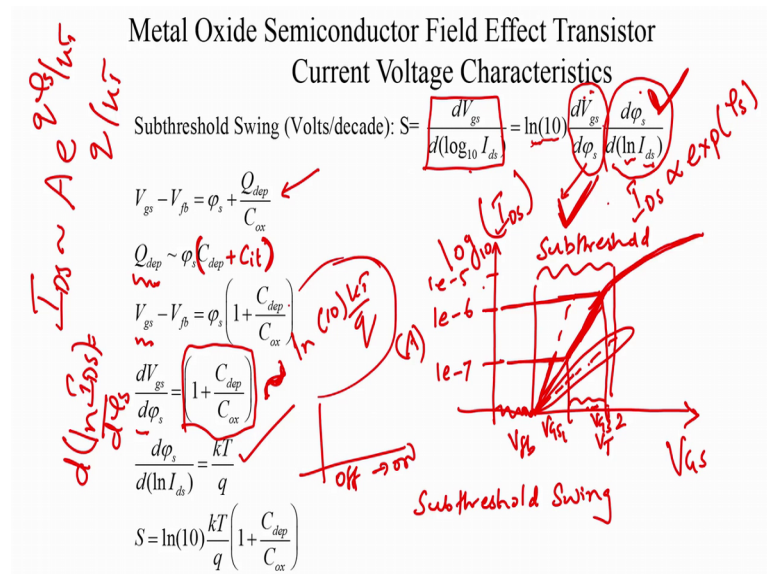


Semiconductor Devices and Circuits
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Lecture - 40
Subthreshold swing, Additional concepts

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So, the way this region is defined or the way the transition is defined is through a term called as a Subthreshold Swing or in fact, the inverse of it which is the Subthreshold Slope ok. And the subthreshold swing or is measured in terms of the volts per decade and I will tell you what; that means, it means that if we plot if you plot the current voltage characteristics on a log scale.

So, we plot the log I DS ok. So, we say to the base 10 let us say log I DS to the base 10 versus V GS I am sorry V GS the gate to source voltage. So, till a certain point it is all in a basically half and then from here from say V fb to V T well it is not off very far below V G V fb just to keep things a little accurate.

So, let us say it is off around this region and from V fb to V T is start having diffusion defining a current and since the current is proportional to the exponential of the surface potential which in turn is related to the gate voltage, you will find that the characteristics takes a linear relation on a log ID versus V GS scale and then beyond V T you start entering the square law dependence of the current.

So, this region is our subthreshold region of operation, now for us to climb one decade. So, it is a log 10 right. So, let us say this is $1e^{-7}$ and then that is $1e^{-6}$ and $1e^{-5}$ and so, on so many amps of current. So, for us to get from one to one decade that is from $1e^{-7}$ that is to climb one order of magnitude ok.

So, $1e^{-7}$ to $1e^{-6}$ we need to apply a certain amount of gate voltage. So, we have gone from say V_{GS1} to V_{GS2} for us to get past one decade. This voltage difference divided by this one decade is essentially the idea behind your subthreshold swing in, how many volts does it take for us to get the current past one decade ok. So, it is the number of volts per decade.

On the other hand the slope is just the inverse right it is the reciprocal of this parameter ok. So, it is good to have a better estimate of what the subthreshold swing or subthreshold slope right.

So, subthreshold swing is defined by $dV_{gs} / d \log_{10} I_{ds}$ which we will rewrite as $dV_{gs} / d \ln I_{ds}$ because V_{gs} is nicely connected to the surface potential into $d\phi_s$ by $d \ln I_{ds}$ and will make a correction for the natural log to base 10 log conversion ok. So, we have redefined this differentially this derivative as with these 2 terms and this coefficient.

Now, how is V_{gs} related to ϕ_s , V_{gs} so, we know this relation. So, all the gate voltage that has been applied is equal to the flat band voltage, plus the depletion charge, plus ϕ_s and when ϕ_s heads here $2\phi_s$ is called as the threshold voltage, like a so, we are now at a ϕ_s below the threshold voltage that is the sub thresholds volt.

So, that is the subthreshold region. So, we know that V_{gs} is equal to $V_{gs} - V_{fb}$ is equal to ϕ_s plus $Q_{depletion} / C_{ox}$. Now what we will do is, we will define an artefact or parameter which is called as a depletion capacitance ok. So, I would not call it an artefact it is it truly does exist, but the way we will define $Q_{depletion}$ we say depletion capacitance effective and the way we will define $Q_{depletion}$ is by saying that it is the surface potential into $C_{depletion}$ ok. So, what does this mean?

So, this means that ok, in you have your insulator, you have your semiconductor and you now have your depletion layer, because of all the applied gate voltage and the electron concentration is very small they just started to appear ok. So, the way we have defined $Q_{depletion}$

depletion is we say that, this is the depletion this is the band bending and therefore, that is my surface potential or in other words the surface potential is basically the potential drop across this region that is the surface potential. And we say that all the charge here is equal to this potential into this effective capacitance of this region it is be treating this as a capacitor.

So, we define $Q_{\text{depletion}}$ as ϕ_s into $C_{\text{depletion}}$ and make the substitution here ok. So, you get you get this relation V_{gs} is equal to V_{fb} plus ϕ_s into 1 plus $C_{\text{depletion}}$ by C_{ox} and therefore, the dV_{gs} by $d\phi_s$ becomes this term, it is 1 plus $C_{\text{depletion}}$ by C_{ox} .

So, we have solved for this part of or subthreshold swing, now in order to get this derivative let us go back to this relation.

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Metal Oxide Semiconductor Field Effect Transistor
Current Voltage Characteristics

$$n(0) = n_{p0} \exp(q\phi_s / kT)$$

$$n(L) = n_{p0} \exp(q(\phi_s - V_{ds}) / kT)$$

$$h \sim \text{Debye Length} \sim \frac{kT}{q\epsilon_s} = \frac{kT}{q\sqrt{2qN_A\phi_s/\epsilon_s}}$$

$$I_{ds} \approx qD_n W \frac{kT}{q\sqrt{2qN_A\phi_s/\epsilon_s}} \frac{n_{p0} \exp(q\phi_s / kT) (1 - \exp(-qV_{ds} / kT))}{L}$$

$$\approx \mu \frac{n_i^2}{N_A} \left(\frac{kT}{q} \right)^2 \frac{W}{L} \sqrt{\frac{q\epsilon_s}{2N_A\phi_s}} \exp(q\phi_s / kT) (1 - \exp(-qV_{ds} / kT))$$

$\sum m = A \exp(q\phi_s / kT)$

So here you have your dependence of I_{ds} on ϕ_s right. So, this is the only dependence on ϕ_s everything else is independent of ϕ_s right. So, all this is independent of ϕ_s . So, let us just say that all this is equal to some a times exponent of $q\phi_s$ by kT that is your I_{DS} ok. So, my writing at there does not make it appear on the screen. So, we just say I_{DS} is some constant times e to the power $q\phi_s$ by kT , which means which implies that my natural log of I_{ds} is the natural log of A plus $q\phi_s$ by kT and therefore, d natural log of I_{DS} by $d\phi_s$ is simply this is a constant and therefore, this is simply q

by kT and the inverse of that is kT by q . So, we can get the second part of this definition which is $d\phi_s$ by $d \ln$ of I_{ds} is equal to kT by q .

So, therefore, the subthreshold swing is given by this term here. So, it takes this many volts sorry it takes this many volts to get the current past one decade. So, what does this mean how do we improve the subthreshold swing, we can improve the subthreshold swing by we cannot do much with the temperature and we cannot do much with the Boltzmann coefficient and you cannot do much with q since we will have to change the universe we live in order to do that. Therefore, the only things that are under our control are the depletion charge and the oxide charge.

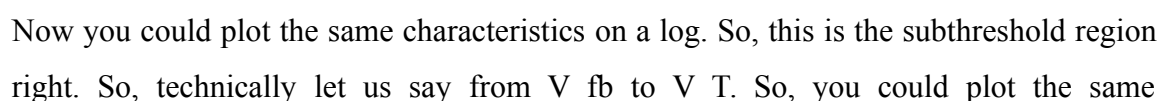
Now we have although you still not defined these things you will realize that as more parasitic starts developing in the device. So, for example, let us say the interface between the semiconductor and insulator will have interfacial trap charge. So, all that will add on because all those regions will trap charge in them and they will all add on to the charge present in the semiconductor.

So, any parasitic capacitance that traps charge or contains charge at the semiconductor insulator interface and which lies in the at the semiconductor in insulator interface will actually add on to these terms in the numerator and all such parasitics will effectively start increasing your subthreshold swing and decreasing your subthreshold slope.

So, the subthreshold slope probably this is a more popular definition is simply the inverse of the subthreshold swing for and therefore, the subthreshold swing will increase which means you need a larger voltage to head towards one decade ok. So, that is a lower subthreshold swing and that is a higher subthreshold swing. So, the subthreshold swing will increase as we have more and more parasitics and the sub thresholds slope will decrease as we have more and more parasitics.

So, this parameter the subthreshold swing is a very key parameter which determines the rate at which the MOSFET goes from the off to the on state and you can see there is got a very fundamental limit. So, let us say $C_{depletion}$ is 0 or you know let us say the no parasitic, somehow you know you built an architecture where in all this is minimal and you can almost ignore this term compared to one. Even then you have a fundamental limit of $\ln 10 kT$ by q being your subthreshold swing, but if let us say $C_{depletion}$ plus C_{ox} is equal to C_{ox} ok.

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characteristics on a $\log I_{DS}$ versus V_{GS} (Refer Time: 13:26) and that is quite useful. So, we will have a complete a lecture on parameter extractions. So, we will actually look at all these things in good detail there, but what are the useful piece of information that you can get from this curve.

From the slope of this curve at very low V_{DS} or even at any other V_{DS} if you know the V_{DS} it is possible to extract this parameter $\mu C_{ox} W$ over L and from that we know what μ is ok, if you know the insulator thickness we can guess what C_{ox} s, but in order to get C_{ox} we can always do a capacitance voltage curve and you will find that you know what C_{ox} s.

So, if you know C_{ox} and you know the aspect ratio because we have designed the transistor and we know the geometry of it we can calculate what μ is from the slope of this curve. But from the $\log I_{DS}$ it is the same data right you take the transfer characteristics the same experimental data sets and you plot it on a $\log I_{DS}$ versus V_{GS} curve and what you will find is that, you will find that the characteristics shows this almost linear relation and then it bends off and this region is the subthreshold region.

And that region is the linear or saturation region of operation that is all above threshold. So, in the subthreshold region you are technically expected to get a straight line because of the purely exponential dependence, but having said that when we talk about non crystalline semiconductors we will see that there are some variations in the subthreshold slope itself because of the existence of states inside the band gap. But for now for this lecture what this lecture tells you is that this should be a straight line because the current is exponentially dependent on the surface potential or there the V_{GS} and then you have a entry into the above threshold region and this μ of the curve sort of defines what your threshold voltages.

So, from this curve we can estimate the subthreshold swing, which tells you which gives you a lot of information on see depletion by C_{ox} and which you can actually correlate with this CV measurement and iterate for this measurement to make sure that you have the right values and therefore, calculate if there are any differential capacitance is or etcetera, etcetera.

Now this $\log$ characteristics also gives you another information if you look at the current here which is below V_{fb} . So, have to say if you look at the current here you will

understand what the leakage current in the mosfet. So, we have still not approached that boundary so, we will do that, but a little later. It tells you what is the current between the source in the drain then we think that the MOSFET is completely turned off which is when we assume the MOSFET is switched off that is there is no V_{GS} present ok.

So, there is still there is V_{DS} , but there is no V_{GS} and what is the current through the MOSFET or the V_{GS} below V_{fb} so, what is the current through the mosfet. So, we will talk about the leakage mechanisms in the MOSFET a little later. And the third plot that we looked at was the output characteristics, which is very useful when we design circuits because it tells us a lot of things about the resistances.

So, this is I_{DS} versus V_{DS} which is the output characteristics or the MOSFET and that had a form like this and as you increased V_{GS} , you found that you can get these different family of curves, you can get these different curves and as we increased V_{DS} we went from the linear region to the saturation region ok. And also we looked at channel length modulation which did not allow these curves to become perfectly flat, but instead gave at dependence on the saturation provided at dependence of V_{DS} on the for the saturation current.

So, this is summary a quick summary of our current state of knowledge in the with regards to the MOSFET current voltage characteristics. Now so, we at the beginning of this lecture we said that there are 2 aspects that we have still not studied, one is the currents below V_T , which we have now you know understood a little better because these are all the subthreshold currents and they are all related, they are all governed by the diffusion of electrons from the source to the drain, the diffusion of minority carriers from one of the electrodes to the other. And second point that we said that we were not we had not covered completely was a variation in the depletion width, which we thought might affect the threshold voltage.

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Metal Oxide Semiconductor Field Effect Transistor
Improving the Model: Bulk Charge Theory

In the derivation of the MOSFET – V_T was treated a constant.
 But V_T is not constant. $V_T = V_{fb} + \frac{qN_A x_d}{C_{ox}} + 2\phi_F$ (Handwritten: $\phi_s = 2\phi_F$, N_A)

But the depletion width varies from source to drain.

$x_d(y=0, \text{next to Source}) = \left(\frac{2\epsilon_s(2\phi_F)}{qN_A} \right)^{1/2}$ (Handwritten: $\phi_s = 2\phi_F$)

$x_d(y=L, \text{next to Drain}) = \left(\frac{2\epsilon_s(2\phi_F + V_{DS})}{qN_A} \right)^{1/2}$ (Handwritten: V_{DS})

$x_d(y) = \left(\frac{2\epsilon_s(2\phi_F + V_c(y))}{qN_A} \right)^{1/2}$ (Handwritten: $V_c(y)$)

And we are going to now address that point. So, what are we doing we have just we have this very basic model for the current voltage characteristics the MOSFET and we are simply improving this model you know with these details. So, the model the first model we had was from V_T till above a 2 gate voltages above V_T , which is all the purely strong inversion characteristics where in we looked at linear and saturation regions of operation. And then we defined the regions the current below V_T which is the subthreshold currents and we defined the subthreshold swing which was an important parameter.

And now, we are defining the current voltage relation a little more accurately ok. Although we will just introduce this introduce the ideas behind this, but not actually use this for many other purposes, but this is a more accurate model. And this model is something called as a bulk charge theory and it takes into count the variation in the depletion width.

So, as we move from the source to the drain the initial model I mean just to re restate the introduction, as we move from the source to the drain not only it is the channel charge decrease, but that depletion width is also supposed to vary. And because that is because the P-N junction here is this P-N junction is more strongly reverse biased as compared to this P-N junction. And this depletion width variation creates an inaccuracy in the way we defined the channel charge.

So, in the previous case we had assumed that the V_{DS} is very small. So, therefore, we got away with many things we said the V_{DS} is very small and therefore, you know the depletion width does not change too much and that is the channel charge and that q_n is simply equal to your C_{ox} into V_{GS} minus V_T minus V_c by h or you know the charge per unit area is C_{ox} into V_S minus V_T minus V_c , where the V_T was treated like as it is a constant, but in reality as if you take a section here and if you take a section here, since the V_{DS} is different or since the V_c is different the depletion width is also different ok.

So, it becomes more prominent in this case if I were to take a section here and a section here, the depletion width is suddenly increased right. So, the depletion width here is x_d 2, there is a depletion width here is x_d 1 and x_d 2 is clearly greater than x_d 1. And since the depletion charge depends upon the x_d , the threshold voltage in this region has to be greater than the threshold voltage here and that variation and threshold voltage is completely missed out in our study, because we went from very low V_{DS} and directly into saturation by simply substituting V_{DS} is equal to V_{GS} minus V_T ok.

So, that is the little gap in our model and what we will do now is, we will completely we will try to see how that gap can be corrected for. So, the V_T is got 3 milestones it is the flat band you first reach the flat band voltage and then you do $Q_{depletion}$ by C_{ox} you deplete the region and then you bring the surface potential to equal to $2\phi_F$. Now the problem is with $Q_{depletion}$ right it is not with ϕ_F is purely dependent on the acceptor on concentration V_{fb} is completely dependent on ϕ_{ms} and it is only the $Q_{depletion}$ that depends on the value of x_d and it is qN_A times x_d by C_{ox} .

So, as x_d varies V_T must vary. So, near the source side there is at x_d at y equal to 0 this is our x_d that is the depletion width and near the drain side you have a larger what you say depletion width and that is given by this term. You are all now already very familiar with you know how in how to define depletion width in terms of the surface potential. And at any point y this is the depletion width it depends upon the potential of the channel there.

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Metal Oxide Semiconductor Field Effect Transistor
Improving the Model: Bulk Charge Theory

Subtracting
bulk charge

Making the correction to the carrier concentration

$$q_n(y) = C_{ox} (V_{gs} - V_T - V_c) - qN_A (x_d(y) - x_d(0))$$

(at y=0)

Corr (V_{gs} - V_T(y) - V_c)
= Corr (V_{gs} - V_T(0) - V_c)

$$I_{ds} = qn\mu E A$$

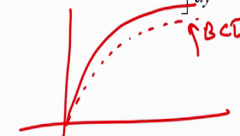
(ds)

$$I_{ds} = W\mu \left[C_{ox} (V_{gs} - V_T - V_c) - qN_A \left(\frac{2\epsilon_s}{qN_A} \right)^{1/2} \left(((2\phi_F + V_c(y)))^{1/2} - (2\phi_F)^{1/2} \right) \right] \frac{dV_c}{dy}$$

↓ V_c

This is called Bulk charge theory

V_{DS} vs V_{GS} - V_T



BCT

Therefore, we can correct for our relation of q_n in this particular manner. So, we will define. So, this should we will try to define this should be q_n into h ok, it is the concentrate that free carrier concentration per unit area ok. So, particularly I am looking at all of been say capital N here ok. It is the inversion carrier concentration per unit area and not per unit volume and that is equal to C_{ox} into V_{gs} minus V_T minus V_c just like the way we defined, but the V_T is computed at the source which means using an x_d value that is present near the source which is close to the equilibrium x_d value of the P-N junctions.

So, we will use that V_T , but then we will come to we will correct for it by saying that instead of you should have we should have actually been using V_{gs} minus V_T of y minus V_c , but we will say that this is equal to C_{ox} into V_{gs} minus V_T at y equal to 0 minus V_c minus qN_A into x_d of y minus x_d of 0. Because all the other terms the ϕ in the ϕ ms and the $2\phi_F$ terms do not change it is only this term that we will correct form ok.

So, essentially we are subtracting $qN_A x_d$ of 0 and instead adding $qN_A x_d$ of y to this relation here to the V_T here. And by making this correction we can now rewrite the differential equation for the drain to source current. We say that I_{DS} is $qn\mu$ electric field into area, where this area is your you know once again it is the W into h where the h cancels off and we say that the current voltage relation or the differential equation that

you need to solve this I_{ds} is equal to W into μ into this particular term into dV_c by dV_y ok. And what does this term this term is nothing, but your corrected channel charge per unit area, upper unit volume because the h has cancelled off.

So, this is the h and that is cancelled off so, that is the channel charge ok. And you saw now you if you see the V_c , the V_c term appears in your differential equation here as well as here. So, earlier we did not have this term and therefore, we ended up with a square law model, but now with this term being present and you have a V_c to the power half and a V_c term. The model is going to be slightly more complicated than a square law, but it is going to be more accurate because it takes into account the variation in x_d .

So, this theory is something called as a bulk charge theory and it generally predicts a lower amount of current ok. So, if that was the square law model the bulk charge theory predicts the current that is slightly lower. And the second point is that the way you define the saturation V_{DS} that is in the square law model the V_{DS} saturation that is the V_{DS} of beyond which the transistor entered saturation was defined as simply any V_{DS} greater than or equal to V_{GS} minus V_T .

Then definition will change a bit because your current voltage characteristics has now changed, but it is up to you to solve for this exact differential equation as a homework. Although in one of the assignments I will put out the exact solution we will have this is an assignment question due to which I and it is due to that reason that I have not given our the answers here, but we will discuss the exact answer when the solutions to the assignments are presented.

So, these are the 2 key points the Subthreshold Slope the understanding of the subthreshold current and the Bulk Charge Theory model with sort of refines the current voltage characteristics that we developed.