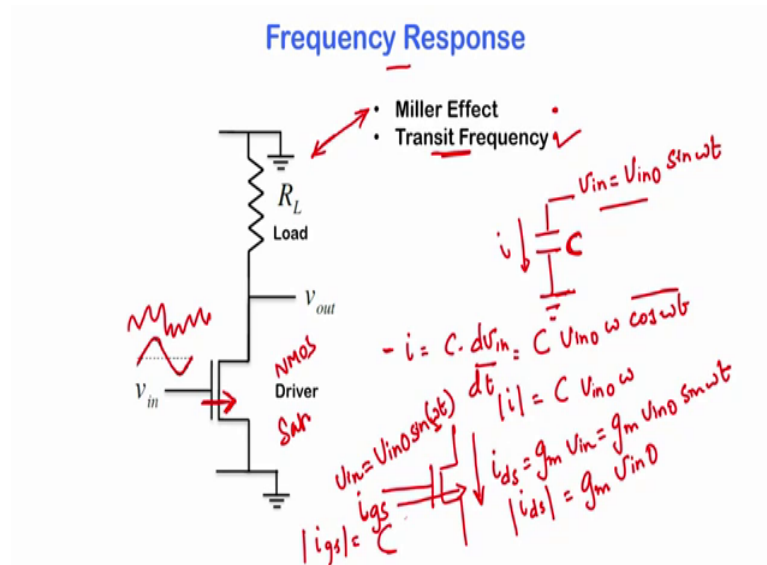


**Semiconductor Devices and Circuits**  
**Prof. Sanjiv Sambandan**  
**Department of Instrumentation and Applied Physics**  
**Indian Institute of Science, Bangalore**

**Lecture – 50**  
**Circuits: Frequency Response, Noise**

(Refer Slide Time: 00:14)



So, let us now come to the topic of the frequency response of circuits and may not going to do this in a very exhaustive manner, but rather look at just a couple of important points, which are related to the engineering of the devices that we use. And these two points that we want to worry about are something called as a Miller effect and something called as a transit frequency.

And to understand the Miller effect we need to get back to using a circuit and we will use a very early circuit of a common source amplifier with an NMOS driver which is on saturation and the and a resistor load. And a transit frequency is a more device level study and therefore we will try to do that first ok.

So, what do you mean by transit frequency? Now, so far we have implicitly assumed that we are talking about input signals that are time varying. We have always said it is a time varying input signal is a small easy waveform, but the frequency has been very low. And why do we say that we have implicitly made that assumption, because we never bothered

about any current going into the gate. So, if you were to take a capacitor and say you have an input  $V_{in}$ , which is say some  $V_{in} \sin \omega t$ . And say the capacitance is called  $C$ , and let us say this end is grounded.

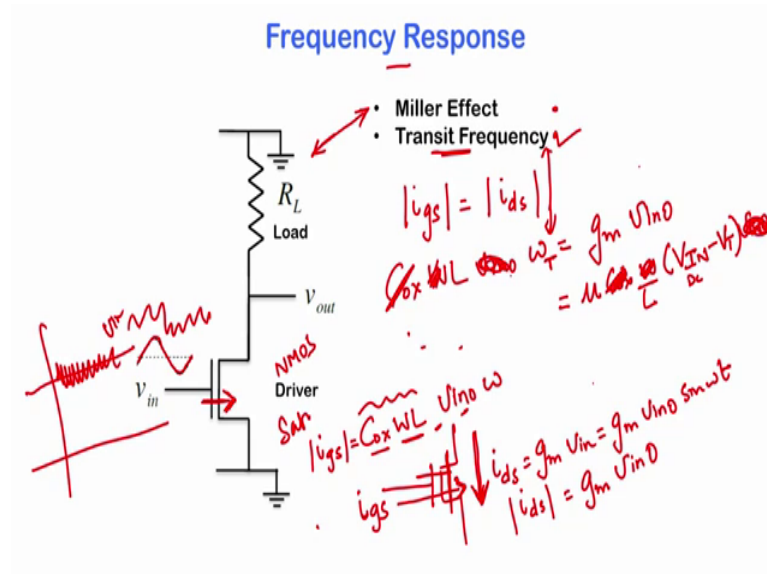
So, what is the current through the capacitor? The current through the capacitor is  $C \frac{dv}{dt}$ . So, in this case it will be  $C \frac{dv_{in}}{dt}$ , which will effectively be  $C \omega V_{in} \cos \omega t$ , where this is basically a phase shift term. So, the magnitude of the current ok, so if you look at the magnitude of the current through the capacitor, it is  $C \omega V_{in}$ , so, as  $\omega$  goes up, the current increases.

And since we never bothered to calculate the current through this gate ever during analysis we were talking about very low frequency signals. So, now we approach a point where the frequencies so large that we need to take these currents seriously and that is where the concept of a transit frequency comes in.

So, let us take a MOS transistor and we apply this input as your the we apply that as your input to the gate of a MOSFET. So, what is the current between the source and drain of the MOSFET? It is  $i_{ds}$ , which is  $g_m V_{in}$ . And what is the magnitude of this current,  $i_{ds}$  magnitude is, so this is nothing but  $g_m V_{in}$ .

And the magnitude of this or which is not capturing the phase information, but just looking at the magnitude, it is  $g_m V_{in}$ , so that is the magnitude of the current through the drain to source, now because this frequency is considerably high we also need to take into account the gate current. So, let us call that as  $i_{gs}$  ok so, what is the gate current ok? So, if I have to use this data we used here, the magnitude of the gate current  $i_{gs}$  is going to be  $C \omega V_{in}$ , which is the capacitance of the MOSFET. So, let me just rewrite it more neatly right here.

(Refer Slide Time: 04:09)



So, the magnitude of  $i_{gs}$  is going to be the capacitance of the gate capacitance which is nothing but  $C_{ox}$  into  $WL$ , so that is going to replace the term  $C$  here.  $C_{ox}$  into  $WL$  let me just remove this, because we do not need that into  $V_{in0}$  into  $\omega$  so that is your total gate capacitance. This is the capacitance per unit area into the area, which is  $W$  into  $L$  into  $V_{in}$  into  $\omega$ , so that is the total gate current.

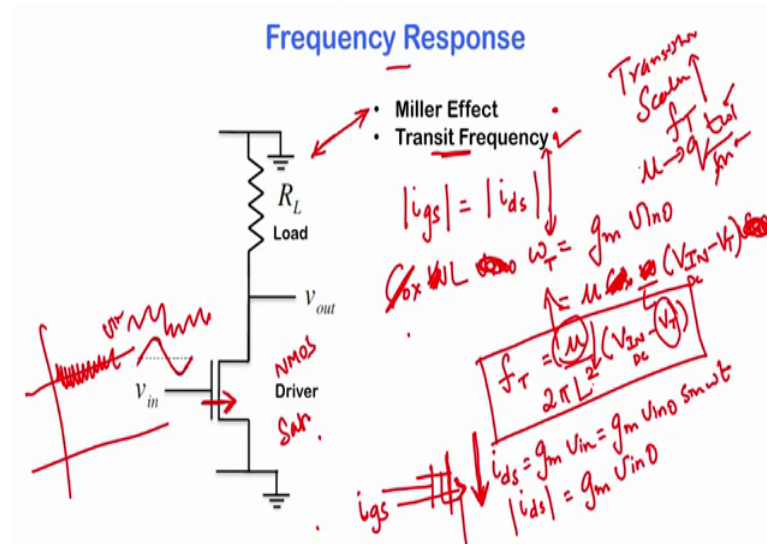
So, you now at high frequencies, you now have a gate current going in and you also have  $i_{ds}$ . Now, the transit frequency is the frequency at which  $i_{gs}$  equals  $i_{ds}$ . And why is that frequency critical, because at that frequency the leakage or the input resistance of the gate has started matching the effective resistance of the source to drain, which is the basic which basically defines the maximum limit or the frequency to which you can push your device to operate.

So, in some sense the technology determines the transit frequency. And it tells you that it is a frequency, when the frequency when the gate to source current, matches the drain to source current in magnitude, so which means that this is when  $C_{ox} W L$  into  $V_{in0}$  into  $\omega$  is equal to  $g_m$  into  $V_{IN0}$ . And what is the  $g_m$ ?  $G_m$  is  $\mu C_{ox} W$  over  $L$  into  $v_{gs}$  or the  $V_{IN0}$  the  $D_c$  value of  $V_{in}$ , which is not shown here.

So, there is a  $D_c$  value of  $V_{in}$ , and on top of that you have this very high frequency  $v_{sn}$  going here. So, the  $V_{IN0}$  minus  $V_T$  into small  $v_{in0}$  and we will give a special symbol to this  $\omega$  we will call it  $\omega_T$ , which is to represent the transit

frequency. So,  $C_{ox}$  cancels on both sides,  $W$  cancels on both sides,  $V_{in}$  cancels on both sides and what we are left with is this definition for the transit frequency, which is  $\omega_T = \mu C_{ox} V_{in,dc} / L^2 - V_T$ .

(Refer Slide Time: 06:39)



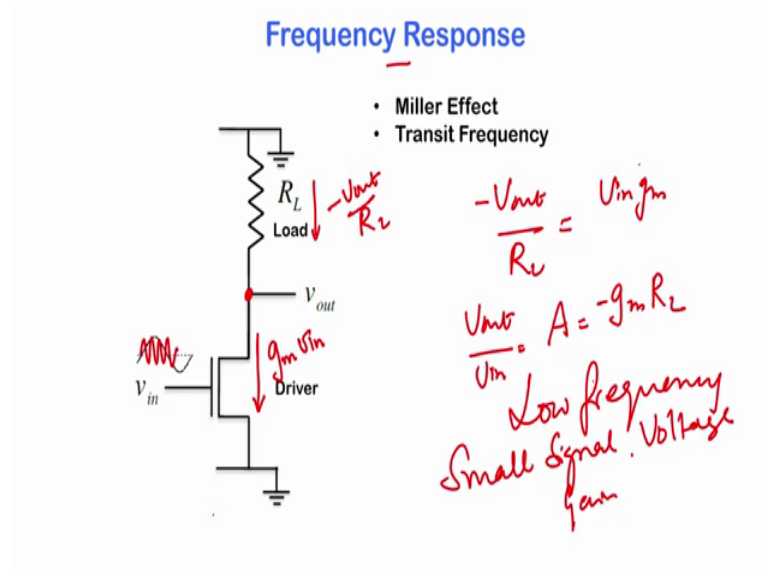
And if you write it in terms of the frequency in hertz, if you say  $f_T$ , then you divide this by  $2\pi$ . So, this is a very key parameter and any technology ok. So, it is telling you that at a certain frequency that the gate input gate current will match the drain to source current and that frequency is given by  $f_T$ , which is  $\mu C_{ox} V_{in,dc} / L^2$ .

So, how do I increase that frequency, we increase that frequency by lowering the channel length. And therefore, transistor scaling plays a great role in improving the  $f_T$ . And the other thing to do is to increase the mobility. And what does mobility depend on; we are already quite comfortable with that it depends on the purity of the semiconductor. In the sense what is the mean collision time, what is the effective mass etcetera ok? And then we have of course, have the threshold voltage. So, this is something called as a transit frequency which I felt was an important concept.

Now, the second concept that we are going to look at is something called as the Miller effect and that is got to do with the circuit that you are using. And in particular it is going to do with circuits that give you high gain voltage amplifiers that give you high gain.

And this circuit that we have used here is a good example of that. So, what is the low frequency gain of the circuit that we have used, so, let us just clear some space here.

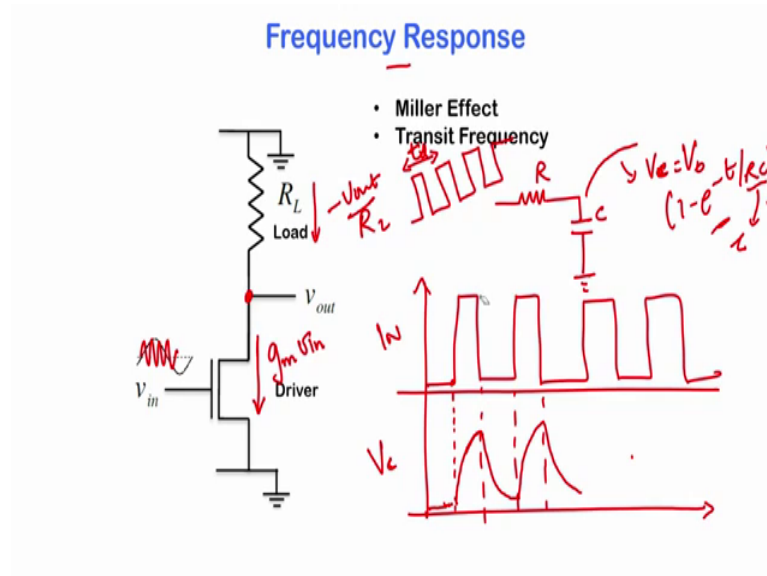
(Refer Slide Time: 08:37)



So, what is the low frequency gain of the circuit, we have already calculated that we said that, so in here we let us ignore channel length modulation you all know how to we use channel length modulation. So, the current through this is  $g_m v_{in}$ , and the current through that is  $\frac{-V_{out}}{R_L}$ . And therefore, by equating these two currents at this node we have  $\frac{-V_{out}}{R_L} = V_{in} g_m$  and therefore,  $\frac{V_{out}}{V_{in}}$  is equal to the gain and  $V_{out}$  is equal to  $-g_m v_{in} R_L$ . And this gain is the low frequency small signal voltage gain of the amplifier.

Now, what happens when the frequency starts getting larger. Now, a key concern is to do with the pole, again what do you mean by a pole for the people, who are not familiar with control systems theory or if you are not taking a control systems course. Let us just take a small detour here. Let us go back to the example of an RC circuit. So, we have an RC circuit.

(Refer Slide Time: 09:44)

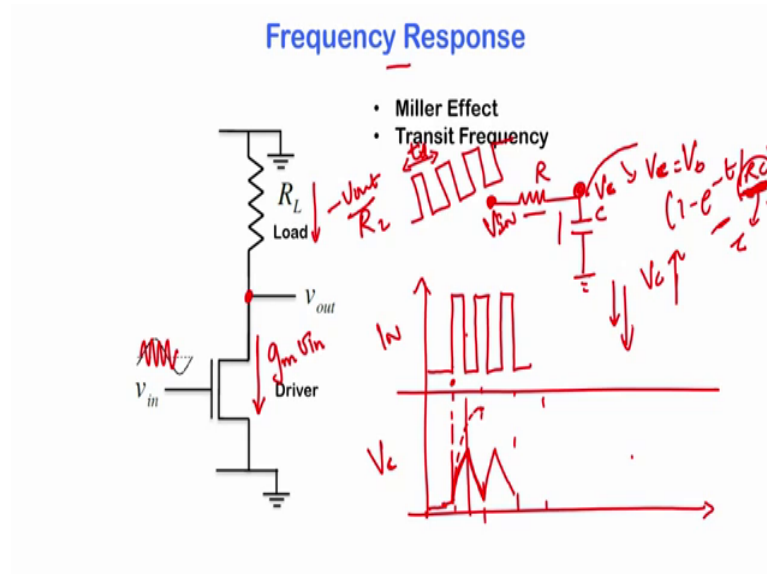


Let us try to define these things in a manner, where we do not require the use of you know Laplace transforms etcetera. So, in an R C circuit, so let us so we already some of the step response of an R C circuit. We said that you know there is going to be the voltage here is going to climb as  $V$  naught into  $1 - e^{-t/RC}$ , where  $t$  was the time constant of the circuit.

Now, imagine in the R C circuit you, now have a pulse, a periodic pulse that beings that is being supplied. And you can control the frequency of this pulse so, you can basically control this delay. So, let us draw the waveform let us say that that is the input pulse and that is the voltage waveform on the capacitor.

How do you think the voltage waveform would be, so let us say this is a starting point. So, the moment the pulse came up let us just start draw these three nodes. Let us say the pulse went up that is the input, the capacitor is going to start charging. It is going to charge to a certain point, it is going to charge to a certain point and then the pulse dropped off. And now the capacitor is going to start discharging. You give it enough time it is going to discharge and then the pulse is started again, it is going to charge and then it is going to discharge and your behavior would look like this. And all this is your exponential functions.

(Refer Slide Time: 11:38)



And now what we do is let us start reducing the width of the pulse ok, so that was what that pulse did, but now what let me just keep that there and now let us start reducing the width of the pulse. So, let us say we now have a new pulse, which is going to be much narrower ok, we not going to give the capacitor so much of time to charge up. So, now, what is going to happen, so now you have a pulse that it is got a higher frequency or increase the frequency input frequency of the pulse.

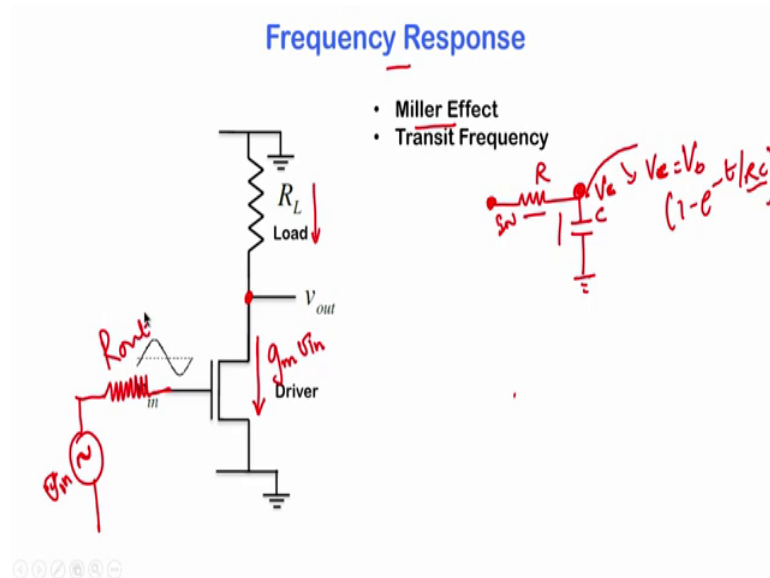
So, now the capacitor tries to take it is time. It is got a time constant of  $R C$ ,  $R C$  has not changed you only change the time period of this pulse. So, it is going to try to climb, but you are not given it enough time to charge fully. So, it is going to climb till here and then it is started to a discharge and then it is going to climb till then and started to discharge.

And therefore, earlier the capacitor was climbing a lot more it is getting charged the lot more, but now we are allowing it to charge only a little. And therefore, if you look at the magnitude of the voltage here and if you take the gain of the circuit the say the  $V_c$  by  $V_{IN}$  and if you take that gain, you will find that the gain has gone down, because  $V_c$  is not climbing up to what it was climbing up to ok, because the frequency has increased. And therefore, if  $R$  or  $C$  increase, then the time constant increases, and the circuit cannot respond to high frequency input pulse.

So, in other sense the gain of the circuit will start going down ok, so that is enough introduction. So, now, let us come back to this amplifier here so, you can actually break

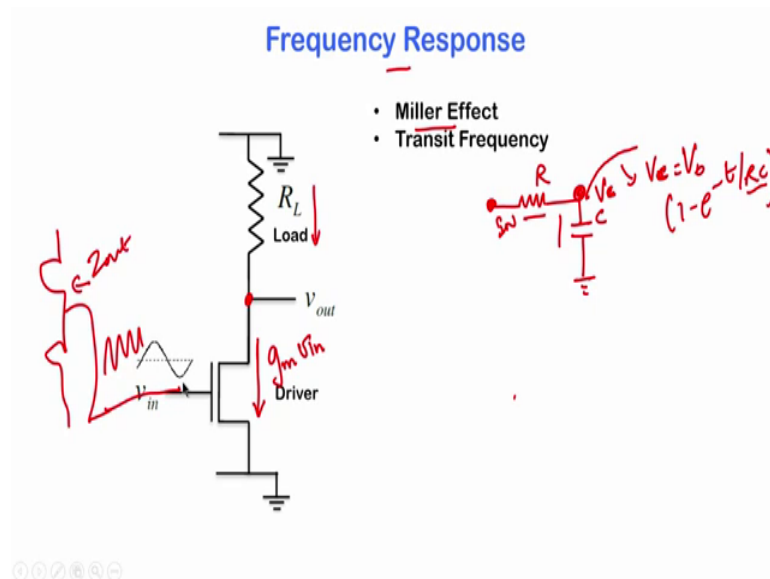
down any circuit into R s and C s series combinations of R s and C s and by just calculating the time constants we will know as to what how whether the circuit can respond quickly or not.

(Refer Slide Time: 13:46)



Now, R interest here in this study on Miller effect has got to do with the input sensor. So, let us say there is a input sensor, which is sending in all your V in signals and those signals are all coming here, but the sensors has got some resistance it is got some output resistance, which is normally the case it could also be a previous amplifier.

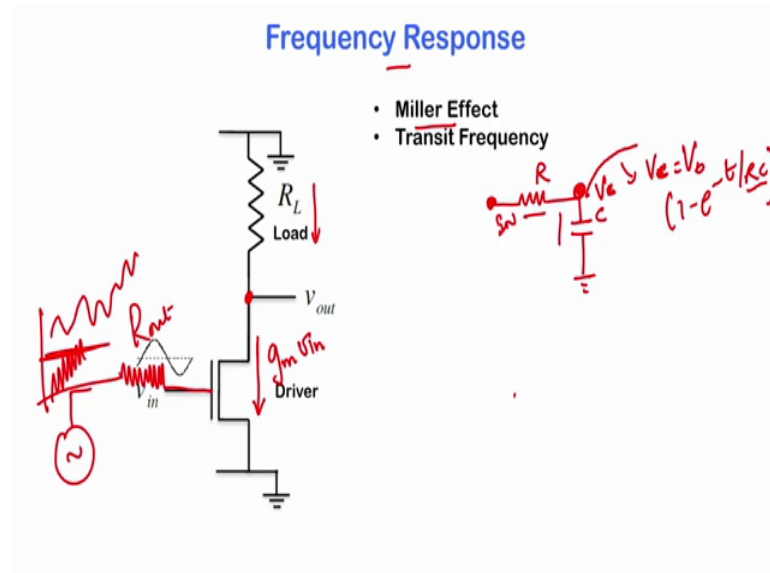
(Refer Slide Time: 14:03)





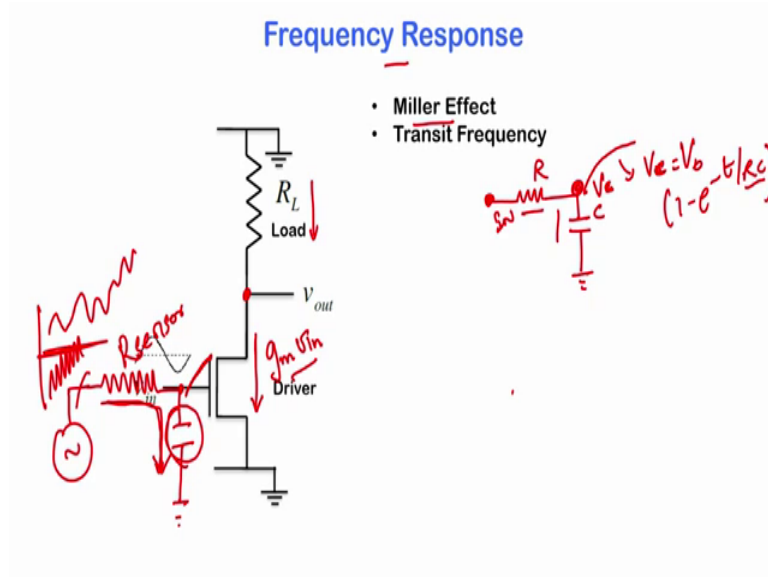
So, let us say there was another amplifier here and we know that we know how to calculate  $Z$  out. So, it has got some output impedance and that amplifiers sending out some signals and it is being sent to this amplifier. So, it could be any situation.

(Refer Slide Time: 14:17)



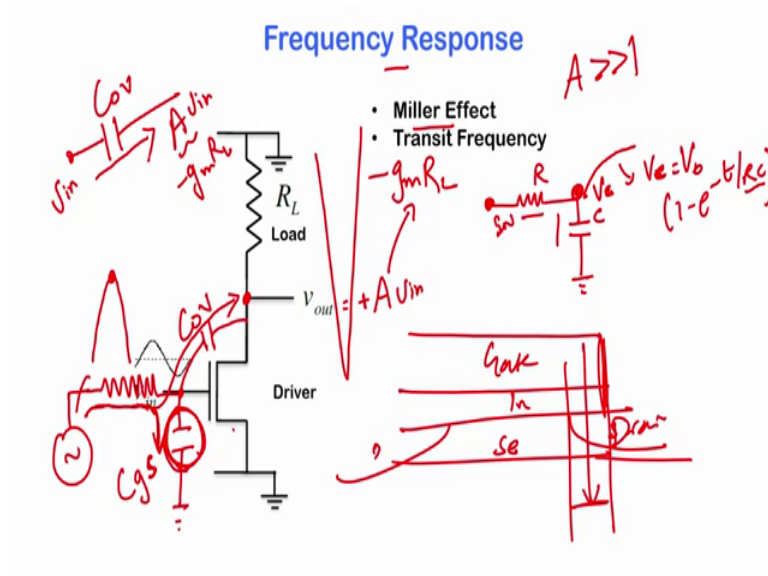
But let us just keep it simple we will say that there is a sensor and the sensor is throwing out you know these high frequency signals, but sitting at a large DC voltage to keep all this in saturation. And the sensors got some output impedance and it is sending these signals here. Now, when these signals were very low frequency, we really did not care about it, because we knew that the MOSFET could easily respond to these low frequencies, but now we need to be very careful.

(Refer Slide Time: 14:48)



And in particular we need to identify what is the input  $R_c$ , so that is the sensor's resistance. And I am now going to be very interested in knowing what the capacitance is at this node, because each time this signal goes up this  $R_c$  node has to charge and it is only then that the gate voltage sees that voltage rise and it is only then that the current here will change. So, the transistor will not respond to the voltage and the gate increases. And the voltage on the gate will have to depend on the charging of this  $R_c$  circuit here ok. And there is a small problem if you use an amplifier what is the problem?

(Refer Slide Time: 15:42)



So, let us say normally what constitutes this capacitance, what does this capacitance comprise of? The way you define this capacitance is to see what are the elements the transistors driving? So, on the face of this capacitance is basically your gate to source capacitance of the MOSFET, it is there is a channel here and there is a gate to source capacitance in the MOSFET. So that is not the problem that capacitance is there in any case ok. Now, the problem arises, because of the gate drain overlap.

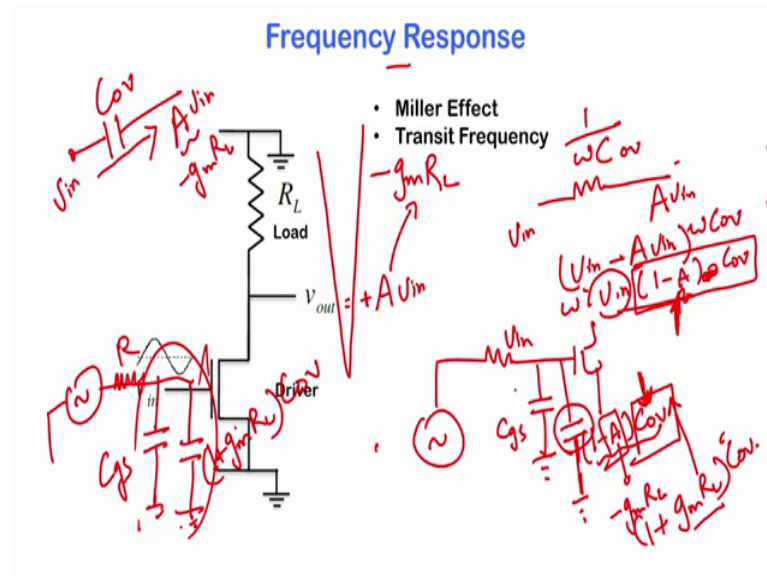
So, if you remember the geometry of the MOSFET, so you had your gate, you had an insulator, you had a semiconductor and let us say that was the gate. And we had a source drain region and there was an overlap, so let us say that is the drain, so there was an overlap between the gate and the drain. And we saw this having an impact when we looked at clock feed through in switches. So, it is a same overlap capacitance that we need to worry about now ok.

So, consider that overlap capacitance there ok. So, now, look at the current sent by the sensor you have an input signal that is going let us say you have an input signal that is going like a blip I am going to exaggerate the size of the signal. And now because the amplifier is giving you again and again or minus  $g_m R_L$  with a phase shift.

The output signal is going to go with a very large negative blip ok. So, what is the current driven by the sensor so, at this point there is it is all AC signals. So, all the capacitors have going to sink current in it is all  $C \frac{dv}{dt}$  so at this point you have a current through this. And you also have a current through this overlap capacitor, but now since the circuit is giving you again your  $V_{out}$  is minus  $A$  times  $V_{in}$  where  $A$  is nothing but your  $g_m R_L$  ok. So, we will just say plus  $A$  times  $V_{in}$  where  $A$  is minus  $g_m R_L$   $A$  is a negative number.

And since  $A$  is designed to be much greater than 1 your  $V_{out}$  is going to be an amplified negative going voltage is compared to  $V_{in}$ . And therefore, the MOSFET is you can imagine this capacitor the  $C_{overlap}$  capacitor has got a voltage which is  $A$  times  $V_{in}$ , where  $A$  is minus  $g_m R_L$  and this end having a voltage  $V_{in}$ . And therefore, it is going to sink in a current.

(Refer Slide Time: 18:40)



And since you can represent you can if you want to represent the capacitor as an effective resistor you can do, so by simply replacing it with it is an impedance of  $\frac{1}{\omega C_{ov}}$  so that is basically your resistance. So, you have a current of  $V_{in}$  in the voltage drop across this is  $V_{in} - A V_{in}$  into  $\omega C_{ov}$  is basically the current through the circuit or in other words you have a current of  $V_{in}$  into  $1 - A$   $\omega C_{ov}$  through the overlap capacitor.

So, from the point of view of the sensor, so if this is the sensor from the point of view of the sensor, the sensor is driving is putting out  $V_{in}$  and that  $V_{in}$  is driving a capacitor  $C_{gs}$ , which is connected to ground, but it is also driving this current through the overlap capacitor. So, from the point of view of the sensor it is like as though you have another capacitor to ground, which is got a capacitance of  $\frac{1}{\omega (1 - A) C_{ov}}$ . So, it is  $V_{in}$  into this capacitance, now the  $V_{in}$   $\omega$  into that capacitance now that is constituting the current.

So, it is like as though you can imagine that overlap capacitor to be another capacitor connected to ground it is got a capacitors a  $\frac{1}{\omega (1 - A) C_{ov}}$ , but what is  $A$ ?  $A$  is minus  $g_m R_L$ . Now, therefore, this capacitor has got a capacitor  $\frac{1}{\omega (1 + g_m R_L) C_{ov}}$ . And what is  $g_m R_L$ ?  $G_m R_L$  is designed to be very large and therefore, this effective capacitor appears to become very large.

So, it is as though your sensor is now driving two capacitors, which are connected to ground it is got some resistor here R and there is the C g s, but there is also this 1 plus g m R L into C overlap. And therefore, your total capacitance is now very large and therefore, your R c time constant is very small and therefore, the amplifier circuit will respond a little slower at high frequency so that is another problem. So, what is the connection to device (Refer Time: 21:16) in the transit frequency we already saw it is the mu by L square term is very important, but here what is the key concern? The key concern is the overlap capacitance.

We cannot compromise the gain, but we must definitely do something about the overlap capacitance so that is why it is important to have processes that are called self aligned processes, which makes sure the way the devices are fabricated ensure that the overlap capacitances minimized it is reduced. So, there are many ways you can make use the Miller effect also, you can artificially create a pole, so as to say to stabilize response of amplifiers etcetera, but nevertheless this is a problem that you must be aware of. And it is not only true for you know MOSFET circuits or MOS capacitor.

(Refer Slide Time: 22:08)

### Miller Effect

Look at the effective input impedance from the input and output nodes.

$$i_{in} = \frac{v_{in} - v_{out}}{Z_f} \approx \frac{v_{in} - Av_{in}}{Z_f}$$

$$Z_{in} = \frac{v_{in}}{i_{in}} = \frac{Z_f}{1-A}$$

If  $Z_f = 1/sC_f$

$$Z_{in} = \frac{1}{s(1-A)C_f}$$
  

$$i_{out} = \frac{v_{out} - v_{in}}{Z_f} \approx \frac{v_{out} - v_{out}/A}{Z_f}$$

$$Z_{out} = \frac{v_{out}}{i_{out}} = \frac{Z_f}{1-(1/A)}$$

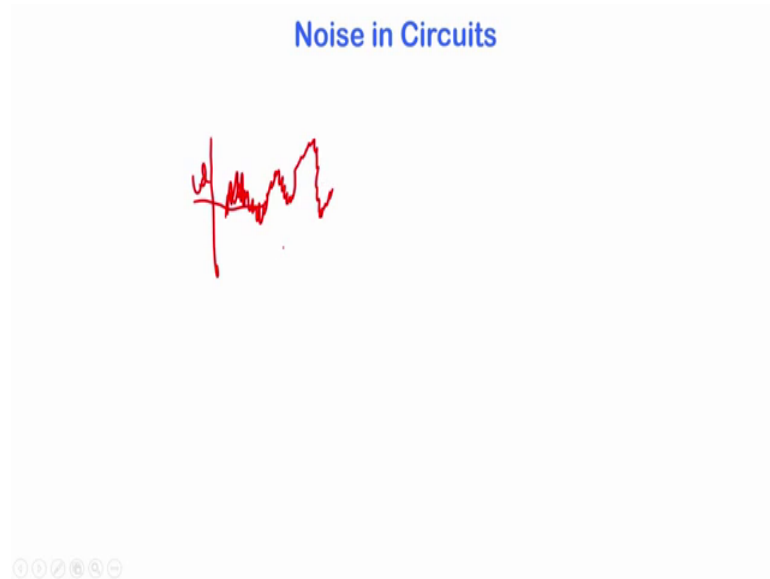
If  $Z_f = 1/sC_f$

$$Z_{out} = \frac{1}{s(1-(1/A))C_f}$$

Helps to associate poles with nodes and get a quick glance into the frequency response.

So, here I have given you a slide where you know any amplifier with any feedback impedance across it well experience this equivalent of a Miller effect. So, if you have the time do go through these arguments, although I will not discuss it right now.

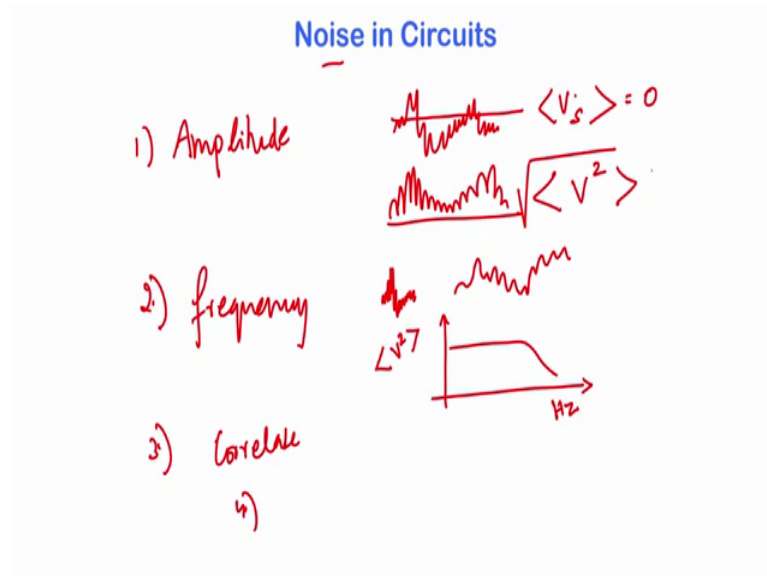
(Refer Slide Time: 22:28)



So, the very last topic we will look at before we close circuits and therefore close the course is related to the noise and circuits. And in particular we will look at the noise and MOSFETs. So, what do we mean by noise in circuits? So, we have several electronic devices and because of the way electrons respond to fields in these devices, the currents through these devices and the voltages across these devices are all. I have got a very predictable component, but there is also a lot of randomness to all this and that is what we mean by electronic noise.

So, we cannot completely predict the currents or voltages to the infinite accuracy, because there is a lot of randomness. So, for example, if we were to measure the current, you will find that there is a lot of fluctuation and randomness in the current a small amount of randomness hopefully. And this is what is meant by noise, but very quickly you know just to you know just to see how to characterize noise. So, let us say you walk into a room and there are many people talking how do we characterize noise?

(Refer Slide Time: 23:45)



The 1st is of course, the amplitude of the noise. Say an what do you mean by amplitude? Since noise is a random signal you cannot end up taking the average, the average of this kind of a random signal would actually be. So, let us say you have some signal and that average will be basically 0, instead you must go ahead and take you must take you must first square the signal. And probably calculate the average of the square of that signal or maybe take the average of the root of the square that is the root mean square of the signal in order to get your estimate of the amplitude. The 2nd estimate has got to do with the frequency content in the noise.

So, let us say you walk into a room and you have many children who are talking as compared to adults. You will find that there is noise and the noise has got more high frequency content as compared to if say adults were talking, where the noise would have more low frequency content. And the frequency content of the noise is very nicely described by something called as the power spectrum of the noise.

So, you look at the power of the noise although we says power it is all measured in whole square per hertz and you look at the distribution of this in frequency. So, this is a very quick summary of noise. And the 3rd has got to do with whether the noise is correlated ok so, if you will work into a classroom and there are lots of kids and let us say you bring in a nice surprise then all the kids will shout out in unison.

So, there all the noise sources will be correlated. And then there are other methods to characterize noise you could say whether the distribution from, which these random points are picked up, whether the distribution is, whether that probability distribution is varying in time or whether it is not. So, whether it is the noise stationary or not stationary? So etcetera.

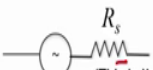
(Refer Slide Time: 25:46)

### Thermal Noise or Johnson Noise

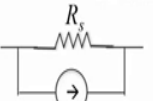
Random motion of electrons in a conductor.

Essentially when there are a large number of electrons flowing through a resistor like circuit - driven by significant drift.

Seen in Resistors, MOSFET in above threshold.

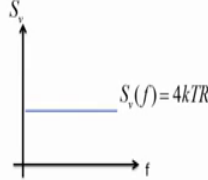


$\overline{v_n^2} = 4kTR_s$  (This is the voltage noise in a 1Hz band)  
Else multiply by  $\Delta f$




$\overline{i_n^2} = \frac{\overline{v_n^2}}{R_s^2} = \frac{4kT}{R_s}$  (This is the current noise in a 1Hz band)

PSD is frequency independent, therefore called white noise.



$S_v(f) = 4kTR_s$



But, we will just very quickly look at some typical noise in circuits. The very first and most important in some sense noise source is noise in the resistor and this noise is simply because of the random movements of electrons in a conductor ok. And this typically happens when you have a conductor and when you have a large number of electrons that are moving through you are not talking about a sparse population. So, since many of you might be familiar with some certain crowded cities I have put a picture of traffic in a very crowded city.

See, if you look at these roads, there completely jam packed with the vehicles ok. And if you were to measure the flux of this traffic, you will find that there is a lot of scattering with the you know with the road itself. There is a scattering between the vehicles and therefore you will see there is some flux in the traffic. And it is quite possible that this will have a thermal noise like behavior although I have I am not very sure, but nevertheless it happens when you have a large number of electrons creating a good amount of flux in the device.




(Refer Slide Time: 27:03)

### Thermal Noise or Johnson Noise

Random motion of electrons in a conductor.

Essentially when there are a large number of electrons flowing through a resistor like circuit – driven by significant drift.

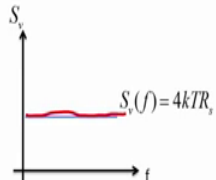
Seen in Resistors, MOSFET in above threshold.



$\overline{v_n^2} = 4kTR$  (This is the voltage noise in a 1Hz band)  
Else multiply by  $\Delta f$

$\overline{i_n^2} = \frac{v_n^2}{R_s^2} = \frac{4kT}{R_s}$  (This is the current noise in a 1Hz band)

PSD is frequency independent, therefore called white noise.



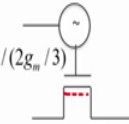
*Handwritten notes:*  $\frac{4kT}{R}$ ,  $\frac{V^2}{R^2}$

And the thermal noise is all the noise is represented as a mean voltage square or a mean current square. And the thermal noise is given by  $4kT$ , which is the Boltzmann coefficient in a temperature into the resistance of the device. And the thermal noise is quite frequency independent, which means that the power of the noise remains constant at all frequencies. And you could also represent the thermal noise in terms of the current, which is to say it is the voltage square divided by the  $R$  squared, which means that this becomes the thermal noise of the resistor.

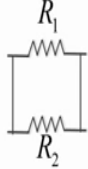
(Refer Slide Time: 27:37)

### Thermal Noise or Johnson Noise

$\overline{v_n^2} \sim 4kT / (2g_m / 3)$



$R_1$   
 $R_2$



$\overline{i_n^2} + \overline{i_{n2}^2} = 4kT(1/R_1 + 1/R_2) = 4kT / (R_1 \parallel R_2)$

$\overline{v_{out,n}^2} = (\overline{i_n^2} + \overline{i_{n2}^2}) Z_o^2 = 4kT(R_1 \parallel R_2)$

$\overline{i_n^2} \sim 4kT(2g_m / 3)$

*Handwritten notes:*  $4kT \sim$ ,  $R$ ,  $4kT \sim$ ,  $2g_m$

Now, in a MOSFET in conduction, you do have thermal noise when you have an inversion channel formation, but once the inversion layer is formed you have a large number of electrons between the source and drain. And therefore the MOSFET has thermal noise through it. So, when we talk about MOSFETs and circuits we must take care of introducing the thermal noise, if you are worried about the noise behavior.

And for a MOSFET in operating and saturation mode above threshold operation, it is got a resistance, which is approximately or you know in this case you have your thermal noise current, which is supposed to be  $4 kT$  by  $R$  right so that was the thermal noise current and that term here is approximately  $4 kT$  into  $2$  by  $3 g_m$ , where  $g_m$  is the trans conductance of the MOSFET. And  $g_m$  therefore determines the amount of noise so, if you want to have smaller amount of current noise, you want to reduce the  $g_m$  of the MOSFET. So, this is the output noise of the MOSFET.

(Refer Slide Time: 28:58)

**Shot Noise**

**Sparse Electron Flow**  
 Variations in the number of charge packets crossing a 'line' in a given interval of time.

$\max(S) = 2qI_{dc}$

$\overline{i_n^2} = 2qI_{dc} \Delta f$

Resistor doesn't show Shot noise, but pn junction does - Need transport of discrete packets of charge - not in metals since the charge is almost continuum.

Important for Photodiodes/photosensors.

If we take into account, the transport delay, the PSD is a sinc

PSD is a sinc function with  $\tau$  the transport delay associated with the charge packet

1005 → 1000.  
 1005 → 900  
 1005 → 980

And then you have something called as short noise. So, I am just going through this just to give you a glance through the devices ok. So, here you can again compare it to traffic, but traffic which is very sparse ok. So, let us say you have a electrons moving in a conductor a moving in a in a device and the electrons are very sparse.

And this is a classic case for example, say in a diode that is a reverse biased or say in a for photodiode applications for measuring to capture the intensity of light. So, here you have electrons moving at some velocity and let us say we put a marker line here. And we

the way we are going to count or the current as we are going to count the number of electrons that cross this marker over a period of time.

So, let us say we say in 100 seconds we counted the number of electrons and we say thousand electrons went past that marker. And then we are going to restart the elec experiment you are going to say that ok. Let me start the count for another 100 seconds and we saw that there were about 900 electrons that went through. And then there were 980 electrons that went through the next time. And therefore, there is some intrinsic fluctuation in this.

So, it was 1900 980 and so on so forth and that is what contributes to noise and that noise is something called a short noise. And it is given by this particular term; here your current noise current is dependent on the DC current through the device. And therefore, this is not seen in resistors it is also not seen in MOSFETs that are you know in strong inversion, but it is very much seen and say devices such as photo diodes or photo sensors, which are you know diodes which are in reverse bias ok.

(Refer Slide Time: 30:43)

**Flicker Noise or 1/f Noise**

Ubiquitous: Earthquakes, typos, stock markets etc.  
Essentially telling you that big events are rare.  
Recommended Reading: *1/f noise: a pedagogical review*  
<https://arxiv.org/abs/physics/0204033>

Found when you have a power law in time domain.

In semiconductors – Two models: charge number variations, mobility variations.

Per unit bandwidth current noise.

$$\overline{i_n^2} \sim \frac{A}{C_{ox}WL} g_m^2 \frac{1}{f}$$

For a band from  $f_1$  to  $f_2$

$$\overline{i_n^2} \sim \int_{f_1}^{f_2} \frac{A}{C_{ox}WL} g_m^2 \frac{1}{f} df = \frac{A}{C_{ox}WL} g_m^2 \log(f_2/f_1)$$

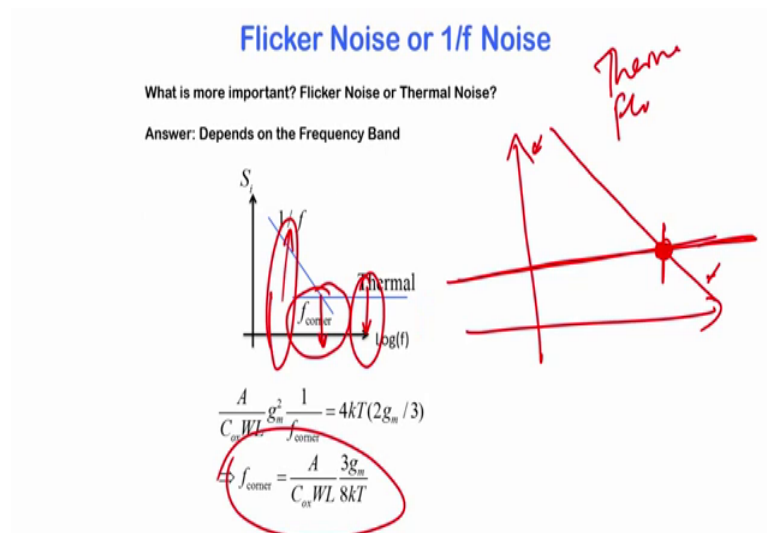
And then finally we will talk about the last noise source which is something called as a flicker noise. And this is the very interesting noise source it is quite ubiquitous and I strongly recommend reading this particular paper that I have linked here. And flicker noise is quite interesting, because it is got a 1 over f responds ok. So, if you plot the

intensity of flicker noise, versus frequency is got it reduces as frequency increases. And this is the flicker noise in the MOSFET ok.

And what is flicker noise due to? Ficker noise is due to typical variations in the number of electrons available for conduction and typical variations in the mobility of these electrons and these two can be linked. So, there are two possible models that are very strongly connected. And they both explain the presence of flicker noise and MOSFETs. And essentially what does flicker noise saying, it is saying that rare events occur big events occur very rarely.

So, at very low frequency the power of the noise is very large. So, large fluctuations occur very rarely and small fluctuations occur very frequently. So, if you think of earthquakes large earthquakes occur very rarely and small tremors occur quite frequently so that is essentially the message of flicker noise. So, this is a flicker noise per hertz and since it is got a frequency dependence you can integrate that to get the total noise in a certain bandwidth.

(Refer Slide Time: 32:18)



And finally, the very last topic is something it is got to do something called as a corner frequency with regards to noise and the MOSFET. And what is corner frequency? We know that thermal noise. So, in a MOSFET there are two key components of noise, we have the thermal noise and we have the flicker noise. And thermal noise is rather frequency independent it is almost constant all frequencies. And therefore, it is called

white, because it is called all frequencies in it, but flicker noise has got a 1 over f dependence. So, it is very strong at low frequencies and very small at high frequencies.

So, the frequency at which the flicker noise power matches the thermal noise power is got a given a special symbol is called as corner frequency. So, if you are designing circuits where the operation is going to be here, then you need to worry about flicker noise. If you design circuits which the operation frequency is very large, then you only worry about thermal noise. So, it is useful to know where the corner noise frequencies.

(Refer Slide Time: 33:23)

**Noise RC Circuit**

$$T(f) = \frac{1}{(1 + (2\pi f R_s C_L)^2)^{1/2}}$$

$$|T(f)|^2 = \frac{1}{(1 + (2\pi f R_s C_L)^2)}$$

$$\overline{v_{in,n}^2} = 4kTR_s$$

$$\overline{v_{out,n}^2} = \int_0^\infty \overline{v_{in,n}^2} |T(f)|^2 df$$

$$\Rightarrow \overline{v_{out,n}^2} = \int_0^\infty 4kTR_s \frac{1}{(1 + (2\pi f R_s C_L)^2)} df$$

$$= \frac{4kTR_s}{2\pi R_s C_L} \tan^{-1} 2\pi f R_s C_L \Big|_0^\infty = \frac{4kTR_s}{2\pi R_s C_L} \frac{\pi}{2} = \frac{kT}{C_L}$$

**Very useful and important result.**  
**kT/C noise is thermal noise through an RC circuit at infinite bandwidth.**

And just to conclude you know I will put up the slide, but we will not go through it. If you have an R C circuit, then the noise on the capacitor of this R C circuit is actually independent of the resistor. Provided the resistor is the only source of noise, which is your thermal noise and provided you are looking at the noise over an infinite bandwidth. You will find that the total noise voltage noise in the capacitor ends up being KT by C and I have given the derivation here, it requires an understanding of certain formulas.

And therefore I will not quiz you or examine you on this point, but nevertheless I felt it is useful enough to put down on a slide and leave it here ok. And this noise is typically just K T by C, it is independent of the resistor. And therefore if you are using a MOSFET as a switch, this is something that you can be aware of. So, with this we will complete noise in the circuits, we complete at discussions on circuits itself and

we if at complete our discussion on the crystalline semiconductor physics part of the course.