

Semiconductor Devices and Circuits
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Lecture - 31
BJT: Non-Idealities and Equivalent Circuit Modeling

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Bipolar Junction Transistors (BJTs)
 Current Voltage Characteristics – Specific Cases

Modes of Operation

Active (i.e. forward active): Used significantly in analog circuits

Emitter-Base (E-B) forward biased —

Collector-Base (C-B) reverse biased —

Saturation: (on state in digital)

Emitter-Base (E-B) forward biased } $V_{eb} > 0$

Collector-Base (C-B) forward biased } $V_{cb} > 0$

Cut-off: (off state in digital)

Emitter-Base (E-B) reverse biased |

Collector-Base (C-B) reverse biased |

Reverse Active:

Emitter-Base (E-B) forward biased

Collector-Base (C-B) reverse biased

So, far we have seen the general IV characteristics for the BJT. So, we have not made any specific assumptions on whether it is active mode or saturation mode or cutoff mode, but we managed to derive the very general expressions for the current-voltage characteristic of the BJT.

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Some notations:

$$\alpha = \frac{I_C}{I_E} = \frac{I_{pE} + I_{nE}}{I_{pE} + I_{nE}}$$

$$\text{Base transport factor} = \alpha_T = \frac{I_{pE}}{I_{pE} + I_{nE}}$$

$$\gamma = \text{Emitter Injection Efficiency} = \frac{I_{pE}}{I_{pE} + I_{nE}}$$

$$\beta = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} = \frac{\alpha}{1 - \alpha}$$

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Noting that

$$n_{p0E} = n_i^2 / N_{AE}, \quad p_{n0B} = n_i^2 / N_{DB}, \quad n_{p0C} = n_i^2 / N_{AC}$$

$$I_E = q A n_i^2 \left\{ \left(\frac{D_{nE}}{L_{nE} N_{AE}} + \frac{D_{pB}}{N_{DB}} \right) (e^{qV_{EB}/kT} - 1) - \left(\frac{D_{pB}}{W N_{DB}} \right) (e^{qV_{CB}/kT} - 1) \right\}$$

$$I_C = q A n_i^2 \left\{ \left(\frac{D_{pB}}{W N_{DB}} \right) (e^{qV_{EB}/kT} - 1) - \left(\frac{D_{nE}}{L_{nE} N_{AE}} + \frac{D_{pB}}{W N_{DB}} \right) (e^{qV_{CB}/kT} - 1) \right\}$$

$$I_B = q A n_i^2 \left\{ \frac{D_{nE}}{L_{nE} N_{AE}} (e^{qV_{EB}/kT} - 1) + \frac{D_{nE}}{L_{nE} N_{AE}} (e^{qV_{CB}/kT} - 1) \right\}$$

And we did this by simply solving the continuity equation and identifying the diffusion currents. So, now, we can take these very general equations and you know make it more specific. So, let us say we are interested in defining the IV characteristics in active mode operation. So, how do we go about doing that?

So, what we do is if you just make use of the different conditions right. So, you have an active mode of operation you have the emitter base junction as is forward biased and collector base junction is reverse biased which means that we can say that V_{EB} is greater

than 0 V_{cb} is going to be less than 0, because it is a pnp device that we have been looking at. So, you have your p, you have your n and you have your p that is your emitter base or p plus emitter base and collector. If my emitter base junction has to be forward biased, then my emitter base voltage should be greater than 0; if my collector base junction has to be reverse biased, then my collector to base voltage has to be less than 0. So, if that is my positive, negative, that is a positive and negative.

So, the moment you make these assumptions watch out for what is happening in to V_{eb} and V_{cb} they all appear in terms of exponentials ok. So, the next obvious step is to say that since this is greater than 0 the exponential of q V_{eb} by k T is much greater than 1. And in the case of V_{cb} less than 0 you can say q V_{cb} by k T my k T is much less than 1 and so on so forth so that is that is the idea. So, we are going to we are going to make use of these different conditions and identify whether these emitter base voltages and collector base voltages are greater than 0 or less than 0 and then start making start throwing in these assumptions into your equations. And you will get a good flavor for what happens in each mode of operation.

So, on the other hand for saturation mode you have both of them to be greater than 0. And therefore, both the exponents are going to be much greater than 1. And in the case of cut off the exponents are going to be less than 1 and so on and so forth.

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Bipolar Junction Transistors (BJTs)
Active Region of Operation

$V_{EB} > 0 \Rightarrow$ forward bias (large diffusion)

$V_{CB} < 0 \Rightarrow$ reverse bias (all diffused carriers drift quickly to collector from emitter).

$\therefore e^{qV_{EB}/kT} \gg 1$
 $e^{qV_{CB}/kT} \ll 1$

$\therefore I_E \approx q A n_i^2 \left[\frac{D_{nE}}{L_{nE} N_{AE}} + \frac{D_{pB}}{W N_{DB}} \right] e^{qV_{EB}/kT}$

$I_C \approx q A n_i^2 \left[\frac{D_{pB}}{W N_{DB}} + \frac{D_{nE}}{L_{nE} N_{AE}} \right] e^{qV_{EB}/kT}$

$I_B \approx q A n_i^2 \frac{D_{nE}}{L_{nE} N_{AE}} e^{qV_{EB}/kT}$

So, for active mode is something we are particularly interested in because we will later use this for doing circuits. So, you say that V_{eb} is greater than 0 V_{cb} is less than 0 because it is reverse biased. And therefore, we have these two assumptions and therefore, the emitter current becomes this. So, by the time the emitter current become this, if you go back to the general equations what we have done is since exponential of $q V_{eb}$ by $k T$ is much greater than 1, we have ignored the 1. And since exponential of $q V_{cb}$ by $k T$ is much less than one we have ignored the exponential term ok, so which means that is the emitter current. And the collector current will have this dependence all right; so which means so these are all approximations, which means that in active mode of operation the emitter current and collector current are more strongly controlled by the base to emitter voltage on the emitter base voltage ok.

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Bipolar Junction Transistors (BJTs)
Summary: *Saturation*

Saturation:

$$I_E \sim q A n_i^2 \left[\underbrace{\frac{D_E}{L_E N_E} e^{q V_{EB}/kT}}_{I_{E1}} + \underbrace{\frac{D_B}{W_B} e^{q V_{EB}/kT}}_{I_{E2}} - \underbrace{\frac{D_B}{W_B} e^{q V_{CB}/kT}}_{I_{E3}} \right]$$

*$V_{EB} \gg 0$
 $V_{CB} \ll 0$*

$$I_C \sim q A n_i^2 \left[\underbrace{\frac{D_B}{W_B} e^{q V_{EB}/kT}}_{I_{C1}} - \underbrace{\frac{D_C}{L_C N_C} e^{q V_{EB}/kT}}_{I_{C2}} - \underbrace{\frac{D_B}{W_B} e^{q V_{CB}/kT}}_{I_{C3}} \right]$$

$$I_B \sim q A n_i^2 \left[\frac{D_E}{L_E N_E} e^{q V_{EB}/kT} + \frac{D_C}{L_C N_C} e^{q V_{CB}/kT} \right]$$

the excess component to active operation

In saturation mode ok, so what is the saturation mode of operation? You say that all these exponentials are going to be much greater than 1 and therefore you have all the exponential terms being present here ok.

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Bipolar Junction Transistors (BJTs) Summary

Curv:

$$\underline{I}_E \sim -g_{\text{Ani}}^2 \frac{D_E}{\epsilon_c \epsilon_E} \quad \checkmark$$

$$\underline{I}_B \sim +g_{\text{Ani}}^2 \left[-\frac{D_i^2}{\epsilon_c \epsilon_E} - \frac{D_c}{\epsilon_c \epsilon_c} \right] \quad \checkmark$$

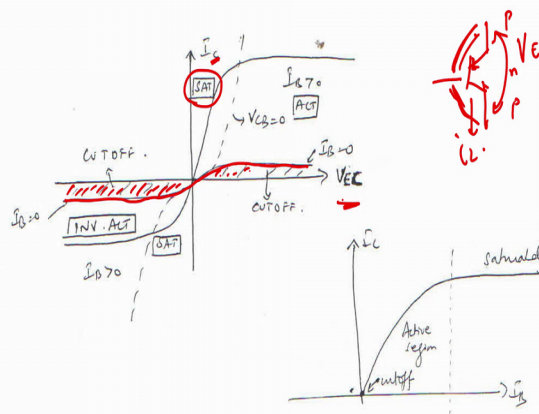
$$\underline{I}_c \sim g_{\text{Ani}}^2 \frac{D_c}{\epsilon_c \epsilon_c} \quad \checkmark$$

And similarly for cutoff we just make use of the fact that all the exponentials are going to be much less than 1 and therefore that is your emitter current that is the collect base current and that is the collector current ok, so that is there is a negative sign there of the emitter so that summarizes you know the three different equations and all this can be plotted off into a nice chart ok, where which gives you this IV characteristics of your BJT. You can take this as a homework and try to build this chart for yourself.

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Bipolar Junction Transistors (BJTs)

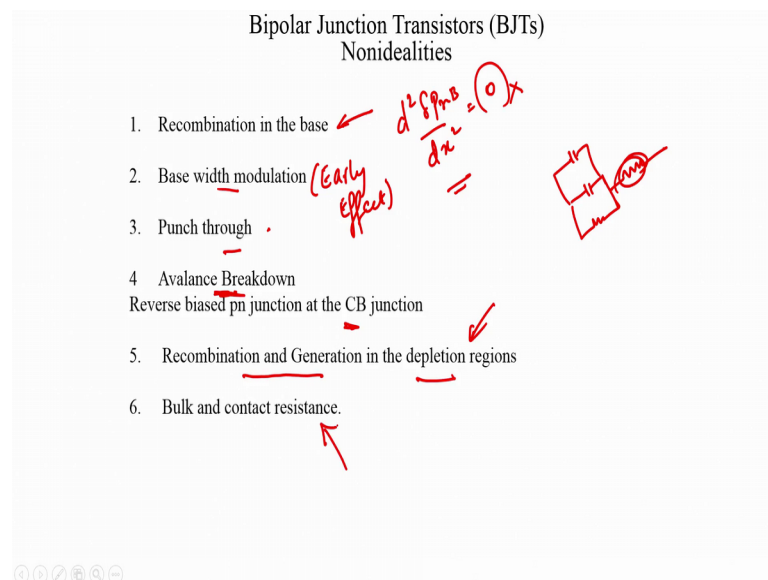
All Regions of Operation



So, you find that you have the saturation mode of operation here because this is the plot of I_C versus V_{EC} that is the emitter to collector voltage. So, you have taken into account both the base to emitter voltage as well as the emitter to collector voltage. So, we are plotting the emitter to collector voltage versus the collector current ok. And this is being plotted for different base current values. So, you see that this curve is for base current of 0. So, here my base current is 0. And everything within this region is the cutoff state ok, there is a no base current flowing.

And as the base current increases, the collector current increases. And for large V_{EC} essentially what we are saying is that the emitter base voltage becomes positive; and the base collector voltage becomes positive which means that the junction is now reverse biased and therefore, you start having a characteristics of this kind. Now, the inverse active operates like the active, but it is going to be much weaker the currents are going to be much smaller because we do not have a p plus n junction anymore instead we have a pn junction. So, this is a summary of all the current-voltage characteristics. And as a homework you could take this ahead and go and plot it for yourselves and convince yourselves as to what the current-voltage characteristics are.

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Now, so far we have been assuming a very ideal BJT ok. And there are several factors that we sort of ignored or we did not really consider when we derive the current-voltage characteristics. And some of these factors are listed here. The very first and the very

obvious one is recombination in the base. We said that there is no recombination in the base and we very conveniently wrote our what we say our equation for the holes in the n type base we conveniently said that was equal to 0, whereas that is not really true because you do have recombination in the base so that is the first non ideality.

The second is something called as a base width modulation and we will explain what that is this is also called as the early effect ok. The third is again got to do the width of the base and something called as a punch through in a BJT I will talk about that. And the fourth is the avalanche breakdown which we discussed in the case of pn junction diodes, but in this particular case in the case of the BJT, it is the reverse heavily reverse bias collector base junction that could cause that could experience an avalanche breakdown.

And then in the case of the pn junction diode we looked at non idealities like the recombination and generation mechanisms in the depletion region that is valid in the case of BJT as well and we have ignored that. And finally, in the case of the pn junction when we built the equivalent circuit you know we said that there is a depletion capacitance, there is a diode resistance and then there is the bulk resistance in forward bias operation which is due to the resistance of the doped regions and the contact and that is another factor that we have ignored in the case of a BJT.

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Bipolar Junction Transistors (BJTs)
Nonidealities: Recombination In the Base

We had assumed no recombination in the base

$$\frac{d^2 \delta p_{nb}}{dx^2} = 0$$

X

In reality

$$\frac{d^2 \delta p_{nb}}{dx^2} = \frac{\delta p_{nb}}{L_p^2}$$

$\Rightarrow \delta p_{nb} = C_1 e^{-x/L_p} + C_2 e^{x/L_p}$

$\delta p_{nb}(0) = C_1 + C_2 = p_{nb} \left(e^{qV_{be}/kT} - 1 \right)$

$\delta p_{nb}(w) = C_1 e^{-w/L_p} + C_2 e^{w/L_p} = p_{nb} \left(e^{qV_{be}/kT} - 1 \right)$

Solve to get obtain the more exact solution.

$x'' \rightarrow \infty$
 $x' \rightarrow \infty$
 $\delta p_{nb}(x \rightarrow \infty)$
 $\delta p_{nb}(x = w)$

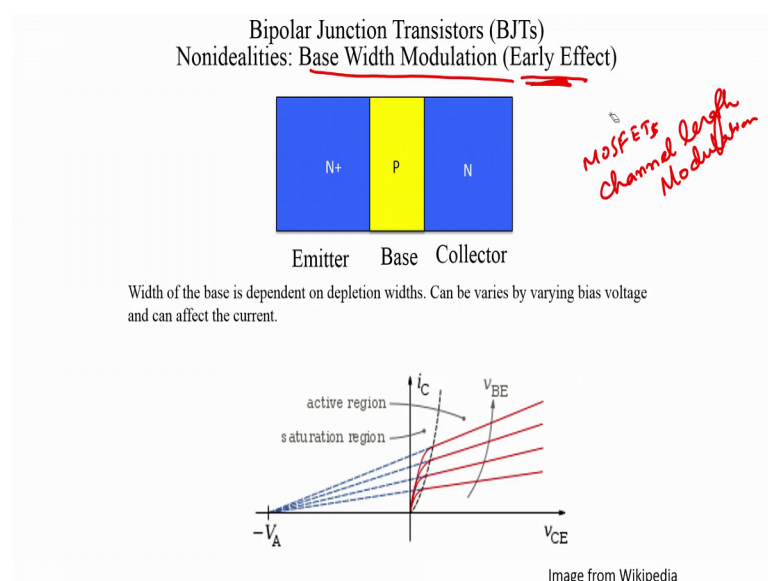
So, first let us look at the recombination in the base. So, we had assumed that there is no recombination and therefore, we had gone ahead and used this equation, but in reality

this equation is not completely correct. There is a small amount of recombination and therefore that the correct differential equation should involve a component like this ok. This is similar to the differential equation we used for the collector and the emitter side.

And therefore, we already know the solution it will have two exponents, but the key here is in the case of the collector emitter we could get rid of one of the coefficients because of the condition that as x double dash went to infinite or when x dash went to infinite you know the excess carriers became 0. But in this case that is not true because you are talking about the base and therefore, we need to have two boundary conditions of p and b at x equal to 0 and Δp and b at x equal to w .

So, we need to have these two boundary conditions, we need to solve these two to get an exact solution. And what is Δp_n be at x equal to 0, it is going to be this it is going to depend on the emitter to base voltage. So, in case that is not that writing is not clear it is the emitter to base voltage. And in the case of Δp and b at x equal to w , it is the same term, but it is going to have the collector to base voltage which is a reverse biased junction ok. So, in this case the collector to base voltage is going to be negative which means it is going to be very small and therefore you are going to have a term that actually goes below this particular value.

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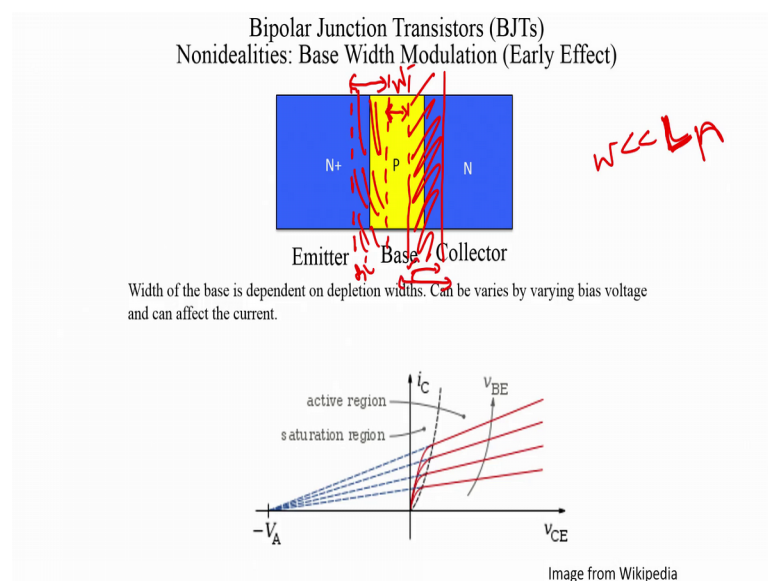


The other non ideality is something called as a base width modulation this quite important from the point of view of circuits and it is got another term it is called as the

early effect it is called as the early effect named after the person who discovered it. And this particular effect has got a counterpart in MOSFETs. So, when we talk about MOSFETs, we will discuss something called as the channel length modulation ok. Now, both these effects are talking about a phenomena wherein the current the current through the device, in this case it is the collector current is dependent on the collector to emitter voltage or in the case of the MOSFET, it will be dependent on the drain to source voltage. We will come to that a little later.

But the effect of these phenomena on circuits is that it leads to the reduction of the output impedance of your device and that impacts your amplifier design and the amplifier gain it is it ok. So, just keep this in mind it is quite important. And the reason the idea behind the base width modulation is actually quite simple. So, what is the width of the base? So, you have let us say here we have taken an npn device ok. And the reason I have taken an npn device is because this plot corresponds to an npn device and I have taken I borrowed this plot from an online website.

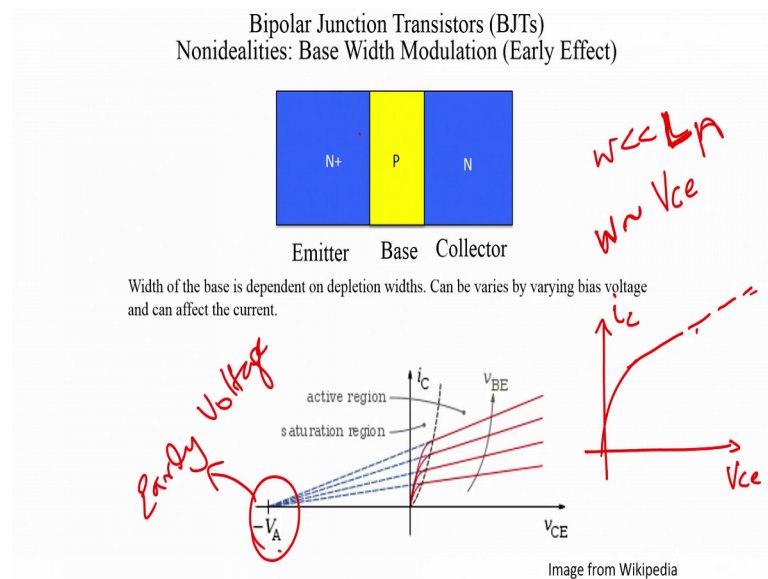
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So, in the case of a in the case of a BJT, you have your emitter base and collector. And you have these depletion regions right, you have a depletion width there, so that is the depletion region because of the base emitter. And then you have a depletion region because of the base and collector these regions are all depleted. The base width is this region here that is your x equal to 0 at x equal to w , so that is the base width.

Now, the base width has to be less than the depletion diffusion length of the minority carriers. So, in this case, it has been less than L_n . Now, the base width clearly depends upon the width of these depletion regions. If these depletion regions become wider, if they become wider, the base width has to decrease; and if they are narrower the base width increases. So, therefore, the width of the base is dependent on the depletion widths which in turn are depend on the bias voltages.

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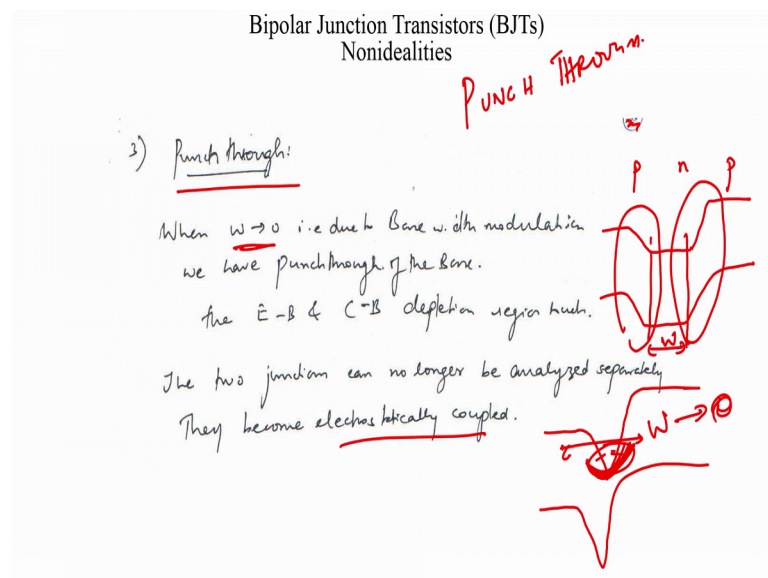
So, if I make my region here more forward biased, then this would decrease; and if I make this region more reverse bias then that would increase. And therefore, essentially we are controlling or modulating the width of the base. So, since it is dependent on effectively these two depletion widths, the width of the base is quite dependent on the collector to emitter voltage ok. Now, ideally in an active mode operation we saw that the collector was not doing anything, it was ideally you should had a collector current that really did not depend too much of the collector voltage.

But because of this width modulation, you will find that the current does depend on the collector voltage ok. It does have a dependent the slope is not 0, it has got some finite value. And therefore, this is the true nature of the IV characteristics. And by taking a tangent to these lines and projecting them backward, you will end up with all of these

lines ideally intersecting at a point. And this voltage this collector to emitter voltage is called as the early voltage.

And why does it modulate the collector current, because if we go back and look at the expression for the collector current. So, let us say active mode operation, so that is the collector current, where is the width you find that the width is there in the denominator right here ok. And therefore if the width changes it is going to quite significantly impact the collector current, so that is the base width modulation or the early effect. And we will I will refer to this once again when we discuss the channel length modulation effect in MOSFETs. So, they are quite important from the circuit's point of view.

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The third non ideality is something called as punch through ok. So, it is not so forgive me for the poor handwriting, but this is something called as punch through. And what this implies is that it is things of it discuss the phenomena when the width of the base starts tending to 0 ok. So, let us say ideally you have let us say your pnp BJT. So, you have a BJT that looks like that ok, so that is the base width that is the width of the base. But now as the width of the base starts heading towards 0 probably due to the way we have doped it or due to the geometrical construct or due to the biasing circuit etcetera for whatever reason let us say that starts heading to 0. You will end up with structure like this, so that becomes your band bending diagram ok.

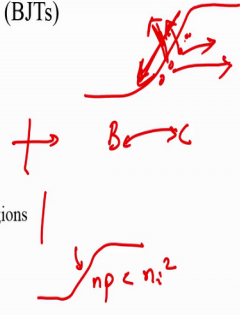
And what happens now is if I further reduce this you know as we start if I as I start approaching W toward 0, we cannot handle these two junctions separately. So, far we handle the base collector junction, base emitter junction separately, we solved for all the minority carrier concentrations there. We handle the base collector junction separately and solved for everything there and we simply summed up all the currents.

So, we cannot do that anymore, because now the emitter is directly interacting with the collector because the base width does not exist anymore ok. So, this is the phenomena where you will start having a significant transport of carriers across the base because the width is approaching 0 and this effect is something called as the punch through phenomena. So, but from the point of view of the analysis you cannot these two are electro statically coupled.

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Bipolar Junction Transistors (BJTs)
Nonidealities

4. Avalanche Breakdown
Reverse biased pn junction at the CB junction
5. Recombination and Generation in the depletion regions
6. Bulk and contact resistance.



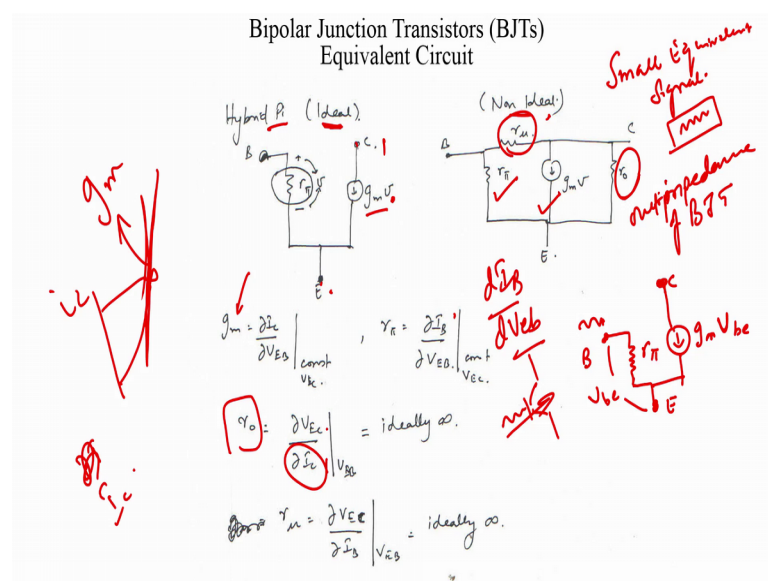
BJT

And finally, the last three points which is the avalanche breakdown in the reverse bias pn junction at the collector base junction, the recombination and generation of the depletion and the bulk contact resistance were already discussed in the case of pn junctions and they are the same ideas here. So, what this means is that if you look at the collector base junction, you have the base, you have the collector. So, in the case of pnp, you have a heavy reverse bias, you have a large field. If I keep reverse biasing this stronger and stronger the field could get so large that the electrons migrating through this junction could have enough energy to create an avalanche of electrons that is pull electrons or rip

the other silicon atoms apart and pull the electrons out of them thereby creating avalanche of current ok, so that is that is the breakdown phenomena there and that is because the reverse biased collector base junction that you might see in the case of active mode operation.

Then recombination and generation and the depletion region; so, once again just in the case of pn junction, since you have a depletion region, so if the depletion if the devices and reverse bias your np is less than ni square and therefore, this encourages generation. On the other hand, if it is forward biased, then it np would be it would encourage recombination mechanisms. And finally, you have the bulk and contact resistance which you have already discussed which is due to the doped regions and due to the metal semiconductor contacts. But on that note since we will head back to circuits, it is useful to talk about an equivalent circuit model or an equivalent circuit representation for a BJT.

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And we have a simple model for that which is something called as the hybrid pi model. So, if you were to open a textbook and look at an equivalent circuit for a BJT this is what you would see. And you know as I mentioned there now you know the course is good to start having a circuits flavor to it ok. And I will just discuss very briefly as to what the different aspects of these models are ok. So, this is the ideal model and what we have here is a non ideal ok.

So, let us focus on the ideal model first. So, what you see here is the BJT has got three terminals, you have the base, you have the collector and you have the emitter. So, the way we have drawn it here it is probably indicated npn in junction, but it does not matter. The base is going to have some resistance, some input resistance which means that the control signal for the transistor is going to impact the signal from the collector to the emitter. So, if you head back to our original discussion on what constitutes a transistor, you will have an implication of what that you will have an understanding you will appreciate what that means.

And you correct your collector to emitter current is dependent on the base to emitter voltage ok. So, let us say that is your V_{be} ok. So, this is going to depend linearly on the base to emitter voltage provided, we have linearized the circuit which means that these are all small signal equivalent circuits. I should have probably mentioned that right at the start. It is a small signal equivalent circuit. So, we have already understood the meaning of small signal which means we are only talking about small fluctuations and V_{be} , V_{ce} etcetera, we are not talking about the large applied bias voltages.

So, under these small fluctuations, your collector current, for example, let us say varies with the base emitter current. So, we have already chosen our dc voltages; and around these dc voltages we are going to have a small signal fluctuation and that fluctuation is going to impact the current. And at this point we can linearize this curve and say that it is got a tangent which has got a slope of g_m and therefore, my i_c is g_m times V_{eb} which in turn decides the current between the collector to the emitter, so that is the message this equivalent circuit is trying to portray. It is all right if you do not understand this now because we will come back to this a little later. So, that is that is the base to emitter voltage.

And therefore, you have a collector current that is that is g_m times the voltage across the base and emitter. And g_m is something called as a transconductance of the device ok, it is a technical term. And r_{pi} it is the pi is got no special significance it is simply because the name of the model, it is called the hybrid pi model probably because this looks like an inverted Greek alphabet pi ok. But the point is that r_{pi} indicates the input impedance of the BJT. So, it is basically τ_{dIB} by dV_{eb} , so that is my small signal input resistance. So, if I fluctuate V_{eb} of the BJT a little how much of base current do I push it, so that is that is the question that is determined by r_{pi} .

Now, ideally the output impedance of the BJT should ideally be infinite, but we already saw that there is an early effect there is a base width modulation and therefore, we have a non ideal circuit which is got the r_{pi} , it is got the current collector emitter current, but it is also got this output impedance which is $\frac{dV_{EC}}{dI_C}$. Now, since ideally I_C was independent of V_{EC} I_C did not dependent on V_{EC} and therefore, r_o was infinite, but since I_C does dependent does depend on V_{EC} a little because of the base width modulation we give a special symbol r_o which is the output impedance of the BJT ok.

And finally, you have r_{mu} which is another non ideality which is the $\frac{dI_B}{dV_{EC}}$ that is how does the collector to emitter voltage impact the base current ok. So, again ideally it should be in infinite, but in reality you have a finite value for that quite large, but finite. So, in that is the summary of the BJT that is that is all we are going to discuss. So, we looked at the geometrical construct, we looked at the device physics and we identified the general IV characteristics and then we looked at the special IV characteristics for the three different regions. It is a lot of derivations not particularly going to be tested in those details, but definitely it is an indicator as to how to approach the understanding of the device physics of a device. And finally, we looked at the equivalent circuit diagram.