Fundamentals of Semiconductor Devices
Prof. Digbijoy N. Nath
Centre for Nano Science and Engineering
Indian Institute of Science, Bangalore

Lecture – 13
Drift-diffusion and trap statistics

Welcome back. So, today we shall continue from our last discussion. So, yesterday you know the last class we had discussed about drift and diffusion equations of current. We told that there started two major components drift happens because of field and diffusion happens because of concentration gradient. We derived their formula and equations.

We also had before that you know we have discussed about your electron mobility hole mobility how scattering effects them right and also the velocity high field and low field regimes hope everything is you know now clear and familiar with you and once you know the concepts of mobility and velocity we are able to understand drift and diffusion equation. Please remember that all the devices eventually will mostly based on diffusion and drift current only and that is why it is very important that you understand them.

So, today we shall introduce another thing it is called continuity equation and before that I shall you know go through something like recombination called recombination and generation. We shall also understand why they are important and you know in real material system recombination is very important just because there is drift and diffusion we cannot exclude the importance of things like recombination ok.

So, now we will come to the whiteboard and we will continue from where we left behind in the last class. So, you see in the last class we have discussed about drift and diffusion equation right.

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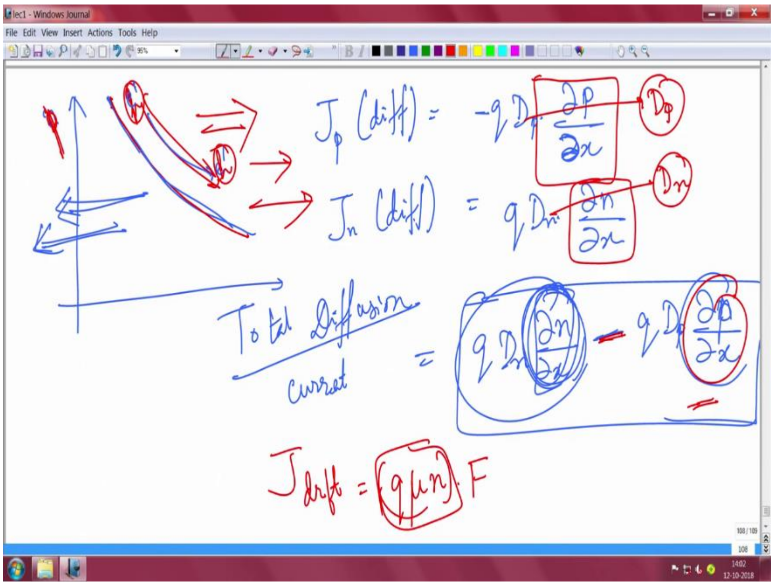

I told you that drift and diffusion equation depends on the slope of the carrier profile rather diffusion equation right. The electron and hole diffusion and the drift current does not depend on that you if you remember drift current you know was purely field dependent. So, that is for electron it is q*μ and which is conductivity times the field ok. So, μ is mobility right.

So, drift and diffusion current they actually have a relation you know in I will come to that very quickly in that that you know this diffusion coefficient that you see here D_p for holes or D_n for electron, these are diffusion coefficient that are dependent on the material, but actually these are related to this conductivity also we will see that how ok. So, once we see that we will understand that a higher diffusion current also means that drift current could be higher ok.

So, today we shall you know discuss first we will come to something called Einstein relation.

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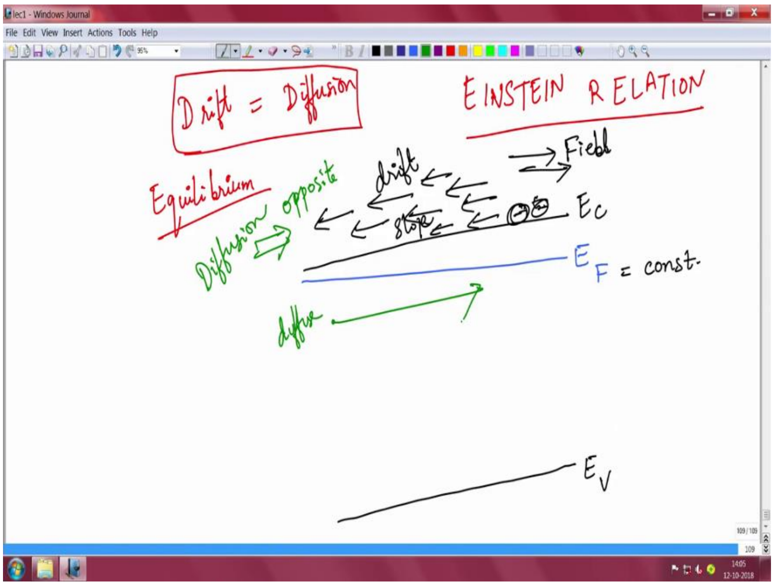

Einstein relation see Einstein not only had worked on relativity and gravity, but also there is a very important relation of Einstein in your semiconductor devices ok. So, suppose I take a sort of a material where both drift and diffusion current are basically going to balance each other.

Suppose there is no net current flowing. So, drift and diffusion current are going to balance each other. So, that there is no net current flowing and I will for simplicity take I will only take an n type dope material, there is only one type of carriers say electron. So, let us imagine an equilibrium situation where there is no current flowing. An equilibrium situation when there is no current flowing which means the Fermi level is constant a constant Fermi level means that its equilibrium there is no current flowing.

But there must be drift without applying field for example. So, there must be a slope in the conduction band and valence band. So, what I will do is that, I will put conduction band something like this slight slope is there in a valence band also has to have the same slope because the band gap is the same everywhere the band gap is same sorry this is valence band. So, you see this is in equilibrium there is no current flowing because Fermi level is constant Fermi level is constant ok.

But there is a slope in conduction band because of this slope you know if there is a field. So, where is the which direction is the field, I told you a very a very simple way to understand where field is you assume that there is an electron here and think of it like as a ball or a marble that is rolling down a slope. So, it will roll down this way know. So, electrons will move this way which means the field is in this direction field is in this direction.

So, there is a field here, but this is equilibrium. Now because there is a field in this direction it will try to push the electrons this way. In other words, the electrons will try to roll down hill because of drift, but this is in equilibrium. So, the Fermi level is constant right.

So, if the electrons are drifting this way then there will be a current know, but there is no current which means there is an exact opposite component there is an exact opposite component which is balanced by you know that this diffusion current and exact value of the drift current will be balanced by diffusion. So, electrons will try to defuse from this point to that point why because you see if I remove this thing here and make it little bit clearer here the Fermi level and conduction band spacing, I told you.

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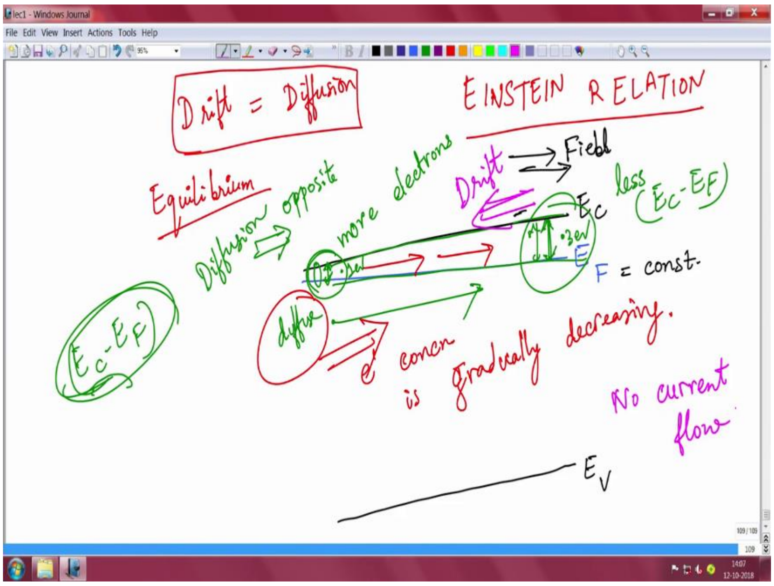

E_C - E_F this spacing tells you how many electrons are there or how heavy the doping is ok.

So, you see on the left side, this spacing this one is small on the right side this spacing is more E_C - E_F is more. Since E_C - E_F very less here, in other words the Fermi level is very close to the conduction band it means here there are more electrons right there are more electrons ok. And because this E_C - E_F spacing is larger maybe here it is 0.1 ev maybe here it is 0.3 ev right that is why it's bigger and its increasing very slowly like a linearly you know every at any point it is increasing its not abrupt increase this one is in smooth increase.

So, here at this part you know the electron concentration is less because your E_C - E_F is more remember this is this is what determines that and of course, as I go from you know left to right actually the electron concentration is gradually decreasing right electron concentration is gradually decreasing electron concentration is gradually decreasing right because it is linearly changing right is gradually decreasing good.

So, in this case electrons will try to defuse from left to right and electrons will try to drift from right to left because of the field. So, they will balance each other and eventually no current will flow eventually it is basically an equilibrium situation that is why the Fermi level is constant ok. So, let me draw it again I will draw it again.

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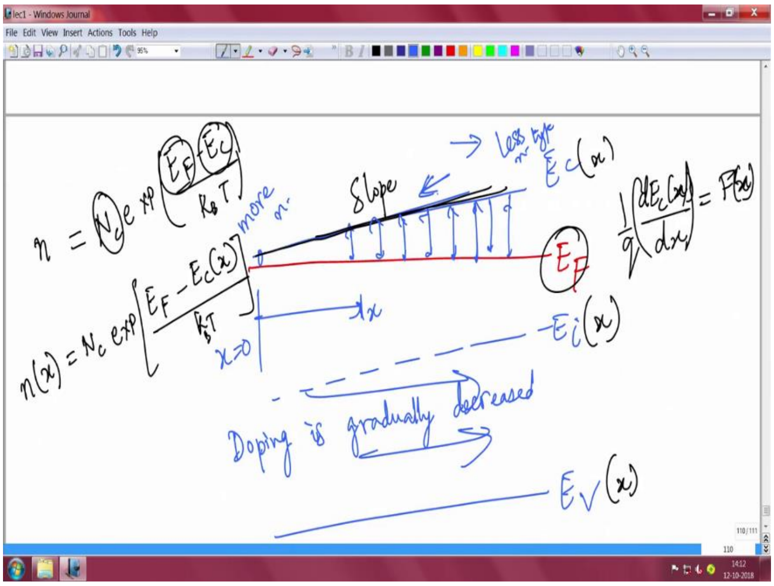

So, I have the Fermi level here ok and I have the conduction band like that and the valence band like that, it is still an n type semiconductor by the way because Fermi level is above the mid gap you know the mid gap will be here.

So, it is not a curved line it is a straight line completely. This is your intrinsic level this is a conduction band this is a valence band. So, that its n-type its more n-type this side and less n-type this side that you know now right. How can you get this kind of a structure you know you can get this kind of a structure if you are doping is gradually decreased from left to right while you are doping the semiconductor in the fabrication unit if you decrease the doping gradually from left to right then you know it this spacing will basically keep increasing that is how it is happening ok.

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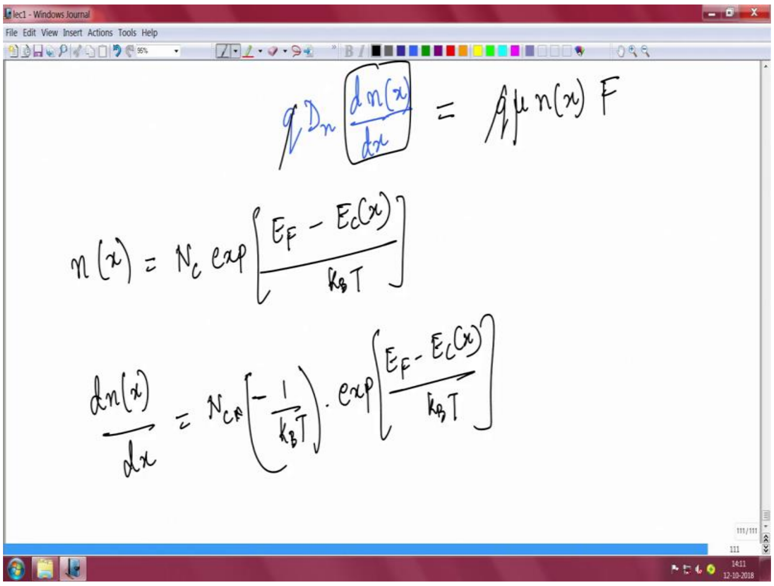

Now in this situation, I told you the net current is 0. So, the drift current has to be equal to diffusion current which means the diffusion current is q*D_n, because its electron dn/dx this n is a function of x why? Because if you look back into the previous diagram here at any point x here, sorry at any point x if I take this x is equal to 0 that at any point x here the electron concentration depends on x because this gap is increasing gradually right.

So, how will you capture that? You remember 'n' this is a moderately doped semiconductor $n = N_c \exp\frac{(E_F - E_c)}{kT}$, this is Boltzmann constant k_BT. Now at any point at every point Nc is constant because this depends on effective mass and temperature which is the same ok. Fermi level is also constant everywhere what is not constant is E_C you see the E_C is not constant because it is changing. So, I can write

$n(x) = N_c \exp\frac{(E_F - E_C)}{k_B T}$ at any this thing ok.

Because $E_C(x)$ is changing, $E_V(x)$ is also changing E_F is not changing E_i is changing ok. If you remember this and you remember that you know the slope of this the slope of this actually gives you the field if you remember I told you once, the slope is actually dE_C/dx

of course, you have to multiply 1/q because you now energy and electron volt and volt. So, this is your slope, and this will be equal to field this will be equal to your field in a semiconductor please remember that ok.

Alright, so, now we come to the next we are looking here. So, this component will be equal to the drift component which is q*µ*n(x) into field, the field is constant because the slope is constant ok. So, q is of course, gone. So, now, what we do is you remember from the previous page $n(x) = N_c \exp\frac{(E_F - E_c)}{k_B T}$ let us take a derivative because this term is a derivative. So, I will take a derivative d(n(x))/dx is equal to Nc, I take a derivative of this what will happen is that, it will be - 1/kT, its multiplied here into its the derivative of $E^{-x/kT}$ actually anyway right.

So, it will be exponential of same thing $\frac{E_F - E_C(x)}{kT}$. So, you know if I take this quantity away on the other side if I take this quantity here $-1/k_BT$ then you see this quantity is actually this can you see that. So, what I can do is that, I can replace this by n(x) I mean is d(n(x))/dx you know what I can do is basically I can remove everything here, I can just put it as n(x) = dn/dx the derivative I can do that ok.

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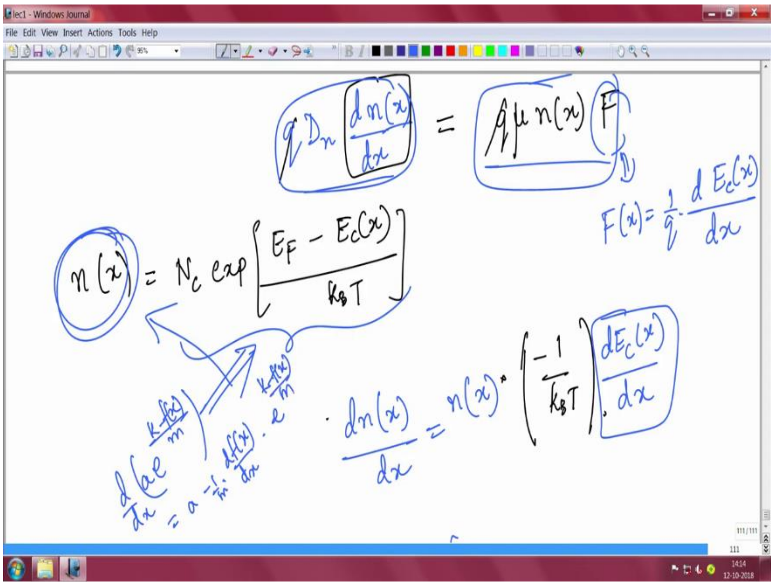

Also if you recall actually in one of the in a previous I think in the last class when we discussed this remember the diffusion current of electron actually is positive here and the drift current actually will be you know the field you have to look at the field here actually.

So, for example, I look at the field here the field is in this direction which means the electrons are actually going to flow in this direction the electrons are going to flow actually in this direction.

So, the current will be in the positive direction, but the direction will not matter so much now here, because we can take the modulus I mean it is the eventually is the value the absolute magnitude of the current, that we are trying to equate because electrons will try to diffuse from this side to this side which means current will be this way and electrons will try to come from this drift from this side to this side which means currents will be this way. So, they will eventually balance out ok. So, the drift current is positive and the diffusion current is negative.

Now, I basically have to equate this quantity with this quantity right and I told you that field the field actually is nothing, but $1/q*[dE_C(x)/dx]$ you agree $dE_C(x)/dx$ that is your field. Here actually I actually have not written one small thing here this actually should be sorry $dE_C(x)$ should be there actually because this expression looks like I would say you know $\alpha e^{k(-x)/m}$ suppose this is constant a. So, if you take a derivative of this function d(dx) and this is suppose f(x) this is something like the f actually.

So, this is the same thing as this function. So, when you take a derivative of n(x) with respect to 'x' you know this there will be an extra this term that was missing actually in my previous equation this term will be there because you know this equation for example, will be a you know $a*-1/m*d(f(x))/dx* e^{k-(f(x))/m}$ (refer slide) it is something like that right.

So, basically you will have a term of e to the power the derivative of the conduction band here which because you are trying to differentiate this expression here. So, that I missed out in my previous expression anyways. So, now, we have this expression for the n(dx) and for field I know this is your equation right.

So, we can now actually substitute the values and go to the next point which is D_n into this value right dn(x) this value.

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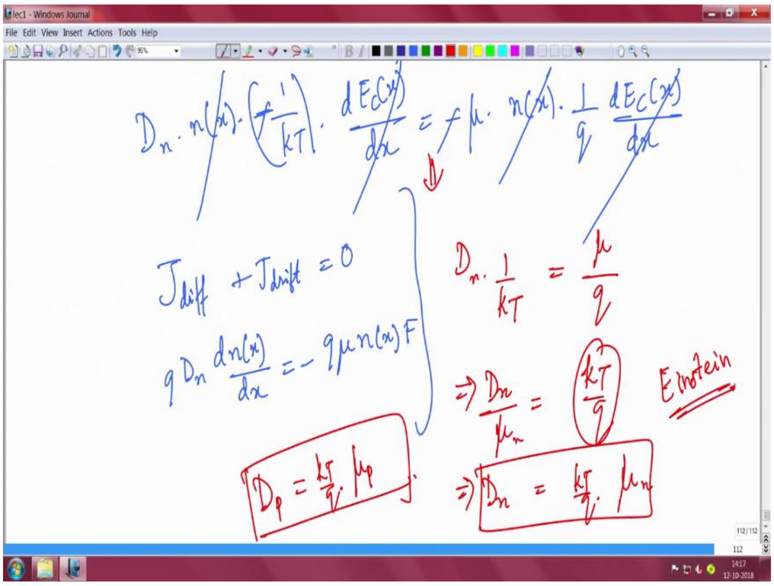

So, I will take that value which is which is sorry n(x) minus let me see again go back to the previous page one sec here. So, it will be -1/kT into the derivative of conduction band ok. So, that will be -1/kT into the derivative of the conduction band here will be equal to if I go back to the previous page you know if I go back to the previous page here this is q is already cancels. So, its μ times the charge n(x) times the field mobility times charge times field. So, that will be mobility times charge times field and field is 1/[dE_C(x)/dx] ok.

So, n(x) and n(x) cancels out dE_C(x) and dE_C(x) cancels out and there is a negative sign actually this negative sign is extra it will also gets cancelled out which I did not cancelled out in the last page because you know the total diffused the total drift plus diffusion J sorry $J_{diffusion}$ + J_{drift} = 0 and $J_{diffusion}$ is q*D_n*dn(x)/dx this is equal to k*μ*n(x) into field = 0.

So, if I basically equate them out here there will be a negative term here which I did not write in the previous slide because anyways we are taking magnitude. So, that sign will not matter so much here so there will be a negative sign here actually. So, that negative will cancel out here ok. So, the negative will cancel out here what remains from this equation above from this equation above is $D_n \times \frac{1}{kT} = \frac{\mu}{q}$.

What it means is $\frac{D}{\mu} = \frac{kT}{q}$, and $\frac{kT}{q}$ is a constant which is the thermal energy at room temperature is 0.026. So, I can write D_n is equal to some constant kT/q times μ of electron of course,μ*(n) is μ of electron, for holes you have the same relation D_p diffusion coefficient of holes is equal to $\frac{kT}{q}$*μ(p) ok. So, this is called Einstein relation and we derived it by equating the drift and diffusion component of the current what it means is that,

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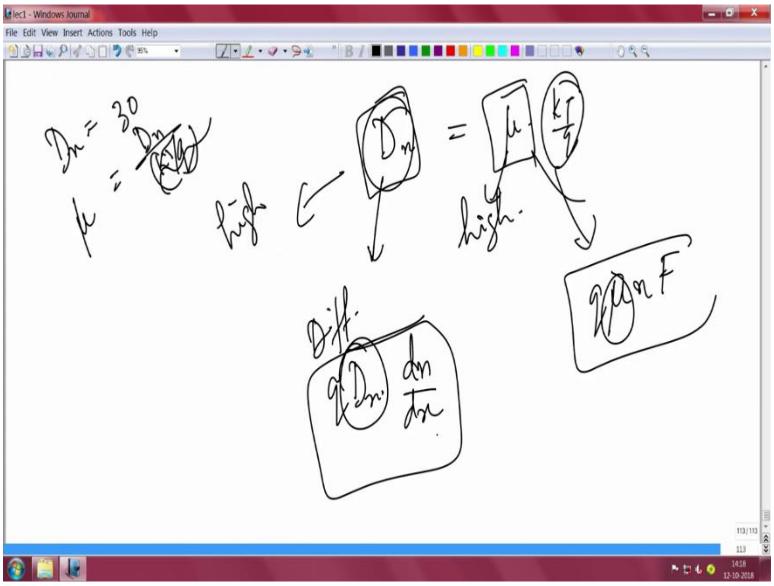

Your diffusion coefficient is proportional to your mobility of course; this is a thermal constant here right.

So, if your diffusion coefficient is high it means your mobility also is high ok. So, this diffusion coefficient comes in diffusion current if you remember it is q*D_n*(dn)/dx and this comes in the drift current q*μ*n*field. So, because your higher diffusion coefficient means the mobility is higher and vice versa a higher diffusion current because of this high also means that the drift current will be higher.

This is called Einstein relation and is very important because if you are given only 'D_n' the value of 'D_n' suppose is given to be you know say 30 units for example, now you can calculate mobility out from there right you can calculate mobility by:

$\frac{D_n}{\frac{kT}{q}}$,   and $\frac{kT}{q}$ is known. So, you can calculate it out.

So, that is a very powerful equation to relate drift and diffusion components ok. So, now, we have done it that ok. So, now, what we have studied in the last couple of lectures.

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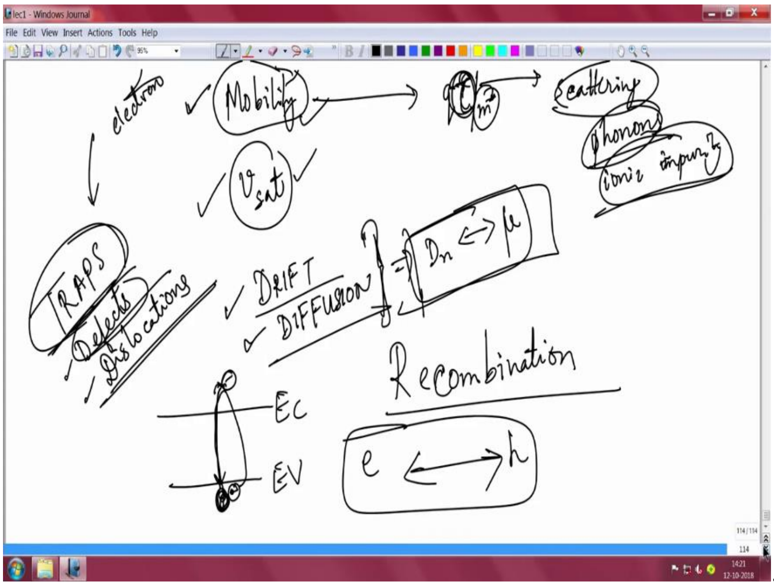

We have talked about carrier mobility this is the recap of what we have learnt we have learnt about carrier mobility. We discussed that carrier mobility is essentially nothing, but the slope of velocity and field relation. If you are applying an external field the electrons will keep accelerating how fast are they accelerating, what is the rate of change of velocity with the field that is your mobility ok after sometime mobility will saturate and you will get a saturation velocity.

So, that means, no matter how higher how high field you are applying your mobility will no longer increase or no longer be there your velocity of the carriers will saturate. Velocity of carriers will saturate and that is called saturation velocity and when the situation velocity kicks in its a high field regime as long as velocity and field are linearly proportional its low field regime called mobility.

Mobility is dictated by something like $\mu = \frac{q*\tau}{m}$, where m is the effective mass.

So, a lower effective mass material will have higher mobility and hence higher current because mobility also dictates current and this 'τ' is actually the mean scattering time, the time between two subsequent scattering, when an electrons move they gets scattered from primarily phonons which are vibrating atoms and also ionized impurities because they have a columbic attraction or repulsion on the carriers.

So, because of ionized impurity and phonons scattering you have a mean scattering time that is decided by that phonon scattering will decrease you know the mobility with temperature ionized scattering will have the opposite effect. So, that we studied mobility, we studied velocity saturation and we studied drift and diffusion current right this is drift this field component diffusion is concentration gradient driven. So, there is a drift and diffusion component there they might co-exist and in equilibrium if they are equal we derived their expression between 'D_n' and the 'μ' this relation is always valid it is not only valid when drift and diffusion are equal, but this is this relation in general is always valid and this called Einstein relation.

So, we have completed that now the next big topic that we have to read and understand is called Recombination. What does recombination mean? Recombination means that if you have an electron and if you have a hole they will recombine that is one thing they will recombine because a valence band from the valence this is conduction band, this is valence band. If electron might go from here to there and leave behind their hole that is the generation of electron hole pair and is electron might fallback and again fill up the hole here and that is called recombination that energy might go out as a light or an energy might go out as a heat to the crystal is called phonon emission ok.

Only electrons and holes do not need to recombine there might be other things that made also help in this recombination ok. A perfect material is not possible there always be something called traps. Traps are actually defects in the crystal defects, dislocations might also lead to traps, dislocations or missing line of atoms. So, all this defects dislocation you know they relate to traps and these traps are actually physically they might be an empty space, they might be a dangling bond, they might be impurities, they might be interstitial vacancies. So, many things are there they might be a defect there right.

So, these traps can also take part in recombination. For example, a trap might be there a physical a trap might be there which might trap an electron. So, it is called an electron trap right. What does it mean actually and these are very important in practical devices.

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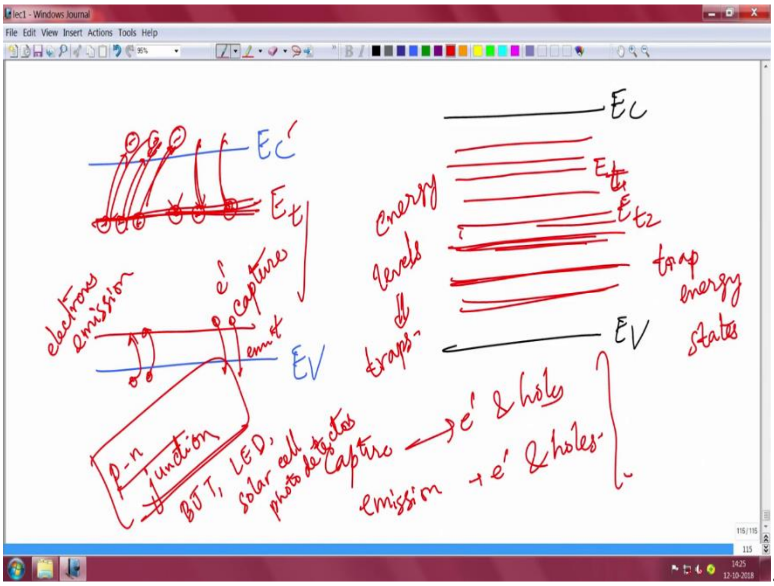

So, for example, I draw the band gap conduction band valence band I am not drawing the Fermi level here, but just conduction band valance band. Ideally this part should be forbidden gap there should not be anything here. It is clean band gap you know it forbidden gap.

But, in reality there might be energy trap energy states here these are called 'E_t' energy trap energy states there might be many trap energy states throughout these are energy trap states what does it mean these are energy levels, these are energy levels that correspond to some traps. So, physically there might be traps in the crystal. There might be a defect they might be an impurity. So, dangling bond and so on those are physical traps. They manifest themselves in the energy band gap as energy states within the band gap they are trap energy states they are trap energy states and they will manifest themselves as some discrete energy levels inside the band gap.

So, suppose we do not talk about so many traps, supposes I have conduction band here, I have a valance band here, but suppose there is a trap energy level here. So, then electron that are there in the conduction band some of them might get trapped here they will become negatively charged and they will get trapped here. So, it will reduce the electron concentration there then what you are desiring? Because they are traps, we call them you know compensating traps they will compensate for example, ok.

There might be traps also which might have already electrons in them and they might emit electrons back in the conduction band. We call them electron emission traps that are an emitting the electrons they are called electron emission traps. They might be capturing the electrons we call them electron capture traps right. This trap state please remember this traps are actually physically they are in the crystal in the form of dangling bonds, impurities, interstitial defects, dislocations and so on. In energy band gap they manifest as some distinct trap level within the band gap and because of their presence electrons and holes might get recombined with this within these traps.

So, you see traps might either capture some electrons or they might emit some electrons. Similarly, they might capture some holes they might capture some holes or they might they might be holes and they might emit some holes. So, in general there is capture of both electrons and holes and there is emission of both electrons and holes.

So electron can be captured hole can be captured, electron can be emitted holes can be emitted by this different trap levels it depends on the position of the trap with respect to the conduction or valence band inside the band gap right and then their trap energy levels you know if for simplicity you might assume that there is only one trap energy level for example, for electron ok. Now these you might ask why it is important? These are important because in real devices and you see many of the real devices are based on p-n junction we will come to p-n junction in another one or two class maybe.

P-n junction is a fundamental building block of things like BJT , things like LED, things like solar cell and things like many of the in the photo detectors ok. So, many things are actually dependent on p-n junction. So, in an operation of a p-n junction this kind of traps that are there are actually very important because they contribute to current which you might you must include while you design the device or understand the device. That is why we have to understand this and that is why I am talking about recombination.

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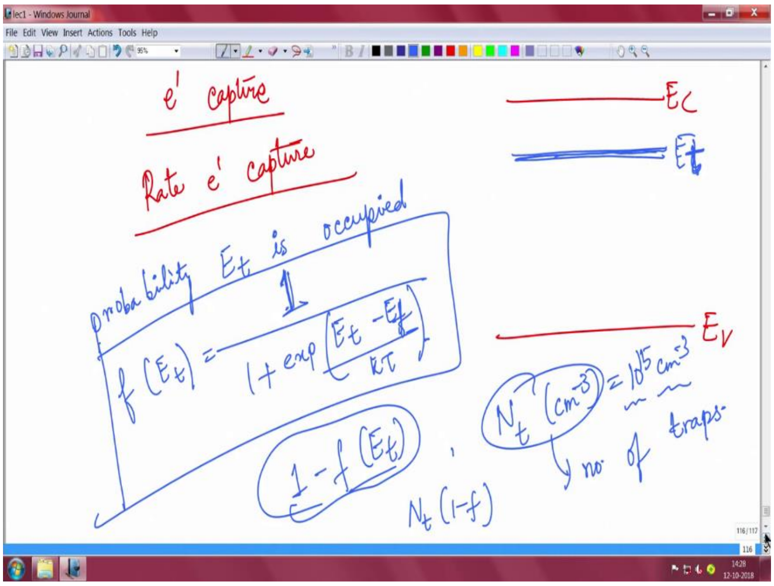

First we will talk about say electron capture electron capture. There is a rate at which electron capture will take place rate of electron capture ok. The rate at which electrons are captured. So, you have conduction band here, you have valance band here right and then you might have a trap energy here ok. So, if there is a trap energy here the probability, I will come to the electron capture right very soon the probability that energy level trap energy level E_t is occupied the probability that the trap energy level is occupied by some electron already is given by $f(E_t) = \frac{1}{1+exp\frac{(E_t-E_f)}{kT}}$.

This gives you the probability that electron is basically you know there is already this trap is occupied by electron for example, this is trap is occupied by electron this is probability. So, $1 – f(E_t)$ will be the probability that the trap is empty the trap is empty and suppose N_t per centimetre cube is the number of traps or the trap density. So, if I say the trap density say 10^{15} /cm^3 in it means in the silicon crystal I have 10^{15} /cm^3 it is this is the density of the traps this is the density of the traps.

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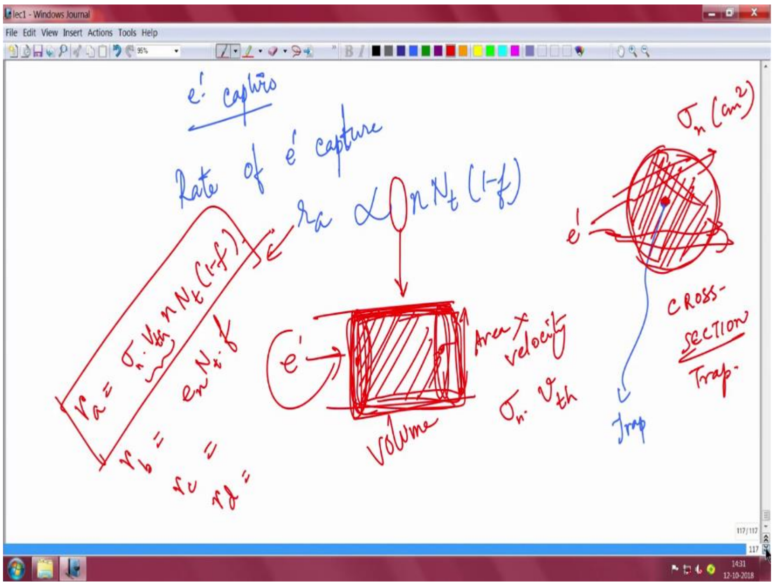

So, you know I was talking about electron capture first. The rate at which electrons will be captured rate of electron capture this rate of electron capture is r_a for example, I will tell it as r_a the rate at which electrons can be captured by this trap states. This must be proportional to few things, first of all this has to be proportional to how many empty trap states are there this is proportional to how many empty trap states are there which can capture. The number of empty trap states is the total number of trap states is N_t and out of it this fraction is unoccupied. So, the total number of trap states which can capture the electron is $N_t(1\text{-}f)$ this I am writing as 'f' ok.

So, the rate at which electron will be captured depends on how many empty traps are there only empty traps can capture. So, that is $N_t(1\text{-}f)$. Of course, this whole thing also depends on how many free electrons are there, if the semiconductor does not have free electron then they will not there is nothing to capture know.

So, it depends on free electron concentration 'n' and the and you know if this is a trap for example, if this is the trap physically I am talking about this will trap the electron then around it there you can define an area a cross sectional area it is a circle, I am not very good at drawing of say σ_n cm^2 this is the area.

This is the area through which if there is any electron that passes it will be trapped here you know this is the influence area you can say this is called the cross section the trap cross section area the cross-section area of the trap ok. This is the cross-section area of the trap and within this if the electron passes through it will be captured by the trap. So, if I talk about this in 3 D then this is the area if I talk about in a sort of a tube tubular structure in one second the volume swept away by the electron is this over through which electron this is the trap for example, electron is coming in one second this is the volume through which the electrons all the ensemble of electrons will pass through and that is given by that is given by the area times the velocity with which electrons are moving.

So, area times the velocity will sweep basically in every unit second this is the volume this is the volume of space which the electron ensemble will sweep through ok. Though this is the area this is the volume through which the electron ensemble will sweep through. So, the proportionality constant here is this, what is the volume it is sweeping through.

So, this is given by area times velocity, area is σ_n and velocity is given by some thermal velocity because it is random. So, I can write this as $r_a = \sigma_n$, which is the area times velocity. So, this is the volume through which the electrons are sweeping through times the free carrier concentration because more the carriers more will be the trapping times how many empty states are there this.

Similarly, I can define 'r_b' which is the rate of electron emission for electron emission it depends on how many filled traps are there, because they will be emitted electron number of filled trap is N_t*f and then there is nothing else it has a constant here I will call it as 'e_n' which is the emission constant similarly I can define 'r_c' which is hole capture and 'r_d' which is hole emission something similar.

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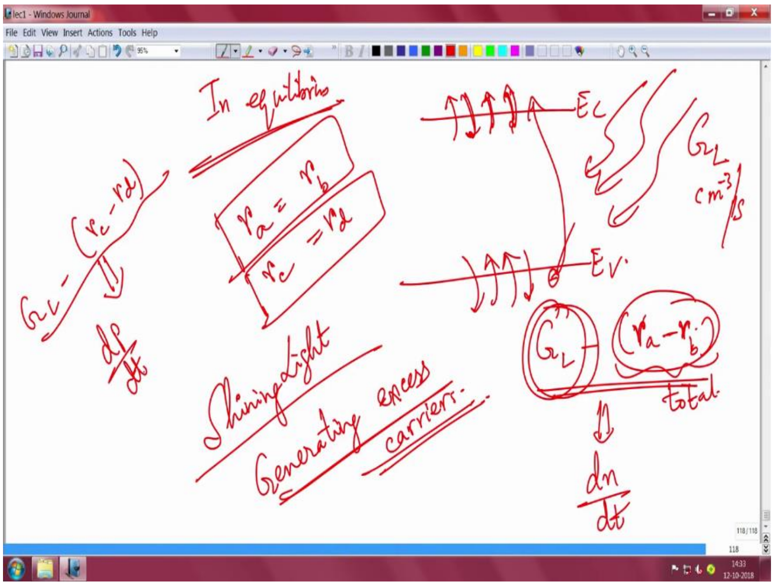

So, in equilibrium in equilibrium when I say in equilibrium it means its equilibrium this you know what is equilibrium. In equilibrium you know we have the conduction band here we have valence band here the rate of electron emission and capture into the conduction band will be the same which means $r_a = r_b$, rate at which electrons are emitted and electrons are captured into the conduction band will be the same, rate at which holes are captured and holes are emitted from the valence band also has to be same. So, $r_c = r_d$. So, you can do these equations and then you can find out the constants excuse me you can find out the constant which was there in the previous slide.

So, here this constant was there known that we can find out and. So, that is one thing. So, you can do some simplification and math and there is another math though you can assume that you are say shining light you are shining light and you are generating excess carrier when you sign shine light you are generating excess carriers and those excess carriers will recombine and decay actually I will come to that.

So, generating excess carrier for example, so, the suppose I am shining light and 'G_L' is the generation in the generation rate at which you are generating the light the photo carriers optically. When you shine light excess electrons will go from valence band to conduction band and clear electron hole pair. The unit of this is per centimetre cube per second. So, this is the rate at which you are generating electron for example, or holes pairs ok.

So, if you are shining light then the total rate of generation $G_L - (r_a - r_b)$ this is the rate at which you are producing electrons this is the total rate of optical producing this is the total rate at which you are losing this is capture minus this is emission this is capture minus this is emission. So, this gives you the total capture or total recombination this gives you the generation. So, this is the rate at which dn/dt it over time how you are generating.

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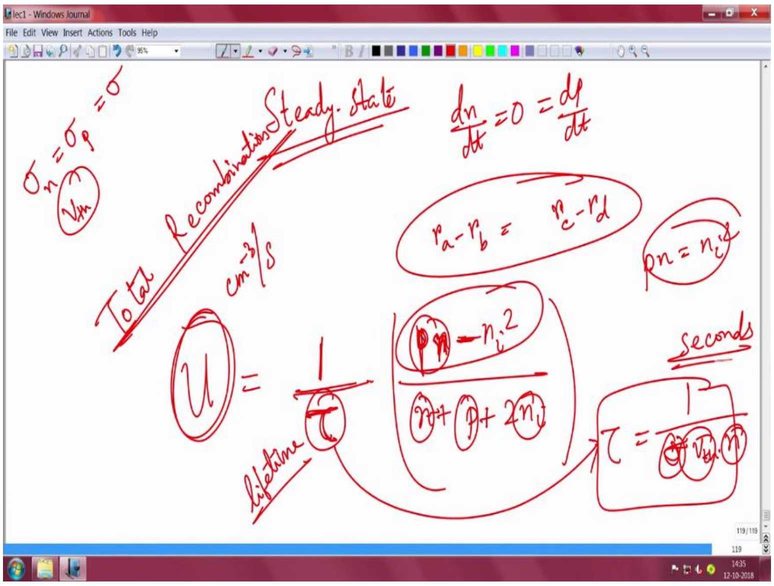

And similarly, for hole it will be $G_L - (r_c - r_d)$ this will be equal to whole thing will be equal to the rate at which you are generating holes any. In thermal steady state in steady state not equilibrium this is steady state which mean steady state means that there is no time variation in steady state there is no time variation. So, dn/dt will be equal to 0 dp/dt also has to be 0 which means, $r_a - r_b = r_c - r_d$ under this case. So, if you do this and you put that in the previous equations that you have obtained here based on this then you can get something like this you can get something like I will explain to that.

P and n, 'n' are the electron total electron total hole concentration this is the intrinsic carrier concentration 'n_i' this is electron concentration this is hole concentration this is intrinsic carrier concentration 'τ' is some kind of lifetime I will come to that very quickly tau is some kind of lifetime and this is the total recombination rate; the rate at which electrons or holes are recombining either with traps or among themselves. But typically, it is in this context they are recombining with traps you know this is the total rate at which they are recombining with traps this is per centimetre cube per second this is equal to

1/lifetime. So, if your lifetime is long, they will recombine fast if your lifetime is long the recombination rate will be small ok.

And then this is pn, p times n and the product of ($pn - n_i^2$). You might said it pn should be equal to n_i^2. So, this should be equal to 0 no actually this p and n are non equilibrium. So, there is light shining and there is excess p excess n. So, it is not equal to n_i^2 here and the last thing today in the class is that this ‘τ’ this is lifetime is defined by $1/(\sigma * v_{th} * n)$.

Here some assumptions are taken the cross-section area of electron trap is equal to cross section area of hole trap is equal to sigma. That is the assumption I am taking here and the velocity or thermal velocity of electron thermal velocity of hole is the same which is am taking here and this is the electron concentration here actually this is the lifetime that is in seconds and it defines many important parameters as we shall see in the next class ok.

So, here I will end the class here today. So, today we have studied about recombination. We introduced the concept of electron and hole capture and so, now, in the next class we will move forward and we will try to do some more analysis and understanding of this recombination this trap related recombination that happens in semiconductor and how they are actually going to find you know application or how they are we have to understand them in the context of device ok. So, I will come to that in the next class ok.

Thank you for your time.