Semiconductor Devices and Circuits
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Lecture – 53
Tutorials Session - 1

Hello everyone, welcome to the course Semiconductor Devices and Circuits. We have headed towards the end of the course.

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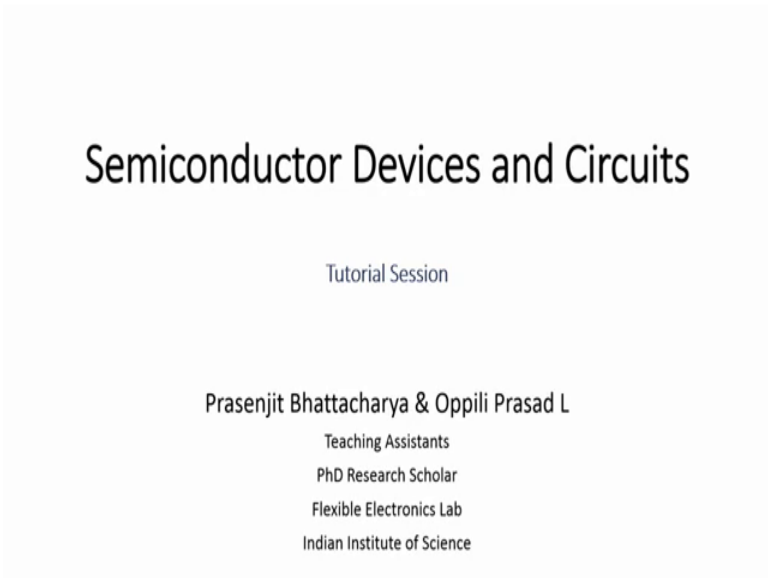

So, we thought we will take a tutorial session for the remaining two or three lectures. Hope you have enjoyed the course.

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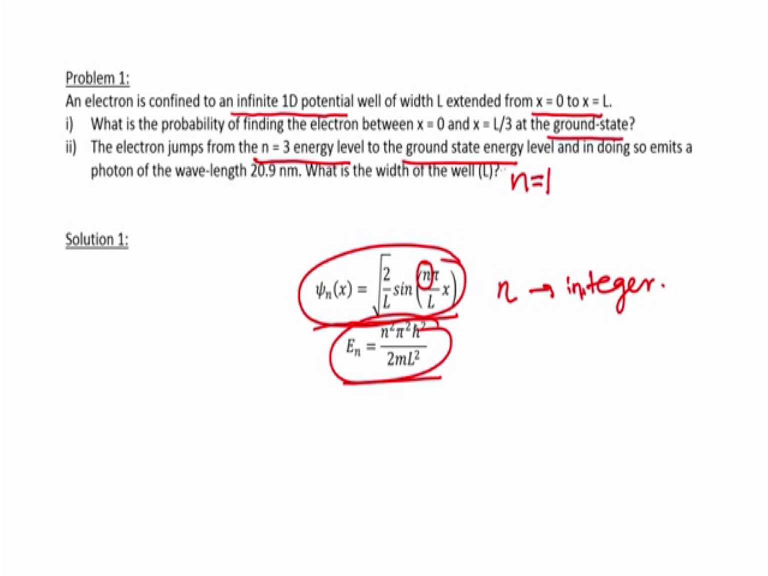

We have designed the Tutorial Session in such a way that you will get many problems from the assignments and we will introduce some new concepts also so that overall it will give you a recap of the entire course. So, let us first jump into the first problem; this from the quantum mechanics.

The problem states that an electron is confined to an infinite 1D potential well of width L extended from x equal to 0 to x is equal to L. So, first is you have to find out the probability of finding the electron between x equal to 0 and x equal to L by 3 at the ground state. And second one is the electron if the electron jumps from the n equal to 3 energy level to the ground state energy level and in doing so, it emits a photon of wavelength 20.9 nanometer then what will be the width of the well?

So, there are few things you have to note down here. First is the infinite 1D potential well which has been covered in the course, in the first topic. That first topic was that quantum mechanics infinite 1D potential well. Second thing is basically the well is extended from x equal to 0 to x equal to L. Now, the first problem states that you have to find the probability of finding the electron between x equal to 0 and x equal to L by 3 at the ground state.

So, just recall the wave function of the 1D potential, infinite 1D potential well. If the particle is at nth state then the wave function is given by psi n x equal to root over 2 by L sin integral multiple of pi by L into x. Where basically, n is any integer and the corresponding energy of the n th sate is given by En is equal to n square pi square h quart square by twice mL square.

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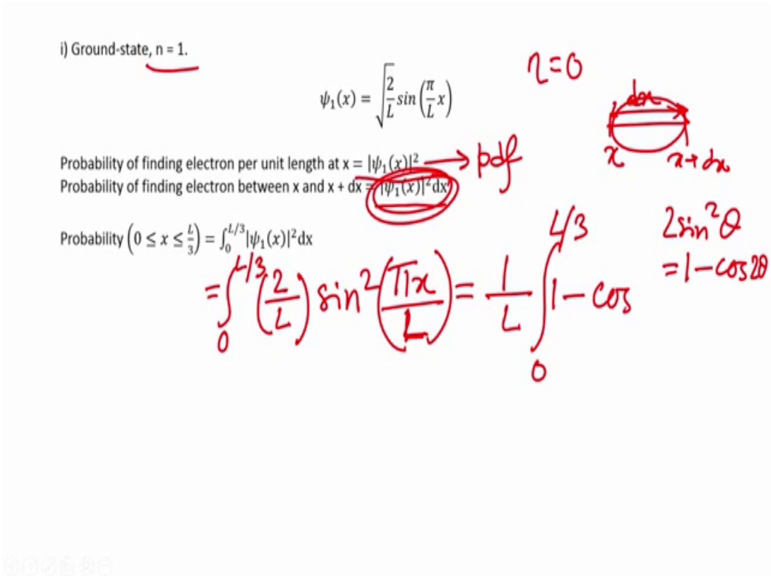

So now, as the; so first thing is the; what will be the ground state? Ground state will correspond to n equal to 1. Just remember n is equal to 0 is not the ground state because if we put n equal to 0 in this equation, here this wave function will vanish because sin 0 is equal to 0. So, and also the energy will become 0. So, n equal to 0 is not any valid state. So, n equal to 1 is your first state and that is why you call it a ground state. Now, probability of finding the electron per unit length at x is given by modulus of psi 1 x whole square and this nothing but your pdf, probability density function.

So, the probability of finding the particle or the electron here between x and x plus dx will be given by mod psi 1 x whole square into dx. That is pdf into dx because you want to find the probability between x and x plus dx. So, this is your dx length. So, what will be the probability of finding the electron between this dx length that is given by this expression? So, the probability of finding the electron between 0 and L by 3, you have to basically integrate this expression from 0 to L by 3.

So, just go through the steps few steps here. So, integrate from 0 to L by 3. So, magnitude of psi 1 x whole square will be given by 2 by L then, sin square pi x by L. So, you have to convert this sin square to cos 2x in order to do the integration. So, that will come as a standard integral. So, 2 by L integration over 0 to L by 3, you can take this 2 inside. So, it will become 2 sin square. So, basically 2 sin square theta that is equal to 1 minus cos 2 theta. So, you have to plug in here 1 minus cos I will just erase this one this mid space

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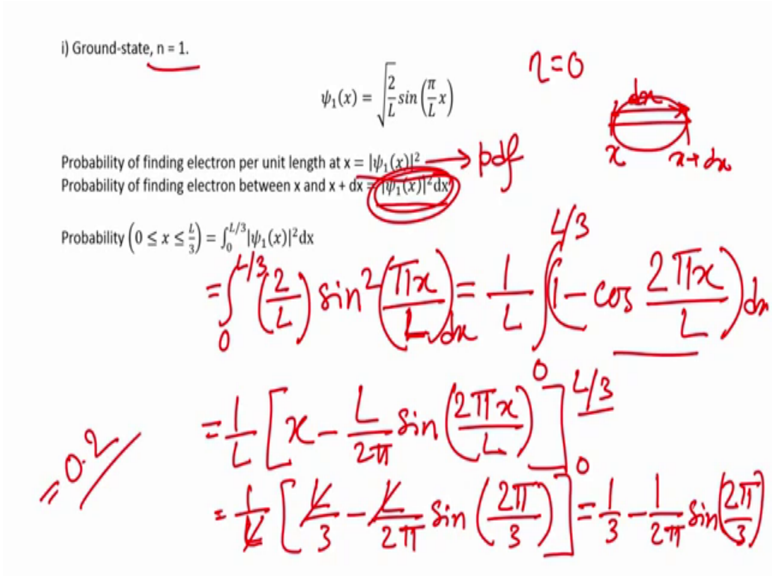

cos 2 pi x by L, there will be one dx. So, if you do this integration it will come as 1 by L. Here it will be x minus. So, you have to do the integration of this. It will be L by 2 pi sin, integral of cos is sin 2 pi x by L.

So, your limit you have to put the limit from 0 to L by 3. So, now just look into this expression, if you put 0 both will be 0; this two term. So, you have to put this L by 3. So, only L by 3, the upper limit will that will give you some value, L by 3 minus L by 2 pi sin 2 pi by 3. So, you just cancel out L here. So, it will come as one-third minus 1 by 2 pi sin 2 pi by 3.

So, if you just calculate this, this probability will come as 0.2; just check this with calculator. So, what is the next problem? It was if the electron jumps from n equal to 3 energy level to the ground state energy level, so, again from n is equal to 3 to n is equal to 1 your ground state corresponds to n equal to 1.

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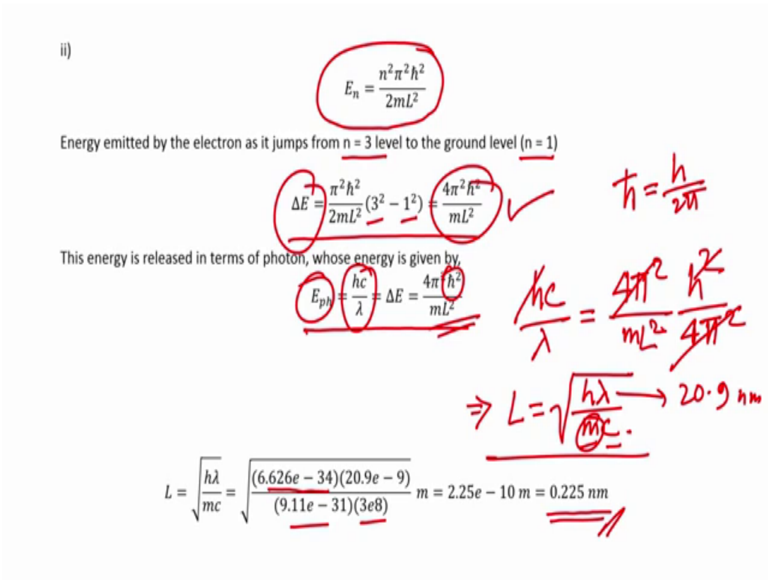

So, basically, En that is given by this expression that is the energy of n-th state. So, if it jumps from n equal to 3 to n equal to 1, it emits an energy which corresponds to del E is equal to pi square h quartz square by twice mL square into n equal to 3 square minus n equal to 1 square. So, it will give you some expression here.

Now, this energy is basically as it emits a photon, so this energy should be equal to the energy of the released photon. So, photon energy then is given by E ph photon energy, this photon energy. This is equal to hc by lambda and that should be equal to the energy of that released energy. So, hc by lambda should be equal to 4 pi square h quarts square by mL square. So, let us just do some simple algebra here; hc by lambda. This is equal to 4 pi square by mL square.

Let us convert this h quart into h by 2 pi; h quart this is just a reduced Planck's constant. So, this is equal to h by 2 pi. Let us plug in this expression here, h square by 4 pi square. So, this will be cancelled out, one h we can cancel here, L will become equal to h lambda by mc ok. So now, you just plug in the numbers given in the problem. So, your wavelength is given as 20.9 nanometers. So, lambda is equal to 20.9 nanometer; h is the Planck's constant whose value is given by 6.626 E minus 3; 34, m will be the mass of the electron; c will be the velocity of light in free space.

So, if you plug in those numbers that L will come as 0.225 nano meter. So, that is the width of your infinite 1D potential well.

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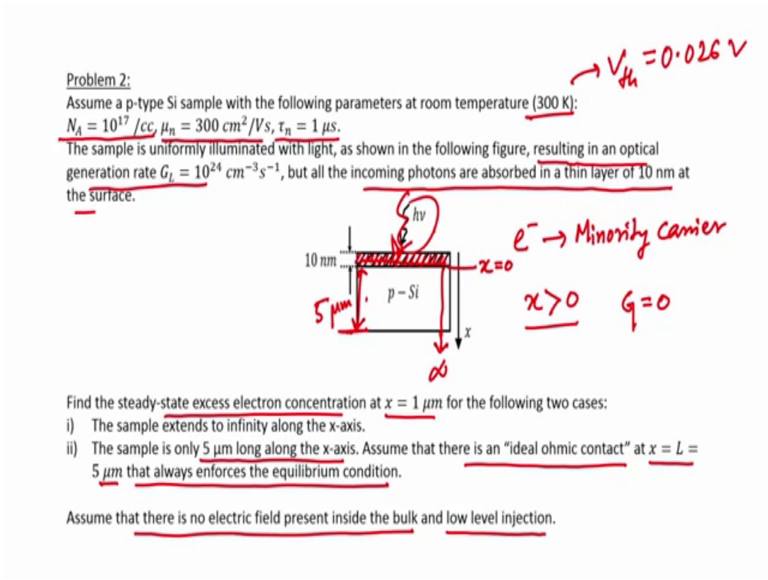

So, now let us come into the next problem. This is from the semiconductor basics and this basically deals with the continuity equation. This also was given in as an assignment problem. So, I thought we could discuss in details that problem here. The problem states that assume a p-type silicon sample with the following parameters at room temperature 300 Kelvin. So, 300 Kelvin means, the thermal voltage here will be equal to 0.026 volt. Some parameters are given for example, the doping density that is NA is equal to 10 to the power 17 per cc. Mobility of the electron is equal to 300 centimeter square per volt second and tau n equal to 1 microsecond. Just remember here electron is your minority carrier.

Because it is a p-type silicon and the sample is uniformly eliminated with light from the top as shown in this figure. So here, light is coming and basically absorbing those photons, electron hole pairs are generating. In this generation rate is also given optical generation rate is GL 10 to the power 24 per centimeter cube per second. So, 1 constraint here is given as the incoming photons are absorbed in a thin layer of 10 nanaometer of the surface. So, if we assume here a 10 nanometer thickness on the surface, all the photons are absorbed over this layer.

So, basically what does it mean that there the electron hole pairs are getting generated over this thin layer only. There is no generation inside the bulk. So, if we consider this is the x axis then just denote this mark as the x equal to 0. So, inside the bulk means for x greater than 0, there is no generation of electron hole pairs. So, basically we can say that this generation rate becomes 0 inside the bulk here. Now, the problem is you have to find out the steady state excess electron concentration at a certain distance 1 micrometer in the 2 following cases.

One is if the sample extends to infinity along the x axis. So, if this dimension a goes to infinite in the first case and the second case states that if the sample is only 5 micrometer long along the x axis. So, if this sample has a finite dimension, so, basically in the second case it is given that this dimension of the device is 5 micrometer and also in the second case it is given that on the other end, after this 5 micrometer length, there is an ideal ohmic contact at x equal to L equal to 5 micrometer that always enforces the equilibrium condition.

We will just discuss this in while solving the problem. So, there 2 assumptions you can consider here. One is the there is no electric field present inside the bulk and you can consider the low level injection. So, this will basically help you reduce the minority carrier diffusion equation.

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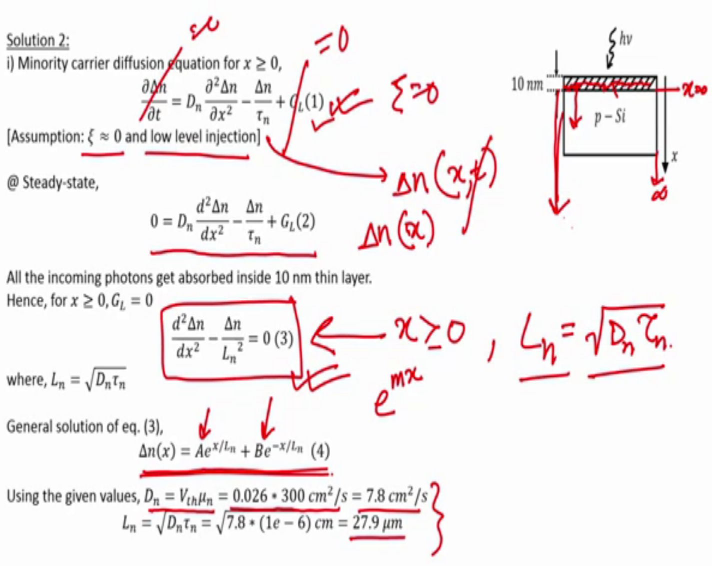

So, let us try to solve this problem. In the first case where this goes to infinity and in that case, we can write down the minority carrier diffusion equation for x greater than equal to 0 as del n t is equal to D n del 2 del n del x square minus del n by tau n plus GL. So, this is the first equation. So, here del n is the excess minority carrier concentration, that is the electron concentration and D n is the diffusion coefficient, tau n is the electron life time and GL is the generation rate.

Now, so, basically what does it state that the electron this electron and holes are generated here and this electrons will diffuse to the bulk according to this following this relation. So, why we have considered this; because our electric field equal to 0 and if you recall the continuity equation, we can reduce in case of electric field equal to 0, we can reduce the equation to only diffusion term. So, that is why this minority carrier diffusion equation looks like this.

So, assumption is electric field equal to 0 and another assumption is low level injection. Under these two assumption, the minority carrier diffusion equation can be reduced to this. Now, the problem ask about the steady state. So, now, when we are talking about the steady state there will not be any change of a carrier concentration with respect to time. So, del n del t will become 0, just one second. So, del n del t will become 0 in case of steady state and other terms will remain.

So, this is the first step where we have reduced for the minority carrier diffusion equation. The next reduction will come when considering the fact that there is no generation of electron hole pairs inside the bulk. So, beyond that 10 nanometer thin layer that generation rate will become 0. So, this will again become 0 for x greater than equal to 0. So, for x greater than equal to 0, the minority carrier diffusion equation can be reduced to this final term d 2 d del n dx square minus del n n square is equal to 0. So, just probably you have noticed one thing that this dou has become now D.

So, basically the partial derivative we have converted to the direct derivative because this del n, here in this first equation it is a function of x and t, but as we are talking about now steady state, so, there is no variation of time. Hence, we are considering now del n as a function of space only. So, there is only a single independent variable and hence we can convert that partial derivative to the direct derivative d 2 dx square.

So, another thing we have just noticed this Ln. This is the diffusion length that is equal to root over Dn tau n; this is the diffusion length of the minority carrier that is the electron here. Now, so basically we have to solve this differential equation under a particular boundary condition. So, what will be the general solution of this equation?

This general solution will be given as A e to the power x by Ln plus B e to the power minus x by Ln. So, you just recall your highest you just recall your undergrad maths where this we consider a trial solution e to the power m x to solve this kind of differential equation and L end up with this general solution.

So, now the neat thing is we have to solve this, we have to find out these 2 coefficients A and B imposing a particular boundary condition. Before that just find out the what are that these Ln and Dn values from the given parameters, Dn that will be that can be found using the Einstein's equation.

Dn is equal to thermal voltage into the mobility and that is given by 0.026 into 300 centimetre square per second is equal to 7.8 centimetre square per second. And what will be the diffusion length? Diffusion length can be found using this equation and that is given if you plug in those numbers, this Ln is coming as 27.9 micrometer. So, we will use this while deriving the final answer ok. So, in the next step we will we will incorporate the different boundary conditions.

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Boundary conditions:

a) $\Delta n(x \rightarrow \infty) = 0$

b) $-D_n \left[\frac{d\Delta n}{dx}\right]_{x=0} = G_s$ where, $G_s = G \Delta x = (1e24) * (1e-6)\ cm^{-2}s^{-1} = 1e18\ cm^{-2}s^{-1}$

$$\Delta n(x) = A\underbrace{e^{x/L_n}}_{=\infty} + B\underbrace{e^{-x/L_n}}_{\rightarrow -\infty}.$$

$$\Delta n(x \rightarrow \infty) = Ae^{\infty} + B\underbrace{e^{-\infty}}_{=0} = 0$$

$$Ae^{\infty} = 0$$

$$A = 0.$$

Using boundary condition (a) to the solution (4),

$$A = 0$$

$$\therefore \boxed{\Delta n(x) = Be^{-x/L_n} \quad (5)}$$

Using boundary condition (b) to the solution (5),

$$B = \frac{G_s}{D_n/L_n}$$

$$\therefore \Delta n(x) = \left(\frac{G_s}{D_n/L_n}\right)e^{-x/L_n} \quad (6)$$

$$\frac{G_s}{D_n/L_n} = \frac{1e18}{7.8/(27.9e-4)}\ cm^{-3} = 3.6e14\ cm^{-3}$$

$$\therefore \Delta n(x\ in\ \mu m) = (3.6e14)e^{-x/27.9}\ cm^{-3}$$

$$\therefore \Delta n(x = 1\ \mu m) = (3.6e14)e^{-1/27.9} = 3.5e14\ cm^{-3}$$

The first boundary condition will come from the fact that at infinity, so as the sample that is extended to infinity, so if we approach to infinity the minority carrier concentration, excess minority carrier concentration will go down to 0 because all these excess minority carriers that are coming from the thin interface layer to the bulk they will recombine with the oppositely charged holes and hence the excess minority carrier concentration will go down to 0. The second boundary condition will coming from the fact that will come from this surface boundaries.

So, it tells that this minus Dn d del n dx at x equal to 0 this is the rate at which the electrons they diffuse away from the thin interface thin surface layer to the bulk and this rate should be equal to the rate at which the electrons are getting generated at the thin interface layer. GS is basically the generation rate, but this is surface generation rate not the volume generation rate. So, basically you have to multiply the volume generation rate with the thickness of that thin layer in order to find what is the number of charge carriers, that is getting generated per unit area inside that thin semiconductor layer.

So, that is you just plug in those numbers and GS will come as this. Now, you have to use these two boundary conditions to solve the to find out that these 2 coefficients A and B in equation 4; del n so that is the general solution del nx equal to A e to the power x by Ln plus B e to the power minus x by Ln.

So, first plug in the first boundary condition, del n x tends to infinity that is equal to A e to the power infinity plus B into e to the power minus infinity and this is equal to 0 according to the boundary condition. Now this term goes down to 0. So, basically it comes as A e to the power infinity is equal to 0. So, your A must be equal to 0. So, only the second term will that will exist, the first term will go down to 0 from the general solution.

So, now the general solution under the first boundary condition is reduced to this term B e to the power minus x by Ln. Now, you will use the second boundary condition; second boundary conditions states that minus Dn d del n dx at x equal to 0 is equal to GS. So, just solve the second boundary condition. So, now, your del nx this is the reduced general solution B to the power minus x by Ln.

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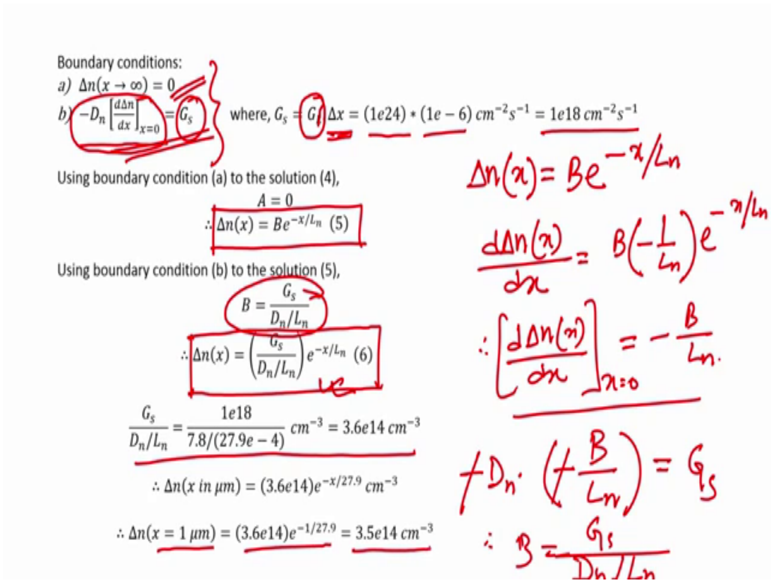

So, d del n x dx this is equal to B minus 1 by Ln e to the power minus x by Ln. Therefore, d del n x dx at x equal to 0 will be equal to minus B by Ln.

So, let us plug in this expression into the second boundary condition minus Dn into minus B by Ln that is equal to GS minus minus will cancel out. Therefore, B will be GS by Dn by Ln. So, that is what B comes under the second boundary condition. So, now, we get the excess carrier concentration profile that is a spatial distribution of the excess electron concentration that is given by this expression. So, now the you have to just plug in the numbers.

GS by Dn Ln you can simply directly you can directly find from the previously calculated values of Dn, Ln and GS. So, you plug in those numbers here; x is given as 1 micrometer. Basically, you have to find the excess electron concentration at 1 micrometer. So, you plug in that number here. You will find that what is the excess electron concentration at x equal to 1 micrometer.

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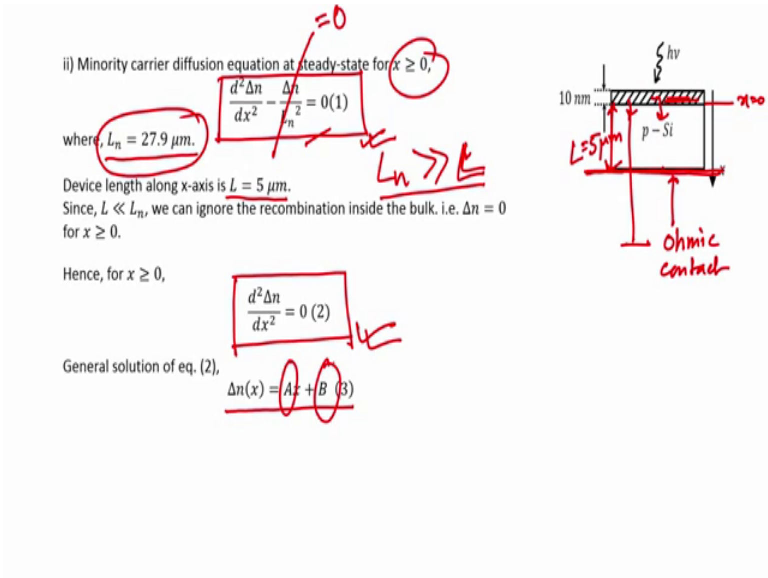

Next part of this problem, where it states that you have to find the excess electron concentration again at x equal to 1 micrometer, but here the boundary condition gets changed because of the finite dimension of the device.

Now, the device dimension becomes L equal to 5 micro meter and this is x equal to 0. So, instead of infinitely extended device, now the device has a finite dimension and that is given by L equal to 5 micrometer.

So, this is your starting point that is a minority carrier diffusion equation at steady state and under the two assumptions that there is no electric field inside the bulk and there is a low level injection. So, this is the differential equation which dictates the transport of the minority carriers inside the bulk that is for x greater than equal to 0. Now, here as calculated previously that the diffusion length of the electron is 27.9 micrometer and the device dimension is given as 5 micrometers. So, if you compare these 2 values, Ln we can consider as much larger much greater than L.

So, what does it signify that Ln is basically the diffusion length that means, on an average after injecting an electron, on an average that electron goes 27.9 micrometer before it recombines with a hole, but we have imposed a boundary at L equal to 5 micrometer. So, we can consider that all the electrons that are coming from the thin interface layer to the bulk they will go to the opposite to the electrode which is placed at l equal to 5 micrometer without recombining with the oppositely charged hole.

So, basically we can ignore any kind of recombination. So, we can reduce this minority carrier diffusion equation further by considering del n is equal to 0; that means, the recombination term there is no recombination. So, this recombination term will go down to 0. So, the minority carrier diffusion equation that gets reduced to this d 2 del n dx square is equal to 0. So, the general solution of this differential equation is given by del nx equal to Ax plus B. That is the linear equation.

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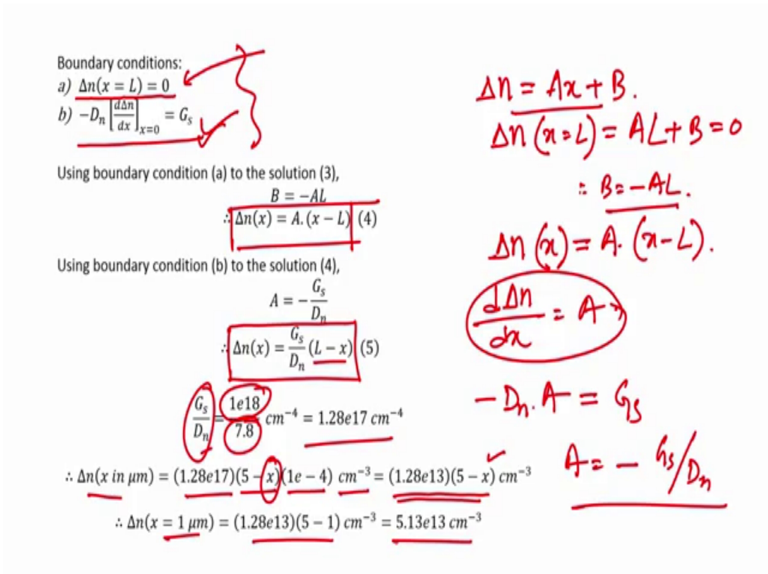

Now the boundary conditions, boundary conditions will be one will be the excess electron concentration at x equal to L will be equal to 0. This is coming from the fact that at x equal to well, we have considered a perfectly ohmic contact. Now, this ensures that whatever excess electrons are coming at this point, they will be immediately taken that will be immediately removed by that electrode. So, that there a thermal equilibrium condition can exist.

So, your del n at x equal to L becomes 0 because of that perfectly ohmic contact. The second boundary condition that is coming from the surface, so, that is similar to the previous case because again at steady state the generation rate of the electrons inside the thin semiconductor layer should be equal to the rate at which that is moving away from the interface to the bulk.

So, this equation gives that boundary condition. So, basically using these two boundary conditions, now we have to solve this basically you have to find out these 2 coefficients

A and B. So, now the general solution is del n equal to Ax plus B. Plug in the first boundary condition. It states that del n at x equal to L that is equal to AL plus B that is equal to 0. Therefore, B is equal to minus AL and if you plug in this into this expression basically you can reduce further the general solution as del n equal to A into x minus L.

Now you plug in the second boundary condition. So, second boundary condition will come from basically you have to differentiate this expression first; A into x minus L. Let us differentiate this expression first that is equal to A. So, let us plug in this expression into the second boundary condition minus Dn. Now, this is a constant. So, it does not matter if whether we are taking at x equal to 0 or at some other x value.

So, you can just plug in directly this number that is equal to GS. So, this will give you the value of A that is equal to minus GS by Dn. So, now the excess electron concentration will come as del nx equal to GS by Dn into L minus x. So, now you have to just simply calculate the numbers. GS by Dn that is given by GS, we have calculated in the previous part. That is 1e 18 Dn that is again we have calculated previously as 7.8. So, GS by Dn is coming with this number. So, now, therefore, del n is given as this GS by Dn.

So plug in that number into L minus x. L is here 5 micrometer, so, 5 minus x. We basically have constructed this expression such a way that x is in micrometer. So, you have to convert this into centimetre in order to get the whole expression in per centimetre cube value. Normally, we express the any concentration as per centimetre cube value. So, I have converted that micrometer into centimetre 1e minus by multiplying with 1e minus 4.

So, this is the final expression where x is in micrometer. So, now, you just plug in at x equal to 1 micrometer and you will get the answer for what is the excess electron concentration at x equal to 1 micrometer.