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**Lecture – 54**  
**Tutorials Session – 2**

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**Problem 3:**

Consider a (chromium) – (p-type Si) metal semiconductor junction. The semiconductor is doped with  $N_A = 10^{17} \text{ cm}^{-3}$ .

- i) Calculate a) the Schottky barrier height (in eV) and b) the built-in potential (in V) at  $T = 300 \text{ K}$ .  
ii) Calculate a) the potential drop across the semiconductor (in V), b) the depletion layer width (in  $\mu\text{m}$ ), c) the magnitude of electric field at the metal semiconductor interface (in  $\text{V/cm}$ ) and d) the junction capacitance per unit area (in  $\text{nF/cm}^2$ ) when the junction is subjected to a 5 V reverse bias.

Use the following parameters if required:

Work-function of Chromium = 4.5 eV

Electron-affinity of Si = 4.05 eV

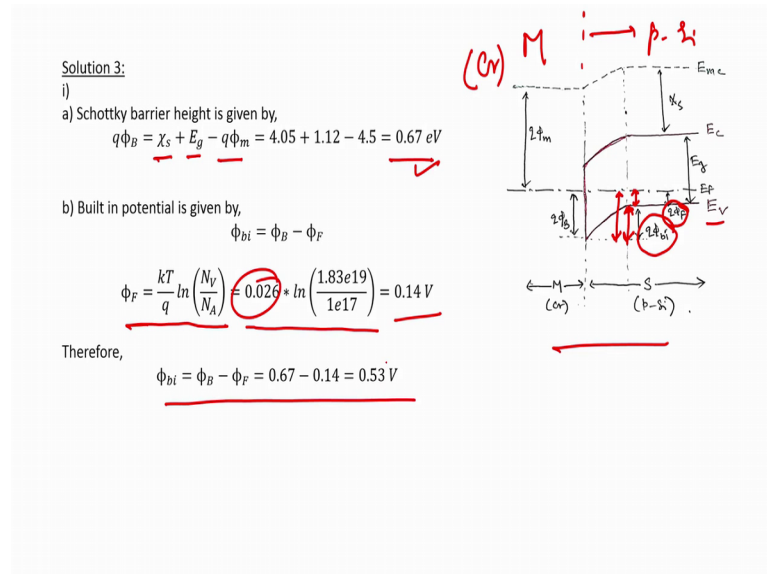
Band-gap of Si = 1.12 eV

For Si,  $N_C = 2.82 \times 10^{19} / \text{cc}$ ,  $N_V = 1.83 \times 10^{19} / \text{cc}$

Dielectric constant of Si = 11.9

So, now let us move into the next problem. So, the problem states that there is a chromium and p-type silicon metal semiconductor junction. The semiconductor is doped with  $N_A$  is equal to  $10^{17}$  per centimeter cube. So, first you have to calculate the Schottky barrier height and the built-in potential at  $T$  is equal to 300 Kelvin. Second you have to calculate the potential drop across the semiconductor in volt. The depletion layer width in micrometer, the magnitude of electric field at the metal semiconductor interface in volt per centimeter which is nothing, but the peak electric field and the junction capacitance per unit area in nano farad per centimeter square all under the case when the junction is subjected to a 5 volt reverse bias. And, some useful parameters are given.

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So, let us try to solve this of the thermal equilibrium case first. Here this is a energy band diagram when the junction is a thermal equilibrium. So, here we have a metal, this side is the metal and this side is a p-type silicon this metal is nothing, but chromium and this p-type silicon. So, at thermal equilibrium the Fermi level will be aligned, this is the Fermi level which is aligned throughout the device and there is a band bending in order to ensure that the there is a continuity of the energy vacuum level. This ensures that the electron affinity of the semiconductor and the metal work function that remains constant throughout the structure.

So, by looking into the energy band diagram you can see that the Schottky barrier this is the Schottky barrier that is given by  $q\phi_B$ , this is the height of the Schottky barrier. So, this is the height of the potential that is basically that will be filled by the electron when it is trying to move from the metal to semiconductor. On the other hand, built in potential is basically the barrier which is failed by the electron when it is trying to move from the semiconductor to the metal. So, this is your built in potential.

Now, how will you find the Schottky barrier height? So, basically if you look into this diagram so, this Schottky barrier height will be given by your, this entire height that is the electron affinity plus the band gap, this total height, minus the metal work function this side. So, that will give you this barrier height. So, that is what here it is written that  $q\phi_B$  is equal to the electron affinity of the semiconductor this side plus the energy band

gap this side minus the metal work function this side. So, that is coming as 0.67 electron volt by plugging in all the numbers given in the problem.

Now, when it is poured is what is the built in potential. So, basically I have to find out the built in potential now. So, the built in potential you can find from the Schottky barrier height. So, basically now we have to find out what is the value of this height and that can be found from the Schottky barrier height minus the minus this much which is written as  $q\phi_F$  which is nothing, but the separation of the Fermi level from the valence bandage eV.

So, basically we know the Schottky barrier height we have to find out now the value of  $\phi_F$  and value of  $\phi_F$  can be easily found by using this expression  $\phi_F$  equal to  $kT$  by  $q \ln N_A$  by  $N_A$  and that is given by 0.026 into  $\ln$  plug in all the numbers 0.026 because it is a 300 Kelvin given in the problem. So, your  $\phi_F$  is coming has 0.14 volt just subtract these two you will find the built in potential that is given by 0.53 volt.

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ii) The MS junction is subjected to a 5 V reverse bias,  $V_a = -5$  V.

a) Potential drop across the semiconductor =  $(\phi_{bi} - V_a) = (0.53 + 5)$  V = 5.53 V.

b) Depletion width,

$$x_d = \sqrt{\frac{2\epsilon_s(\phi_{bi} - V_a)}{qN_A}}$$

$$= \sqrt{\frac{2 \times 11.9 \times (8.85e - 14) \times (0.53 + 5)}{(1.6e - 19) \times (1e17)}} \text{ cm} = 0.27 \mu\text{m}$$

c) Electric field at the M-S interface,

$$\xi(x=0) = \frac{qN_A x_d}{\epsilon_s} = \frac{(1.6e - 19) \times (1e17) \times (0.27e - 4)}{11.9 \times (8.85e - 14)} \text{ V/cm} = 4.1e5 \text{ V/cm}$$

d) Junction-capacitance per unit area,

$$C_j = \frac{\epsilon_s}{x_d} = \frac{11.9 \times 8.85e - 14}{0.27e - 4} \text{ F/cm}^2 = 3.9e - 8 \text{ F/cm}^2 = 39 \text{ nF/cm}^2$$

$C_j \approx C_{dep} = \epsilon_s / x_d =$

So, let us jump into the next part where it is given that now the junction is reverse bias. So, basically you have this metal semiconductor junction which is a p-type silicon the semiconductor. Now, the metal in case of reverse bias, the metal will be at higher bias and semiconductor will be at lower potential. So, this is how the junction or the diode is biased 5 volt.

So, in usual notation we can write this applied bias is equal to minus 5 volt. So,  $V_a$  if it is positive it signifies that it is forward biased when  $V_a$  is negative it signifies that it is a reverse bias phase. So, in this particular case  $V_a$  is negative. So,  $V_a$  is equal to minus 5 volt. This semiconductor is at a lower potential this Fermi level will basically rise. So, there will be a separation of the Fermi levels between the metal Fermi level and the semiconductor Fermi level. So, this is a separation now between these two energy levels.

And, this separation is given by the amount of the applied bias. So,  $q$  into magnitude of  $V_a$  is the separation between these two Fermi levels. So, the result is the more band bending more amount of band bending because Schottky barrier height cannot change. What will change effectively the height of the potential that is seen by the electron when it is trying to move from the semiconductor to the metal, that is the built in potential was that barrier in case of thermal equilibrium. Now, that built in potential will increase by built in potential will not change, but effectively the electron will see  $\phi_{bi}$  plus magnitude of  $V_a$ ; that is the applied bias. This is the resultant voltage that is observed by the electron from when it is trying to move from the semiconductor to the metal.

So, the first part of this question was what is the potential drop across the semiconductor and that is given by  $\phi_{bi} - V_a$ , because now this that this entire amount of band bending is basically that corresponds to the total amount of voltage that drops across the semiconductor. So, that is equal to  $\phi_{bi} + \text{magnitude of } V_a$ ; because here  $V_a$  is negative. So, if you plug in those numbers you can find that potential drop across the semiconductor is now 5.53 volt.

Now, the second is that what is the depletion width this again straight forward you just plug in those numbers. The expression of the depletion width is  $x_d = \sqrt{\frac{2\epsilon_s \phi_{bi} - V_a}{qN_a}}$ . So, you just plug in all the numbers given in the problem, we will find the depletion width as 0.27 micrometer. Now, what is the electric field at the middle semiconductor interface this is nothing, but your peak electric field whose expression is given by  $qN_a x_d / \epsilon_s$  plug in all the numbers again you will find the electric field at the metal semiconductor interface.

Now, the fourth point was what was the junction what is a junction capacitance per unit area in this under this reverse bias situation? The junction capacitance basically if you recall the junction capacitance that constitutes of two capacitances; one is the depletion

So, again plug in all the numbers depletion width already we have found here plug in all the numbers you will get the junction capacitance at 39 nano farad per centimeter square.

Problem 4:

Consider a linearly graded PN junction, where the doping profile linearly varies with the distance from the interface as,  $N_D - N_A = ax$ . Calculate i) the peak electric field, ii) the built-in potential and iii) the depletion width at thermal equilibrium.

Consider that the intrinsic carrier concentration  $= n_i$  and the dielectric permittivity of the semiconductor  $= \epsilon_s$ . Use the fully depletion approximation and assume that all the dopant atoms are ionized inside the depletion region.

$a = \text{proportionality const.}$

$N_D^+ \approx N_D$

$N_A^- \approx N_A$

Now, here this problem discusses about the linearly graded PN junction. Now, what is the linearly graded PN junction? In case of abrupt PN junction what you have consider that the doping concentration on p-side and inside they remain constant uniform throughout the structure. But, in case of linearly graded PN junction the doping profile linearly varies with the distance from the interface and this expression of the doping profile is given as  $N_D - N_A = ax$ , where  $a$  is nothing, but your proportionality constant.

So, basically the here the doping spatial distribution of doping profile is a linear function of the space is a linear function of the space. Now, here you have to find out the peak electric field, built-in potential and depletion width at thermal equilibrium. So, basically you have to do that entire analysis of electrostatics at thermal equilibrium in this spatial kind of doping profile. So, some parameters are given like the intrinsic carrier concentration is equal to  $n_i$  the dielectric permittivity of the semiconductor  $\epsilon_s$ .

And some approximations you can use in your analysis, first is the fully depletion approximation and the second one is the, you can consider that all the dopant atoms inside the depletion width in inside the depletion region they are completely ionize. So, you can consider that  $N_D$  plus is nearly equal to  $N_D$  and  $N_A$  minus is nearly equal to  $N_A$ .

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Solution 4:  
Doping:  $N_D - N_A = ax$

Poisson equation, for  $-\frac{x_d}{2} \leq x \leq \frac{x_d}{2}$

$$\frac{d\xi}{dx} = -\frac{d^2\phi}{dx^2} = \frac{\rho(x)}{\epsilon_s} = \frac{qax}{\epsilon_s} \quad (1)$$

Integrating eq. (1),

$$\xi(x) = \frac{qax^2}{2\epsilon_s} + C_1 \quad (2)$$

Boundary condition:

$$\xi\left(x = \frac{x_d}{2}\right) = 0$$

$$C_1 = -\frac{qax_d^2}{8\epsilon_s}$$

$$\xi(x) = -\frac{d\phi}{dx} = \frac{qa}{2\epsilon_s} \left[ x^2 - \left(\frac{x_d}{2}\right)^2 \right] \quad \text{for, } -\frac{x_d}{2} \leq x \leq \frac{x_d}{2} \quad (3)$$

$$\xi_{max} = \xi\left(x = 0\right) = -\frac{qax_d^2}{8\epsilon_s} = -\xi_m$$

$$\xi_m = \frac{qax_d^2}{8\epsilon_s}$$

Let us try to first solve this problem. So, the doping profile is given by  $N_D$  minus  $N_A$  is equal to  $ax$ . So, the doping profile picture will look like this is the doping profile. This is the linear doping profile so, with  $a$ . So, basically the slope of this line is equal to  $a$  and. So, first we have to solve the Poisson's equation.

So, between minus  $x_d$  by 2 to  $x_d$  by 2 we are considering here that  $x_d$  is the depletion width and we are considering in symmetry that minus  $x_d$  by 2 is on the p side  $x_d$  by 2 on the inside and the depletion layer is extended between these two regions between these two lines. So, let us try to first solve the Poisson's equation between these two

points minus  $x/d$  by 2 and  $x/d$  by 2. So, here Poisson's equation is given by  $d$  electric field  $dx$ . So, let us write that  $d$  electric field  $dx$ . This is equal to minus  $d^2 \phi/dx^2$  which is equal to your charge per unit volume on the semiconductor by semiconductor permittivity.

Now, this  $\rho(x)$  can be written as  $q(N_D - N_A)$  plus  $p - n$ , right. Now, here the first assumption was the, you can consider it is a fully depleted region this region is a fully depleted region. So, we can consider that  $p$  and  $n$  they can be consider as 0. The next assumption you can consider is a fully ionization of the dopant atoms. So, we can consider now  $N_D$  plus is equal to  $N_D$  and  $N_A$  minus equal to  $N_A$ . So, this can be further reduced to  $q(\epsilon_s N_D - N_A)$ .

Now, this is given by this doping profile  $N_D - N_A$  equal to  $ax$ . So, we can write it as  $qax/\epsilon_s$ . So, this is what given here the Poisson's equation. So, basically now we have to solve this Poisson's equation. If you solve this Poisson's equation so, first integrate this one time, so, we will get the electric field expression this is a straightforward integration this electric field is coming as a quadratic function of  $x$  along with the integration constant.

Now, this integration constant can be found from the boundary condition the boundary condition is that electric field that goes down to 0 beyond this the depletion width. So, we can consider that  $x$  equal to plus or minus  $x_d/2$ , the electric field goes down to 0. If you plug in this boundary condition here in this expression you will find the proportionality constant  $C_1$  as minus  $qax_d^2/8\epsilon_s$ .

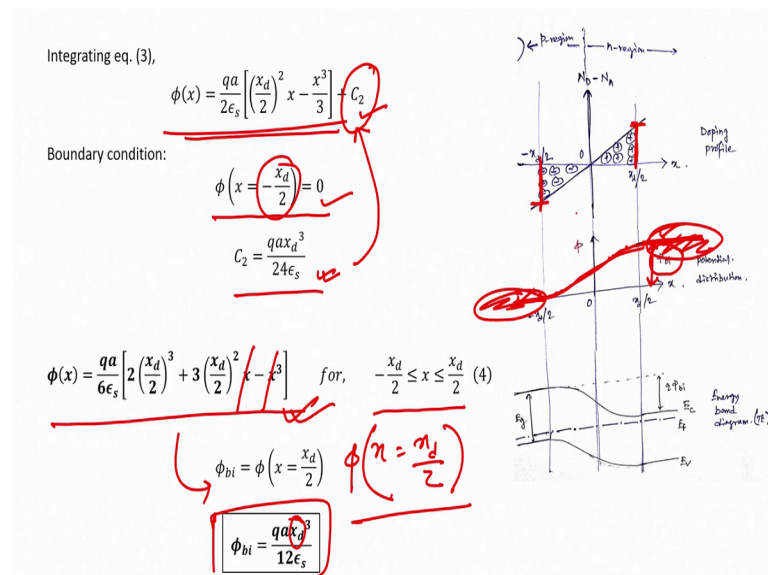
So, here just observe that here you get basically two boundary conditions, one is at plus another one is at minus both will give you the same result because of the symmetry because this is a this an even function because of that symmetry will get the same value of  $C_1$ . Now, you plug in this expression of  $C_1$  into equation 2 you will find the total electric field expression that is equal to minus  $d\phi/dx$  equal to this expression. So, this is the electric field for a minus I mean this is a electric field inside the region minus  $x_d/2$  to  $x_d/2$  beyond that electric field will become 0.

So, from this electric field expression we can find out the maximum electric field that maximum electric field will be at the interface or at the junction exactly at the junction. So, maximum electric field will be when  $x$  equal to 0 just plug in that  $x$  equal to 0 here.

So, this term will be canceled. So, this expression will be minus  $qa \times d$  square by 8 epsilon s. So, it is a negative this is a negative field because the field that will be directed from n region to p region. So, electric field here is a negative and the peak electric field hence also is a towards the negative axis. So, you have to performed you can find the magnitude of the peak electric field as this.

So, this is the diagram which basically shows the electric field profile this you can see a quadratic profile these quadratic profile and this is the peak electric field and its magnitude is epsilon m. So, minus that maximum electric field is this value, ok.

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So, next you have to integrate that electric field in order to find the potential. So, what this is this was our electric field. Electric field was that can be written as minus  $d\phi/dx$ , let us write the equation first  $qa$  by twice epsilon s  $x$  square minus  $x_d$  square by 2  $x$  square by 4, sorry. So, now, you integrate this expression to find out the potential or potential  $\phi$ . So, if you integrate this again a straightforward integration you will end up with  $qa$  by twice epsilon s. Here this term will give  $x$  is  $q$  by 3 and this term will give  $x$   $d$  square by 4 into  $x$ . So, this is what comes here along with an integration constant  $C_2$ .

So, now we have to find out this integration constant  $C_2$  under the boundary condition that the potential is 0 at  $x$  equal to minus  $x_d$  by 2 that is a reference level. So, just look into the diagram of the potential distribution along with the doping profile if we match the potential distribution profile. So, it will look like this these are a non-linear



expression cubical expression you have. So, this is a potential profile. We can consider that this is your reference level with respect to the reference level the potential at this point is a constant potential here is consider the that is termed as a built in potential  $\phi_{bi}$ .

So, we can consider this as a reference level that is why we have kept  $\phi$  at  $x$  equal to minus  $x_d$  by 2 equal to 0, that is the first boundary condition. That is the only boundary condition we need to solve this expression because there is only a single integration constant. So, if you plug in that number here you will find  $C_2$  is equal to  $q_a x_d^3$  by  $24 \epsilon_s$ . Now, if you now just plug in that expression of  $C_2$  into this equation you will find the entire expression of the potential.

So, again this is the potential profile valid within this range because beyond the depletion range again this will for example, beyond  $x$  equal to minus  $x_d$  by 2 it will be 0, that is the reference level and beyond  $x$  equal to  $x_d$  by 2 it will be  $\phi_{bi}$  that is a built in potential. So, within this range this expression is valid. So, how we will you find built in potential you have to plug in that  $x$  equal to  $x_d$  by 2. So, if you find the potential at  $x$  equal to  $x_d$  by 2 that will give you the built in potential. So, just plug in that  $x$  equal to  $x_d$  by 2 here, here you will end up with this expression this expression built in potential equal to  $q_a x_d^3$  by  $12 \epsilon_s$ .

So, just observe that here we have find maximum electric field in terms of the depletion width, here we have found built in potential in terms of the depletion width again. So, now, the only thing is remaining to find the depletion width at this  $x$  equal to minus  $x_d$  by 2 and at  $x$  equal to  $x_d$  by 2 by magnitude this doping concentration will be equal.

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The image shows a slide with three printed equations and several handwritten red annotations:

- Printed equation 1: 
$$\phi_{bi} \approx \frac{kT}{q} \ln \left[ \frac{(ax_d/2)(ax_d/2)}{n_i^2} \right]$$
- Printed equation 2: 
$$\phi_{bi} \approx \frac{2kT}{q} \ln \left( \frac{ax_d}{2n_i} \right)$$
- Printed equation 3: 
$$\phi_{bi} = \frac{qax_d^3}{12\epsilon_s}$$
- Handwritten red annotations:
  - A box around  $a \cdot \left(\frac{x_d}{2}\right)$  with the note  $N_D - N_A = ax$ .
  - The note  $x = \pm \left(\frac{x_d}{2}\right)$ .
  - The derivation  $\phi_{bi} = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$ .
  - The substitution  $\phi_{bi} \approx \frac{kT}{q} \ln \left( \frac{\left(\frac{ax_d}{2}\right) \cdot \left(\frac{ax_d}{2}\right)}{n_i^2} \right)$ .
  - The final simplified form  $\phi_{bi} \approx \frac{kT}{q} \ln \left( \frac{ax_d^2}{2n_i} \right)$ .

And, that doping concentration is given by  $a$  into  $x_d$  by 2 because  $N_D$  minus  $N_A$  is equal to  $ax$ . So, you just plug in if  $x$  equal to plus minus  $x_d$  by 2. So, the magnitude of the doping density at  $x$  equal to  $x_d$  by 2 or minus  $x_d$  by 2 that is at the edges of the depletion layer that value is given by this  $a$  into  $x_d$  by 2. So, at these two edges the doping densities are equal.

Now, we will use an approximate formula what we have used earlier in case of abrupt junction that the built-in potential is equal to  $kT$  by  $q$  in case of abrupt junction this formula is you we have used this formula  $kT$  by  $\ln N_A N_D$  by  $n_i$  square. So, in this particular case also we are using an approximate formula this not exact formula, but this an approximate relation we are using here, built-in potential equal to  $kT$  by  $q$   $\ln$  instead of any  $N_D$  now we will use the doping density what we obtained at the two edges of the depletion width and that is given by  $a x_d$  by 2 into  $a x_d$  by 2 by  $n_i$  square.

So, this will be equal to  $kT$  by  $q$   $\ln a x_d$  by 2  $n_i$  whole square. So, just take this 2 out. So, now your built-in potential will be nearly equal to  $2 kT$  by  $q$   $\ln a x_d$  by 2  $n_i$ . So, this is another relation.

So, now we have two relations between  $x_d$  and built-in potential; one is this relation which is an approximate relation and another one this relation this relation. So, basically if you look into these two equations, these two are transcendental equations. So, analytical it is not possible to solve these two equations. So, you have to sort for some numerical

analysis either by simulator or by some numerical technique you have to solve this built in potential and depletion which separate together.

So, once you solve for the depletion width you plug in the depletion width number here in order to find the maximum electric field. So, this is how this linearly graded a PN junction electrostatics can be taken care.