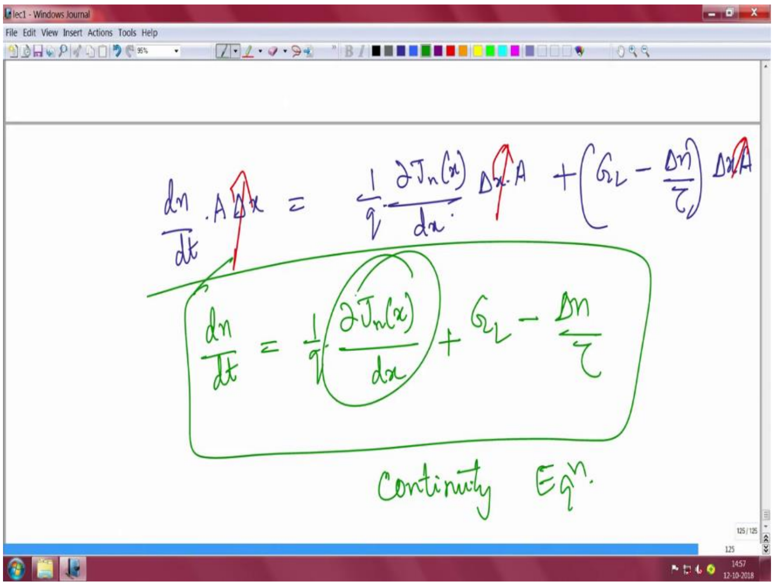

I can write dn/dt, of course there will be A into Δx, is equal to 1 by q dn(x) by dx into *Δ*x into A plus G_L minus Δn by τ Δx by A. So, of course, this quantity can go away this will be cancelled out everywhere ok. If you remember, I mean this derivative of the current how does it come, if you recall, actually this quantity comes because of this, it is like you know if I if you recall in the last class about the diffusion current also I had said the same thing, f(x + Δx) – f(x) is given by df(x)/dx into Δx, that is what I am doing here ok.

So, I can do that. dn/dt is equal to 1 by q $dJ_n(x)/dx$ plus G_L minus Δn by τ, this is called continuity equation. I will make it simplified also again little bit more, this is called continuity equation that tells you how currents will actually flow. Now you look at this equation, this term carefully current.

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} + G_L - \frac{\Delta n}{\tau}$$

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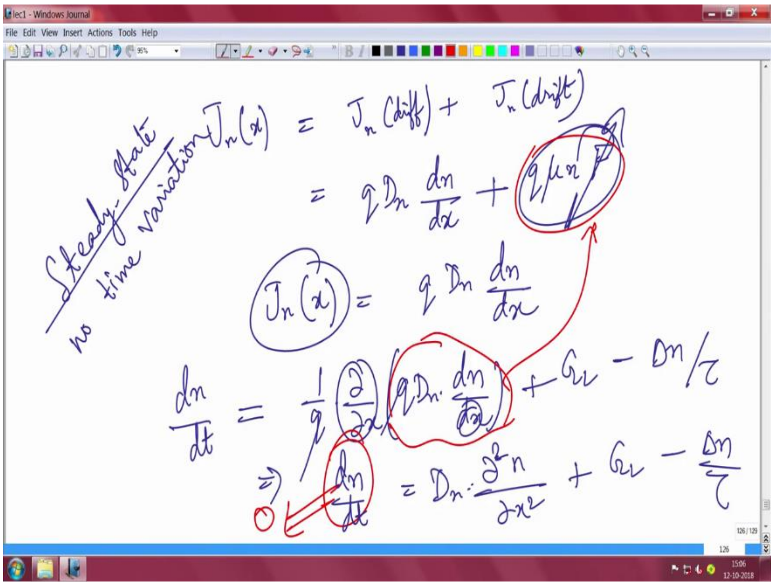

Look at the term current, J_n it has two component electron current one is diffusion current, one is drift current. Diffusion current is given by qDn(dn/dx),

$$qD_n\frac{dn}{dx}$$

drift current is given by qμn field. Suppose there is no field then this component goes, then what remains is $J_n(x)$ will be equal to qDn(dn/dx). You substitute that in the continuity equation. If you remember the continuity equation,

$$\frac{dn}{dt} = \frac{1}{q}\frac{dJ_n}{dx} + G_L - \frac{\Delta n}{\tau}$$

what it will be, dn/dt is equal to D_n, D_n sorry here I will write D_n.

You have two derivative now this is one there is already one. So, $d^2n(x)$ actually it will be n(x, t), but I will just write it as n for example, by $dx^2 + G_L - \Delta n/\tau$. If any it if it is in steady state, if we talk about steady state, in steady state, there is no time variation, there is no time variation, which means this quantity is zero, in steady state this quantity will be zero. Then what will happen there.

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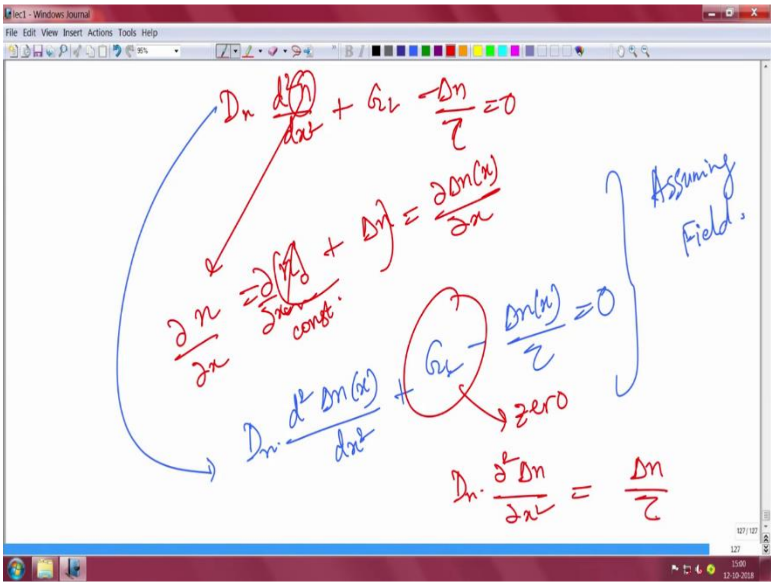

Then you will have D_n d^2n/dx^2 plus G_L minus $\Delta n/\tau$ is equal to 0.

$$\frac{dn}{dt} = D_n \frac{d^2 \Delta n(x)}{dx^2} + G_L - \frac{\Delta n(x)}{\tau}$$

now this n is actually the background concentration n_0 I mean the equilibrium concentration n_0 which is constant; this is constant plus the excess carrier concentration that your generating whatever means it is and dn/dx will be equal to d of this. But this is constant right. So, the derivative will also be only $\Delta n(x)$ by dx.

So, I can write this expression again as $D_n d^2 \Delta n(x)$ by dx^2 plus G_L minus $\Delta n(x)$ by tau is equal to 0 ok i.e.

$$D_n \frac{d^2 \Delta n(x)}{dx^2} + G_L - \frac{\Delta n(x)}{\tau} = 0$$

This is a simplified form of continuity equation assuming there is no field, because if there is a field there is a drift component that I have ignored here. If there is no optical generation, then this quantity will go to 0. If there is no optical generation, then your equation will look like

$$D_n \frac{d^2 \Delta n(x)}{dx^2} = \frac{\Delta n(x)}{\tau}$$

You see this is a second order differential equation.

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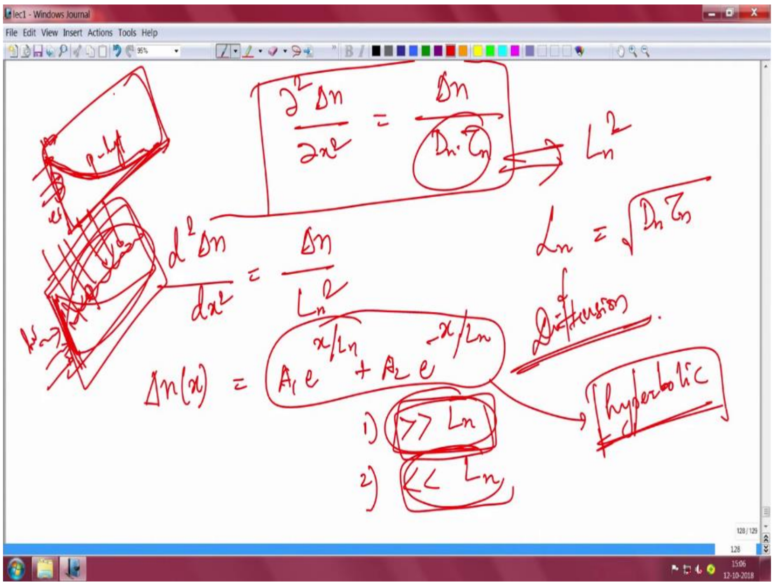

You can write it as

$$\frac{d^2 \Delta n(x)}{dx^2} = \frac{\Delta n(x)}{D_n \tau}$$

and this quantity can be written as L_n^2, in other words I can write

$L_n = \sqrt{D_n \tau_n}$, its called the diffusion mean diffusion length mean free path of diffusion, you can say its or you can just say it is the diffusion length actually that is the average length over which electrons and holes or electrons will diffuse ok. So, I can write it as

$$\frac{d^2 \Delta n(x)}{dx^2} = \frac{\Delta n(x)}{{L_n}^2}$$

this is a it is like you know, d^2y/dx^2 equal to y by some constant, how do you solve it.

Actually, it has a generic solution which is $\Delta n(x)$ will be equal to

$\Delta n(x) = A_1 exp^{\frac{x}{Ln}} + A_2 exp^{\frac{-x}{Ln}}$

You have to take some boundary condition and do things like that, but we can do two approximation: one is there can be two situation, not approximation, two situations where we can approximate, one is that , number one is that, your the semiconductor you are talking about the length of the semiconductor the length of the semiconductor is much larger than the diffusion length L_n and in one case, the length of the semiconductor is much shorter than the diffusion length L_n.

Because, the length of the semiconductor will come because you are talking about a finite block of semiconductor in which you are going to recombine the things. So, in one case of semiconductor can be much larger than the diffusion length, in one case of semiconductor length can be much smaller than diffusion length, in which case this can be. Actually this solution, this solution will give you a hyperbolic equation a hyperbolic sine cosine equation. We do not have to memorize or you know derive that that a directly here, but we can simplify that actually when you have two unique situation when either the length of the semiconductor is much larger than the diffusion length and or the length of the semiconductor is much smaller than the diffusion length.

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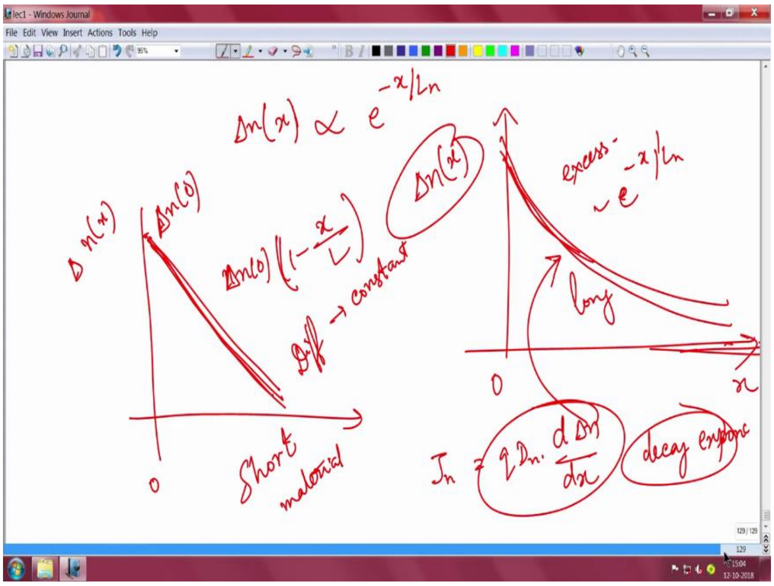

So, if your length of the semiconductor is much smaller than the, is much larger than the diffusion length, then you know this is a solution, $\Delta n(x)$ will have an exponential decay. It will basically it will decay exponentially as e to the power minus x by L_n, which means this is suppose Δn, excess carrier concentration at x equal to 0 it will be here over distance x it will decay exponentially like this, this is excess carrier remember this is excess carrier not the total carrier the total carrier will be the excess carrier plus the baseline equilibrium concentration ok.

This excess carrier decay exponentially like this as e to the power minus x by L_n e to the power minus x by L_n. And if the semiconductor length is smaller than the diffusion length, then, if the semiconductor length is smaller, then the diffusion length then this excess carrier $\Delta n(x)$ at x will be, it will decay linearly, it will decay linearly. So, it will be like n of if this is point is del sorry if this point is a $\Delta n(0)$ and it will be something like $\Delta n(0)$, 1 minus something like the, x by the length of the semiconductor L for example, so, it will decay linearly and if it is a short diode, it is a shot, in a shot of a material a very short we call it as a short diode or something. It is a long diode or long semiconductor it will decay exponentially it will decay exponentially.

And this has a lot of important consequences for device. If the delta x decays like that ,then the current diffusion current will also go as $qD_n\frac{d\Delta n}{dx}$ and if this decays exponentially, then this also will decay exponentially, which means the diffusion current also will decay exponentially, the diffusion current also will decay exponentially. In this case the, this is a straight line, thus the derivative of this is constant. So, the diffusion current also will be constant and in this case, the diffusion current will decay exponentially. So, they are very important things actually that you know this how the decay has a lot of role to play actually, whether it is a long diode or it is a short diode and this decay of excess carriers comes from solving the continuity equation.

What are the assumptions we made, we made some very big assumptions, one of them is that there is no field and that is why you are able to derive this equation, if there is a field then you know there will be drift component that you are neglecting. And, also we are neglecting that the optical generation rate, only in that case we are able to put this equation up, the second order differential equation that you are solving to get this decaying profile. So, we will see in real devices actually when you inject carriers, you know you will have this excess carriers will decay for example, if you have a a p type semiconductor and you are injecting electron somehow, you are injecting electron. So, this electron will decay exponentially like this is position or if it is a long one, if it is a shorter it will decay linearly right.

Because the electrons will decay exponentially like this, the derivative also will decay exponentially which means the diffusion current also will decay exponentially. So, they are important things actually, important implication for real device applications that will come how it you know when you do it. For example, if you have a p type semiconductor you are shining light here, we will we will do this all in the next class of course, but if you shine light here you will generate excess carriers here, this excess carriers also will diffuse ok.

So, that is one thing, you might shine light everywhere uniformly. If you shine light everywhere uniformly then everywhere it will be generated, then nothing will diffuse because everywhere the concentration will be same. Remember in this case there is no field its is only there are the diffusion of excess carriers that is happening. So, continuity equation is very important and if you have seen that continuity equation we have conveniently neglected the drift current.

If you can, if you want to take the drift current then this expression will have an extra term because if you recall that the way we have derived it is this we have neglected. So, if you take that, then in this expression in this expression you have to also take that expression. So, then it will become a little longer expression and that can be done of course, but not we should not do it right now if an when a necessity arises, then we can definitely do that ok. So, the things that we have learnt right now the things that we have learnt right now I can tabulate it again here, we have learnt about the sorry.

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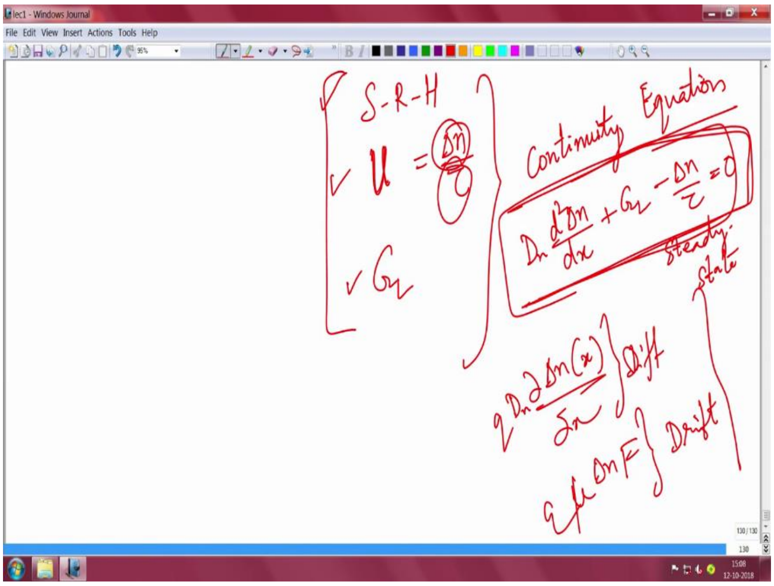

We have learnt about the Shockley Read Hall statistics and how recombination takes place. I told you the total recommendation rate delta n by tau there is a tau is a lifetime this is the excess carrier you have, this is a recombination rate you have, you might shine light and generate some carriers which is optical generation rate. In the presence of all of this, you can actually solve and get the continuity equation and this continuity equation will be our very important starting point in many of the semiconductor devices.

This continuity equation tells you that you know how you solve the equation, how you solve for the carrier movement or a carrier distribution or the current flow it will basically depend on $D_n d^2\Delta n(x)$ for n type for p type it will be Δp plus G_L optical minus Δn by τ is equal to 0 in steady state. So, similarly for holes, you will get this for p type and this is without consideration of field and in any case you can always solve this equation to get the carrier profile delta n. Once you get the carrier profile delta n, you can get qD_nd of this you will get the diffusion current, and not only that if you get delta n by multiplying mu times field times q you can also get drift current. So, both drift diffusion current will come out.

So, that is why we need to solve this equation; that is why we need to solve this equation here. So, we will end the lecture here today. We have solved continuity equation and we now can move to the next topic after this, which is, you know more or less p-n junction, we will discuss that in the next class. And, before p-n junction, we start p-n junction one small thing, we have to learn in probably in the start of the next class and that is about few examples; a few examples of how we apply the continuity equation in some special and unique cases like shining light from one side, applying voltage to a sample and shining light.

So, if we can learn those things, then we will exactly understand how continuity equation helps you get the profile of carrier Δn or Δp, and that Δn or Δp will give you the drift and diffusion current that we already discussed. So, after we are armed with all this after we are familiar all this concept, we will immediately started p-n junction. And, in p-n junction you will see that drift diffusion current, the Einstein relation, the carrier statistics everything that you have learnt till now will be used in p-n junction. Also, whatever we learnt in high school things like Poisson equation, you know charge neutrality all this things also will be used in p-n junction. And, once you know p-n junction, things like LED, photo detector, solar cell are very easy because they all p-n junctions.

So, we will end up the class here. Next class we will start with some examples of how to apply continuity equation and depending on time we will move to p-n junction from there ok.

Thank you.